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Hydrographic and U. S. Land Surveying

132 Illustrations

Prepared Under Supervision of

C. K. SMOLEY

DIRECTOR, SCHOOLS OF CIVIL AND STRUCTURAL ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS

HYDROGRAPHIC SURVEYING

UNITED STATES LAND SURVEYS

MAPPING

DETERMINATION OF TRUE MERIDIAN

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

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CONTENTS

NOTE.—This volume is made up of a number of separate Sections, the page numbers of which usually begin with 1. To enable the reader to distinguish between the different Sections, each one is designated by a number preceded by a Section mark (§), which appears at the top of each page, opposite the page number. In this list of contents the Section number is given following the title of the Section, and under each title appears a full synopsis of the subjects treated. This table of contents will enable the reader to find readily any topic covered.

UNITED STATES LAND SURVEYS—(*Continued*)

HYDROGRAPHIC SURVEYING

SURVEY OF OUTLINE OF A BODY OF WATER

1. Definition and Object of Hydrographic Surveying.—**Hydrographic surveying** comprises all surveys of lakes, streams, reservoirs, or other bodies of water. Its object may be:

1. To obtain sufficient information from which to draw an outline map of the body of water surveyed.

2. To determine, in addition, the elevations of a sufficient number of points on the bottom below the water surface to define the subaqueous contours of the containing valley or basin.

3. To determine the form of a portion of the bottom of the sea, a bay, harbor, or navigable river, for purposes of navigation. In this case, it is necessary to locate the navigable channels, and the obstructions to navigation, such as shoals, rocks, sunken wrecks, etc.

2. Limitations.—Hydrographic surveying consists merely in making the measurements necessary for acquiring such information as is outlined above. The measurement of the velocity and discharge of rivers and streams, and the planning and execution of works of improvement, such as the reclamation of submerged areas or the construction of breakwaters, sea walls, dams, etc., belong to hydraulic engineering, and will not be treated here.

3. Traverse Survey.—The survey of a body of water to determine its outline may be conducted as an ordinary traverse by any of the methods described in *Transit Surveying.*

The courses of the traverse are run at convenient distances from the water's edge and the shore line is determined by measurement from the line of the survey. The position of the ordinary low-water line is usually defined, but in many cases the high-water line is also determined and noted.

A good way to make an outline survey of a body of water is by means of a deflection traverse, using a transit and a

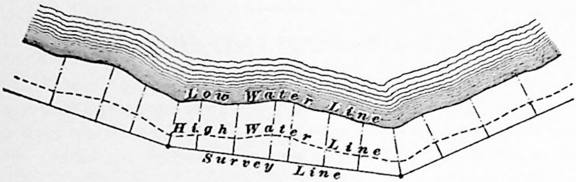

FIG. 1

chain or tape. This method is commonly used and is satisfactory for ordinary surveys of this kind. The outline survey of a body of water can also be made by the transit and stadia; this method of surveying is fully described in *Stadia and Plane-Table Surveying*. The entire survey of a small

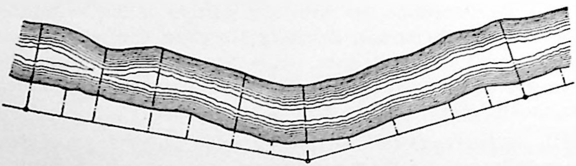

FIG. 2

river or stream, including the location of soundings, can be made with the stadia.

Prominent objects on shore may be located by direct measurement from the line of survey. If at a considerable distance, they may be located by triangulation, by taking sights from two known points. The distances from the line of the survey to the high- and low-water lines are usually measured by offsets, as illustrated in Fig. 1. If it is not necessary to obtain a close approximation of the shore outline,

the offset measurements can be omitted and the shore line between the survey stations sketched in by the eye.

In the case of a small stream, a traverse run along one bank is usually sufficient. The offsets should be measured to the edge of the water, and the width of the stream also measured at sufficiently close intervals to give the required information, as shown in Fig. 2. In the case of a small lake, the traverse is run entirely around it and closed on the point of beginning.

4. Triangulation.—A triangulation survey probably affords the best means for determining the outlines of large rivers, lakes, and other large bodies of water. Triangulation, as applied to hydrographic surveying, consists: (1) in locating distant objects from a measured base; (2) in determining the surface outlines of a river or other body of water by a system of triangles referred to a measured base.

The base line should be measured on fairly level ground in a location convenient for making the angular measurements from its ends. It should be not less than 500 feet long and as much longer as practicable. The ends should be marked with substantial stakes or with stone monuments. The line should be measured carefully with a steel tape.

5. Locating Distant Objects.—For locating points of reference and other distant objects, the angle formed by the intersection of the base line and the line of sight to the object, at each end of the base line, is measured. From the values of these angles and the length and azimuth of the base line, the lengths and azimuths of the lines to the object can be calculated and the object located. Let $A B$, Fig. 3, represent a base line and C a distant object whose position is to be determined. The base line $A B$ is measured accurately, and the angles $A B C$ and $B A C$ are measured with a transit or

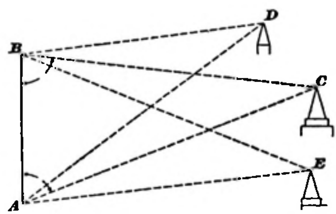

FIG. 3

sextant. Then, in the triangle ABC, the side AB and the adjacent angles ABC and BAC are known, from which the sides AC and BC can be calculated by trigonometry, and the point C located. The points D and E can be located in a similar manner from the same base line.

6. Triangulation of River.—For the survey of a river or other body of water by triangulation, points are selected on both sides for the vertexes of the triangles. Such points are called **triangulation stations**. They should be so located as to give triangles of advantageous form, in which no angle will be less than 30° or greater than 120°. Fig. 4 illustrates the triangulation of a river for the purpose of determining the outline of its shores. Some convenient line, as the line *1–3*, is taken as the base, its length is carefully measured, and its azimuth is either determined or assumed. The angles from *1* and *3* to *2*, and the angle *2*, are carefully measured. Their sum should not differ from 180° by more than 1 minute. The difference between that sum and 180° is distributed equally among the three angles of the triangle, one-third of it being added to, or subtracted from, each angle, as may be necessary to bring the sum to 180°. The same applies to the other triangles, in each of which each angle should be measured directly.

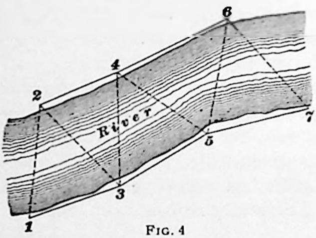

Fig. 4

Knowing the angles of the triangle *1–2–3*, and the length of *1–3*, the lengths of *1–2* and *3–2* are computed by trigonometry. Then, in the triangle *2–3–4*, the angles and the side *2–3* are known, and the other two sides are computed; and so on with the other triangles. At the end of the chain of triangles another line, as *5–7*, whose length has been calculated, is measured, as a check on the work.

The shore line between triangulation stations can be sketched in approximately, or, if it is desired to determine its outline more closely, the more important points can be located with transit and stadia, or by intersections from two triangulation stations when these are so situated as to give satisfactory intersections. If more detailed information is desired, a traverse can be run between adjacent stations and the shore line located by offsets at such intervals as may be desired.

SURVEY OF A SUBMERGED AREA

7. Purpose of Survey.—A hydrographic survey to determine the topography of the bottom of the basin or channel containing a body of water may be made for one or more of the following purposes:

1. To determine what changes it is desirable or necessary to make in the configuration of the channel or basin under consideration.
2. To indicate where material should be removed by dredging or blasting and where it may be deposited for filling, and to measure the quantity of material removed or the extent of the filling.
3. To obtain the information necessary for planning the construction of sea walls, jetties, lighthouses, docks, bridge piers, etc.
4. To construct a map or chart of the channel or basin for navigation purposes.
5. To determine the volume of the body of water, or capacity of the containing basin.

In making the survey of a submerged area, it is first necessary to make an outline survey in order to determine the shore line and locate points of reference. The points of reference are usually on shore and may be located by direct measurement or triangulation, as may be more expedient. In some cases, buoys are anchored in the water and used for reference points; as they are inaccessible, their distances from other points of reference must be determined by computation.

FIG. 5

SOUNDING

8. Soundings.—The shore line having been determined and the reference points located, the next step is to measure the depths, below the water surface, of a sufficient number of points to show the configuration of the bottom; such measurements are called **soundings.** For depths of 18 feet or less, soundings are made with a graduated wooden rod called a **sounding pole.** For greater depths, a line having a weight attached is necessary; this is called a **lead line.**

9. Sounding Pole.—The sounding pole may be of any sound, straight-grained wood. It should be well seasoned to prevent warping, and the bottom end should be provided with a disk-shaped iron shoe, not less than 5 inches in diameter, to prevent the rod from sinking into the soft mud of the bottom.

A good form of sounding pole is illustrated in Fig. 5. Poles of this kind are usually made of white pine finished smooth and round. The length is usually from 15 to 20 feet, and the diameter from 3 to $3\frac{1}{2}$ inches at the lower end and from 2 to $2\frac{1}{2}$ inches at the upper end. The lower end of the pole is formed by an iron shoe that terminates in a disk, as shown.

The pole is painted white and is graduated to feet and tenths, the zero of the graduation being at the bottom of the shoe. Each foot division is marked by a red band about $\frac{3}{8}$ inch wide, and each tenth division by a black band about $\frac{3}{16}$ inch wide; the bands extend entirely around the pole. The graduations are numbered by two sets of figures placed on the pole diametrically opposite each other. The

numbers designating feet are painted in red and those designating tenths in black.

The bottom of the shoe is sometimes hollowed out cup-shaped for the purpose of bringing up samples of the bottom over which the soundings are taken. When samples of the bottom are desired, the cavity is lined with grease or tallow, to which particles of the sand or mud of the bottom will adhere.

10. Lead.—The weight attached to a sounding line is called the **lead**, because it is usually made of that material. It should be long and slender, and should taper slightly toward the upper end, so as to reduce its resistance to being raised in the water. The form shown in Fig. 6 is frequently used. An iron rod *R* has molded around it the lead *L*, which is usually square in cross-section, as shown at *S*, and of sufficient size to give the requisite weight. Small cross-bars attached to the rod prevent the lead from slipping. At the lower end of the rod is attached the cup *C*, which is covered with a leather washer *W* that slides freely on the rod between the cup and the lead. When the lead is lowered to the bottom, the cup sinks far enough into the bottom to fill, and the leather cover prevents the contents from being washed out while the lead is being drawn to the surface. In some cases the cup is omitted and the bottom of the lead hollowed out in conical form. When it is not desired to know the composition of the bottom, a plain lead of nearly cylindrical form, but tapering toward the upper end, will answer the purpose. For still, shallow water, a lead weighing about 5 pounds is satisfactory. A lead weighing 10 pounds is suitable for depths under 40 feet in reasonably quiet water. For greater depths and in strong currents the weight of the lead should be from 15 to 20 pounds.

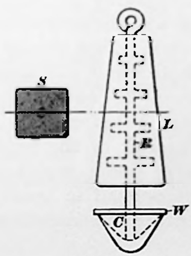

FIG. 6

11. Sounding Line.—Preferably, the sounding line should be of strong, closely plaited linen or twisted hemp.

Sometimes a cotton rope or a wire chain is used, but the use of such materials is not recommended. The line should be of a size suited to the weight of the lead; for ordinary river or lake soundings, about $\frac{3}{8}$ inch in diameter is a good size. It is marked with leather or cloth tags, which are inserted between the strands of the line. For river and harbor surveys, the tags are placed at intervals of 1 foot. At every fifth or tenth interval a conspicuous tag, usually of a bright color, is used. The zero of the graduation is the bottom of the lead.

Before being measured and marked, the line should be thoroughly stretched. This is done by stretching it tightly between two posts or trees, or wrapping it closely around a post or smooth-barked tree, then fastening both ends, wetting thoroughly and allowing it to dry. The slack is then taken up and the operation repeated until the line shows no further slack. Care should be taken not to stretch it too much, as in that case it will shorten in use. The length of the lead line, from the end of the lead to each 10-foot mark, should be tested before and after each day's use, and the results entered in the notebook. The line should preferably be kept under water when not in use; if it is not, it should be soaked in water for $\frac{1}{2}$ hour and then tested for length before the sounding is commenced.

12. Sounding Party and Equipment.—When the soundings are located by means of observations made with instruments stationed on the shore, the sounding party may consist of the recorder, leadsman, and boat crew. If the soundings are located by the stadia method, a stadia rodman is added to the sounding party. When the soundings are located from the boat, the sounding party is usually composed of two observers, a recorder, a leadsman, and the boat crew.

The usual equipment of a sounding party consists of a sounding pole or lead line and two signal flags, one white and one red. The flags are used to signal to the instrumentman on shore when a sounding is being taken, if the soundings are located by an instrument on shore. The white flag

is shown for each sounding except every fifth one, when the red flag is shown. The recorder is provided with a notebook in which to enter depths of soundings, nature of bottom, etc.

13. Making the Soundings.—If the depth of the water does not exceed about 75 feet, the soundings can usually be made while the boat is in motion. When the soundings are made at long intervals and the depth of the water does not exceed about 30 feet, it is usually more advantageous to withdraw the lead from the water after each sounding. In this case the lead is cast far enough ahead of the boat, as each sounding is made, for the line to become vertical when the lead reaches the bottom. If the depth of the water is too great for this method, the soundings can be made at intervals, as the boat moves, without drawing up the lead farther between soundings than is necessary to free it from the bottom.

As the soundings are made, the leadsman calls out the observed depth of each sounding to the recorder, who repeats the depth to prevent mistakes and then enters it in his notebook, together with the time and the number of the sounding. The character of the bottom is observed and noted at such intervals as may be desired, and all changes in the material of the bottom are noted.

GAUGES

14. Tide Gauges.—The bottom depths, as determined by the soundings, are measured from the surface of the water, the elevation of which varies considerably in river and tidal waters. In order to reduce the observed depths to the same surface of reference or to the datum of the survey, it is necessary to know the water level at the time each sounding is made. For this purpose a gauge that will show the height of the water surface should be established at some convenient place. An ordinary graduated board or staff is best for temporary use. This may consist of a board about 6 inches wide, 1 inch thick, and of a length somewhat greater than the variation in the height of the water, painted white and

graduated to feet and tenths in black. Such gauges are used very commonly for this purpose, and are called **staff gauges.** A simple form of staff gauge is shown in Fig. 7. For facilitating the reading of the gauge, a float, consisting of a small board painted white, may be so placed as to rest on the water surface in front of the gauge. This float moves up or down with the water surface as it rises or falls, and indicates at once the gauge reading.

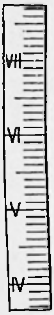

FIG. 7

15. Location of Gauges.—A tide gauge may be attached to a dock, quay wall, pile, stake, tree, or any other stationary object that is in a convenient position and to which the gauge can be secured in a vertical position. Sometimes the gauge is divided into sections and fastened to different objects, as trees, according to their heights, as illustrated in Fig. 8. Each section should slightly overlap the other, so as to afford a continuous gauge reading. In some cases, the gauge may be conveniently set at any suitable inclination and attached to stakes driven firmly in the bank. It may be made in sections and fastened to stakes in such a manner as to conform to the slope of the bank, as illustrated in Fig. 9. Each section should consist of a straight, well-seasoned board from 4 to 6 inches wide and 1 inch thick, and should be painted white. The divisions are determined with the level and should be marked by nails or tacks driven into the face of the board. Such a gauge should be located where the bank is not changing by caving off or filling up, and should be in

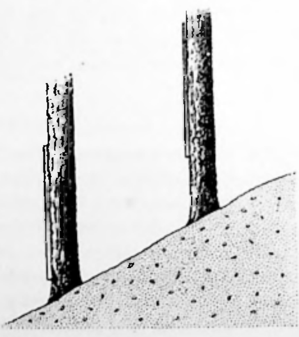

FIG. 8

a location where it will not be disturbed by floating drift at periods of high water. A gauge so placed can be easily observed from the bank at any height of the water.

When a continuous record of the fluctuations of the water surface in tidal waters is desired, a self-registering gauge should be used. This consists essentially of a float that rises and falls with the tide. The float is protected by a perforated box and is so arranged that its motion is recorded by a stylus or pencil on a roll of paper, which passes over a cylinder that is revolved at a uniform speed by clockwork. The path of the pencil on the paper indicates the stage of the water at any given time.

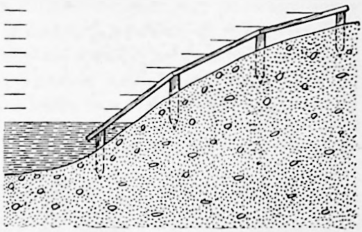

Fig. 9

THE SEXTANT

16. Law of Reflection.—Let AB, Fig. 10, be a plane mirror; PO, a ray of light meeting the mirror at O; OP', the direction taken by the ray after it strikes the mirror; and ON, a line perpendicular to the plane of the mirror. This perpendicular is called the **normal** to the mirror at O. The ray PO coming to the mirror is called the **incident ray**, and the angle NOP that it makes with the normal is called the **angle of incidence**. The ray OP' leaving the mirror is called the **reflected ray**, and the angle NOP' that it makes with the normal is called the **angle of reflection**. It is a general law of physics that the angle of incidence is always equal to the angle of reflection; that is, $NOP = NOP'$. It follows that the angles POA and $P'OB$ are also equal.

Fig. 10

17. Description of Sextant.—The sextant is a hand instrument for measuring angles. By means of it the angle between two lines of sight can be measured by a single operation. The angle between two lines of sight directed to two objects is commonly spoken of as the **angular distance** between the objects. When making the observations, the instrument is held in the hand, and successive angular measurements can be made with great rapidity. It is therefore especially adapted for use in a boat on the water, where the motion renders the use of fixed instruments impracticable.

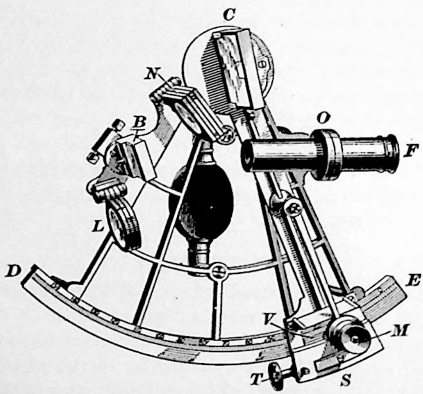

Fig. 11

As the sextant is frequently used in the location of soundings, a description of the instrument and a discussion of its theory are given here.

A sextant, as represented in Fig. 11, consists of a metal frame CDE, and an arm CS, called the **index arm**, which is fitted with a vernier and rotates about the center C of the sextant; to this index arm is also attached the **index mirror** BC, Fig. 12. To the arm CD is fixed the **horizon glass** A, half of the back of which is silvered, while the other half is transparent. The arm CE carries a **telescope** T directed

toward the horizon glass *A*. Thus, while the telescope is directed to an object *H*, the rays of light from another body *S* are reflected first from the mirror *B C* to the silvered half of the mirror *A*, and then from this mirror to the telescope in the direction *H A T*. The observer will thus see both of the bodies *H* and *S* in the field of the telescope together.

In order that the ray of light *S C* may enter the telescope after reflection, the index arm *B I* must be turned about the pivot *C* until the mirror *B C* is brought into the proper position with reference to *S C* and *A*. When *I* is at *E*, the two mirrors are parallel; and when *I* has been moved forwards until the rays of light *S C*, after two reflections, enter the telescope, the arc *E I* over which the arm *B I* carrying the mirror *B C* has been moved will be exactly equal to one-half the angle between *S C* and *H T*. Thus, the angle between the rays of light coming from two distant objects *H* and *S* may be measured as follows: The observer points the telescope directly toward *H* and then moves the arm *I* along the arc *E D* until the reflected ray *S C A T* also passes along the line *H A T* and the image of the second object enters the telescope. The arc *E I*, which is equal to the angle, called the **index angle**, between the surfaces of the index mirror *C B* and the horizon glass *A*, will then be one-half the angle between *S C* and *H A*, that is, one-half the apparent angular distance of the bodies *H* and *S*.

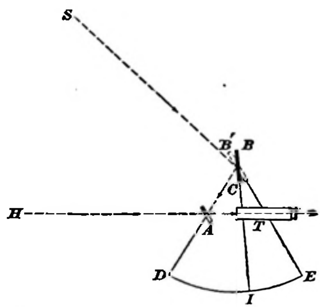

FIG. 12

Since the angle *E C I* is one-half the true angle between *S C* and *H A*, the arc *E D* has each half degree marked as a whole degree, so that *E D*, which is an arc of 60°, is divided into 120 equal parts and each part marked 1°. This is done merely to spare the observer the trouble of multiplying the reading by 2.

18. Theory of the Sextant.—That the index angle, or angle between the index mirror BC and the horizon glass A is equal to one-half the angle between SC and HA, Fig. 13, will now be proved. With the index arm in any position, produce the planes of the mirrors to their intersection at O, and produce the perpendiculars to the mirrors, pC and $p'A$, to their intersection at N. Since CE is the zero position of the index arm and CO its given position, the angle ECO is the index angle. Since CE is parallel to AO, $ECO = COA$, and since pC and $p'A$ are perpendicular to CO and AO, respectively, $COA = pNA$. Hence, $pNA = N$ is equal to the index angle.

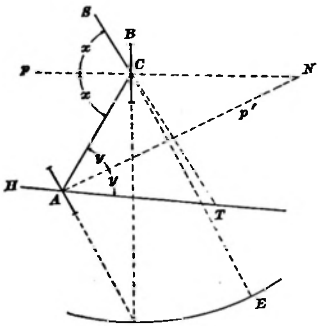

FIG. 13

It will be remembered that the ray of light SC, before reflection, makes the same angle with the perpendicular Cp as the ray of light CA after reflection. Hence, $SCp = pCA$, and similarly $CAp' = p'AT$. Calling each of the former angles x, for brevity, and each of the latter angles y, we have from the triangle ACN,

$$N = x - y$$

The observed angle is $STH = T$. From the triangle ACT,

$$T = 2x - 2y = 2(x - y) = 2N$$

whence, $$N = \tfrac{1}{2}T$$

That is, the angle between the two mirrors, or the index angle, is equal to one-half the angular distance between the two objects.

The vernier plate V, Fig. 11, is attached to the index bar immediately below the graduations on the limb. The index bar can be clamped to the limb by means of a clamp screw (not shown in the figure), and when so clamped can be

moved very slowly by means of the tangent screw *T*. The magnifying glass *M* is for reading the graduations on the limb and vernier. The two sets of colored-glass shades *N* and *L* are used to prevent the glare of the light from the observed body affecting the eye of the observer; they are attached to the frame by hinges in such manner that the shades *N* can be turned into the path of the reflected ray and the shades *L* into the path of the direct ray.

19. The Vernier.—As stated previously, the divisions on the limb of a sextant correspond to degrees, and each division is subdivided into three, four, or six parts, according to the instrument. These divisions are numbered as shown in Fig. 14, the upper portion of which represents part of the limb of a sextant. The divisions representing degrees on the limb there shown are subdivided into three parts, each part representing a third of a degree, or 20 minutes. By means of the vernier shown in the lower part of the figure, however, the angles can be read to half minutes, or 30 seconds. The vernier of a sextant is substantially the same as that of a transit, except that it is single instead of double; that is, it reads in only one direction, to the left, from the zero point, instead of both to the left and to the right. The first line at the extreme right of the sextant vernier, which is usually designated by a spear-shaped mark, is the zero point or index mark of the vernier. The vernier shown in Fig. 14 is divided into forty equal parts, which together are equal to thirty-nine divisions on the limb. Since the least reading is equal to a scale division divided by the number of parts in the vernier, it is $\frac{20}{40}$ minute, or 30 seconds. The numbering of the vernier corresponds to whole

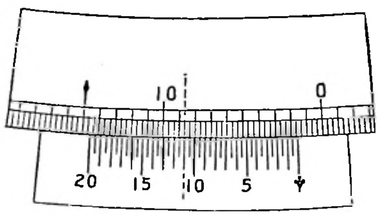

FIG. 14

minutes, which are represented by the longer graduations, the shorter graduations represent half minutes. Some sextants are graduated to read to 20 seconds and some to 10 seconds.

The graduations of the limb of a sextant usually extend a few degrees to the right of the zero mark, and it is sometimes convenient to make an observation that requires the vernier to be read when the index mark stands to the right of the zero mark. Such readings are said to be *off the arc.* When readings are made off the arc, the graduations are counted and the vernier is read to the right instead of to the left. The number of degrees and minutes are counted to the right from the zero mark to the first graduation at the left of the index mark; to this is added the number of minutes indicated by the coincident line of the vernier, as counted to the right from the graduation at the extreme left of the vernier. Thus, in Fig. 15, the index mark of the vernier stands to the right of the zero mark of the limb and the reading is off the arc. The angle is read by counting to the right on both the limb and vernier, as just described.

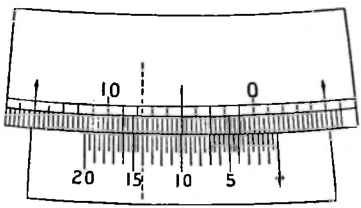

Fig. 15

20. Adjustments of the Sextant.—There are four adjustments of the sextant, as follows:

1. To make the plane of the index glass perpendicular to the plane of the limb.
2. To make the plane of the horizon glass perpendicular to the plane of the limb.
3. To make the line of collimation of the telescope parallel to the plane of the limb.
4. To make the planes of the mirrors parallel when the index reading is zero.

21. To Adjust the Index Glass.—Place the index bar near the middle of the limb; with the eye near the plane of the limb, observe whether the limb as seen directly and its image as reflected in the index glass form a smooth continuous curve; if they do, the glass is perpendicular to the plane of the limb and the adjustment is correct. But if the reflected limb appears to be above that part of the limb seen directly, the glass leans forwards; if it appears to be below, it leans backwards. In either case it is made perpendicular to the plane of the limb by means of the adjusting screws at its base.

22. To Adjust the Horizon Glass.—Look through the telescope and horizon glass toward a star or other well-defined distant object. Move the index bar slowly until the reflected image passes over the image seen directly. If these images coincide, the horizon glass is perpendicular to the plane of the limb. If they do not coincide, the horizon glass is adjusted by an adjusting screw placed under, behind, or beside the glass, according to the construction of the instrument.

23. To Adjust the Telescope.—Place the sextant in a horizontal position on a table or other support, and direct the telescope at some well-defined point or mark about 20 feet away. Place two small blocks of equal height on the limb, one near each extremity. These blocks should be of exactly equal height, so that a line of sight over their tops will be parallel to the plane of the limb, and should be at the same height above the limb as the center of the telescope. Some sextants are provided with two small brass sights that can be placed on the limb for this purpose. Sight over the tops of the two blocks or through the sights, as the case may be, in the direction of the point or mark sighted through the telescope, and note if the line of sight intersects the mark. If it does not, but falls above or below the mark, the telescope is not parallel to the limb. It can be made parallel to the limb by means of the screws in the collar that holds the telescope. This adjustment, however, is not usually made

unless the error is considerable, since a slight lack of parallelism between the line of sight and the plane of the limb does not appreciably affect the angular measurements on the limb.

24. To Adjust the Index—Index Error.—Set the index at zero, look through the telescope toward a star and note whether the direct and reflected images of the star coincide. If they do, the adjustment is correct. If they do not, move the index bar until they do coincide, and clamp it in this position. The reading of the index when in this position is called the **index error.** This error can be corrected by means of screws at the back of the index glass, which cause it to revolve about an axis perpendicular to the plane of the limb. To make the correction, set the index bar at zero and, by turning the screws, revolve the index glass until the two images exactly coincide. This adjustment will usually disturb the previous adjustment of the index glass, and, as a rule, it is not made unless the index error is greater than 3 minutes.

When the index error is less than 3 minutes, it is usually applied as a correction to all observations. If the error is *off* the arc, that is, if the index is to the right of the zero mark, it is additive or plus and should be added to all readings. If the error is *on* the arc, that is, if the index is to the left of the zero mark, the error is subtractive or minus and should be subtracted from all readings.

25. Method of Using the Sextant.—To measure an angle between two objects with a sextant, hold the plane of the limb in the plane of the two objects, look through the telescope toward the less distinct object, and move the index bar until the reflected image of the brighter object comes in contact with the direct image of the less distinct object. Clamp the index bar, and, with the tangent screw, bring the two images exactly together. Note the reading of the vernier and apply the correction for the index error.

In order to have the plane of the limb in the plane of the two objects when the telescope is directed toward the less

distinct object, it may sometimes be necessary to hold the sextant upside down. In locating soundings, the measured angles should lie in planes that are almost horizontal.

EXAMPLE.—The angular distance between two objects, as measured with a sextant, reads on the vernier 35° 36′ 30″; what is the true angular distance if the index error of the sextant is: (*a*) +1′ 20″; (*b*) −1′ 40″?

SOLUTION.—(*a*) Since the vernier reading is 35° 36′ 30″ and the index error is +1′ 20″, the true angular distance is equal to 35° 36′ 30″ + 1′ 20″ = 35° 37′ 50″. Ans.

(*b*) Since in this case the index error is −1′ 40″, the true angular distance is equal to 35° 36′ 30″ − 1′ 40″ = 35° 34′ 50″. Ans.

LOCATING SOUNDINGS

RANGES

26. Preliminary Remarks.—Before starting the sounding work, the stations, triangulation points, and ranges should be carefully located. The work should be so arranged that the soundings may be made and located as rapidly as possible, especially when the area to be sounded is large, or many soundings are to be made. The position of the sun should be considered, so that clear, distinct sights may be had without interference from glare. The observer should be so stationed that while making observations the sun is not directly in his face, but preferably on his back or overhead. If practicable, the order of work should be so arranged that observations toward the west may be taken in the forenoon, and toward the east in the afternoon. In tidal waters, the range of the tide should be considered. If the difference in elevation between high and low tide is very great, sounding work should preferably begin after the tide has fallen to a level about half way between high and low tide, or after half ebb, and cease when it has risen to the same level or at half flood. Usually, however, sounding work can be done at all stages of the tide.

In all sounding operations where simultaneous measurements are to be made, the recorder and the various observers

should have watches set accurately to the same time. Soundings are usually made at regular intervals of time, but there is no fixed rule regarding this. The length of time between successive soundings will depend on the depth of the water, the method of observation, and the distance between adjacent soundings. When the soundings are located from the shore with a transit, the observations are commonly made at intervals of 1 minute. In this case, intermediate soundings may be located by interpolation. When soundings are located from a sounding boat by sextant observations, as many as three observations per minute can be made, since angles can be measured more rapidly with a sextant than with a transit.

27. Sounding Ranges.—Soundings are usually made on well-defined lines or courses whose positions are known. These lines are called **ranges.** They are usually laid out on shore and prolonged across the area of water surface to be sounded. In such cases, two points on each range are selected at which poles or other signals are placed to serve as guides to the sounding party in determining the range. These points should be a considerable distance apart, and should be carefully located in order to accurately establish the direction of the range passing through them. The point near the shore is called the **front range point,** or **front signal,** and that back from the shore is called the **back range point,** or **back signal.** The steersman determines the course of the sounding boat by sighting along the range and keeping the two range points in line.

The ranges should be laid out and arranged with reference to the extent of the area to be sounded and also the method of locating the soundings. In localities where the area to be sounded is comparatively small, the arrangement illustrated in Fig. 16 can be used. Two parallel rows of stakes, as CD and EF, are established. The stakes in each row through which the parallel ranges pass are usually spaced at regular intervals. The length of the interval or the distance between adjacent ranges will depend on the frequency with which the soundings are to be made. The distance between the front

and back range points should be such that well-defined ranges can be established by sighting from the boat to two range points in line.

In Fig. 16, the dotted parallel lines represent the sounding ranges. *A* and *B* are observation stations so situated as to offer a clear view over the area to be sounded, and preferably visible one from the other; the distance between them should be accurately determined.

This system of range lines controls the positions of the soundings, which are made on the range line and distributed over the area to be sounded in such manner as may be desired, thus avoiding unequal distribution, making two or more soundings at or near the same place, or allowing too great an interval between soundings. For such purpose, the range lines are advantageous in nearly all soundings. When a boat is on a range the observer always knows its approximate position. Soundings made on range lines are platted with greater facility than those not made on ranges. The system of range lines is especially adapted for use when the soundings are located from shore by means of angles measured with a transit at one extremity of the base line. One angle for each sounding is measured between the base line and the line of sight to the boat. Before commencing the soundings, the angles made by the range lines with the base line should be measured, and the distance along the base line from the instrument to the intersection of each range line with the base should be determined. In some cases, the soundings are located by means of two angles measured simultaneously from both extremities of the base to the sounding boat; in this case each range is used as a guide for the sounding boat and also as a check on the accuracy of the locations made on it.

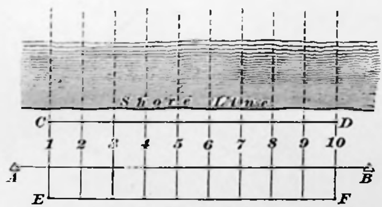

FIG. 16

28. Ranges Marked by Buoys.—If the shore is heavily wooded or rocky and precipitous, it may be impracticable to establish two rows of stakes at a sufficient distance apart to serve as front and back range points. In such a case, buoys may be used for signals to mark the front range points. Each range will then consist of a point on shore and a buoy in the water, as illustrated in Fig. 17. In this figure, A, B, C, and D are points or stations on shore, which may be located by direct measurement from point to point, while *1*, *2*, *3*, and *4* are the corresponding buoys. The buoys are located by measuring two angles in each triangle, the angles being read from the shore stations. Thus, the buoy at *1* can be located by measuring the angles $BA1$ and $AB1$; the buoy

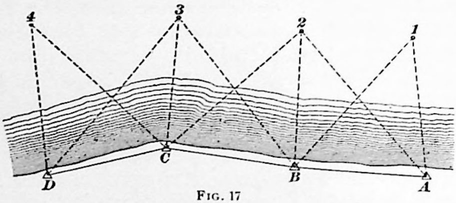

FIG. 17

at *2* by measuring the angles $AB2$ and $BA2$, or the angles $CB2$ and $BC2$. The buoys at *3* and *4* are located in a similar manner.

Buoys may be used for points of reference in connection with fixed points on shore. The position of each buoy will vary within small limits according to the stage of the tide, but such variations will not usually be sufficient to cause appreciable error in the platted positions of the soundings.

29. Radial Ranges.—Where topographical conditions will permit, the front range points are located close to the shore line, and some prominent natural object, such as a tall tree, a church spire, a windmill, or the cupola of a building, is selected for a back range signal. In such a case, the distance from the shore line to the back range signal should be such that radial range lines from this point

through the several front range signals will cover the area to be sounded. The distance between adjacent ranges at their extremities should not usually be greater than the average distance between successive soundings on a range. Such an arrangement is illustrated in Fig. 18. In this figure *A B* is a base line on which the front range points are located; these are represented by small dots. The broken lines numbered *1*, *2*, *3*, etc. are the sounding ranges. The front range points should be at known distances apart, the distance between adjacent range points being carefully measured with a steel tape, and each point marked by a stake. The points *A* and *B* at the extremities of the base line, and

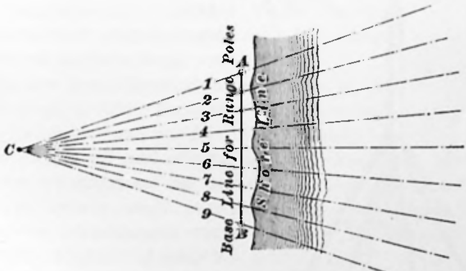

FIG. 18

the back range signal *C*, can be located from other points on the outline survey whose positions are known.

30. Ranges Across Streams.—When soundings are to be made across rivers or streams of considerable magnitude, sounding ranges are usually run across in directions perpendicular to the axis of the stream. Such ranges are marked by range signals placed either on one or both banks, according to the width of the stream.

RANGE SIGNALS

31. Range Poles.—In designating the ranges, it is important to have the front and back range points marked by poles, or other objects, sufficiently conspicuous to be

easily visible from the sounding boat, and designated in such a manner as to distinguish the different ranges, each from the others. Such objects are called **signals**; they may consist of poles or rods, or of natural objects on the shore. When a range point is in the water, the signal marking it usually consists of a buoy, as has been previously stated. In shallow water, signals similar to those used on the shore are often used, being set on, or driven into, the bottom.

Range signals on shore usually consist of poles of suitable height and dimensions. When the sounding ranges are short, ordinary transit sight poles are frequently used for signals. In such cases an assistant holds the rod in position on the stake marking the range point, or it is placed in a vertical position by the stake.

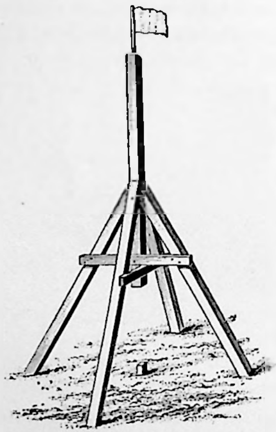

Fig. 19

If the ranges are of considerable length and soundings are to be made at some distance from shore, larger and more conspicuous poles should be used for signals. These may consist of pieces of 4″ × 4″ scantling, of suitable lengths. They should be set vertically and may be supported by being firmly driven into the ground, their lower ends being sharpened, or by being placed in holes dug for the purpose. If the ground is hard or rocky, the poles may be supported by braces or by stones piled around their bases. A form of range signal often used is illustrated in Fig. 19. A pole, consisting of a piece of 4″ × 4″ scantling of suitable length is supported in a vertical position by four inclined braces. The upper ends of these are nailed to the vertical piece, which is also held in place at the bottom by two horizontal strips of wood extending between the opposite braces, as

shown. The lower end of the vertical pole may be set at any convenient height above the ground; a hole may be bored into its upper end to receive a flagstaff, or the latter may be nailed to the top of the pole. This form of range signal can be set directly over a stake or hub, as shown in the illustration.

FIG. 20

Range poles should be whitewashed so as to be conspicuous against the background of the shore. When a number of adjacent ranges are used, they may be designated by attaching colored flags to the poles or signals. In such cases each range is known by some distinctive color or combination of colors. Thus, the flag for range No. *1* may be red; that for range No. *2*, red and white, etc.

Ranges are sometimes designated by strips of wood, such as laths or barrel staves, nailed to the poles or signals. The strips should be arranged in the form of Roman numerals, as shown in Fig. 20. In such cases the numerals denote the numbers of the ranges; the arrangement shown in Fig. 20 represents range No. *6*. The strips and the pole should be whitewashed so as to be conspicuous.

FIG. 21

32. Targets.—When sounding ranges are to be projected a considerable distance from shore, the range poles or signals should be provided with targets in order to be conspicuous. Such targets should be sufficiently large to be visible from the sounding boat at the most distant point on the range, and of suitable designs or colors to enable the steersman to distinguish readily the different ranges. No rigid rule can be laid down specifying any particular form or design for range targets. They may be made of such forms as are best suited for the purpose, and composed of such materials as are available in each case.

A good form of target for range signals is shown in Fig. 21. The target is lozenge shaped and can be made in the following manner: A strip of wood, about 3 inches wide, 1 inch thick, and 3 feet long, is nailed to the face of the range pole about 3 feet below the top, care being taken to center the strip and to make it perpendicular to the pole. To the framework thus formed, a square piece of white or colored cloth is tacked in such a manner that two diagonally opposite corners of the cloth are at the two extremities of the cross-piece, the two other corners being on the center line of the range pole. Such a target is quickly and economically made and is very effective when the sights are directly in front and the full size of the target is visible.

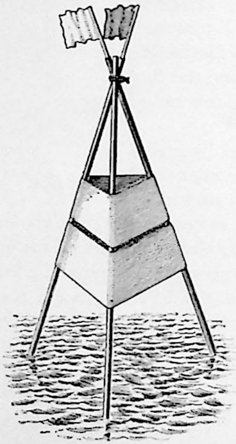

Fig. 22

33. Range Signals in the Water.—When a range point is in the water, the signal marking it usually consists of a buoy; in shallow water, however, stationary range signals are often used. A form of signal used by the United States Coast and Geodetic Survey is shown in Fig. 22. This consists of a tripod about 10 or 12 feet high, each leg being made of a piece of gas pipe about $1\frac{1}{2}$ inches in diameter. The legs are forced firmly into the mud or sand of the bottom, at suitable distances apart, and inclined toward the center. They are lashed together near the top, and flags about $1\frac{1}{2}$ feet square attached to poles just large enough to fit inside the pipes are placed at their upper ends as shown. Two strips of cloth, each about $\frac{1}{2}$ yard wide, are wrapped around the tripod, about half way between its top and bottom. These strips serve as a target.

34. Buoys.—A buoy is a float of wood or other suitable material, or a hollow air-tight vessel, anchored in place by a heavy weight to which it is attached by a rope or chain. Buoys are used to mark certain places or points on the water surface. They are usually employed to designate the limits of a channel or some submerged object in connection with navigation. They are also used as points of reference and for range points in hydrographic surveying, and only such as are suitable for such purposes will be considered here.

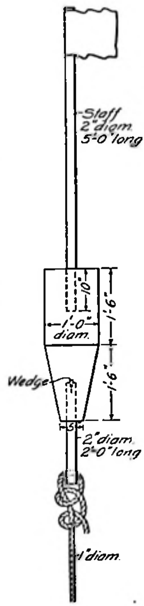

FIG. 23

A form of buoy that has been much used in hydrographic surveying is illustrated in Fig. 23. It consists essentially of a round log of cedar or other light wood, about 1 foot in diameter and 3 feet in length, sawed square at ends. The lower half is trimmed in the shape of a truncated cone, tapering to about 5 inches in diameter at the lower end. A hole about 2 inches in diameter and 9 inches deep is bored into the lower end on the axis of the log, into which a pole 2 inches in diameter is driven. The upper end of the pole is split, and a wedge inserted in the cleft, which is driven up and tightens the pole as it is driven to the end of the socket, thus preventing the pole from pulling out. A hole sufficiently large for the anchor rope to pass through is bored through the pole a few inches above its lower end. The anchor rope is preferably of manila hemp and is about 1 inch in diameter. A good form of knot with which to tie the rope is shown in the figure; it is shown open in order to illustrate the method of forming the knot. This knot is called the **two half-hitches.** A hole about 2 inches in diameter and 10 inches deep is bored into the top of the buoy along its axis. This serves as a socket into which is inserted a staff

about 2 inches in diameter and 5 feet long. At the upper end of this staff is fastened a flag about 1 foot square and of suitable design or color.

Another form of buoy is illustrated in Fig. 24. This consists of a log or round piece of light wood, about 3 feet long, sawed square at both ends. This log is trimmed in the shape of a truncated cone, tapering from about 8 or 10 inches in diameter at the upper end to a diameter of about 4 or 5 inches at the lower end. A hole about 2 inches in diameter is bored completely through the log, the center of the hole coinciding with the axis of the log, and a round pole large enough to fit the hole closely is passed through the log and wedged tightly in place. This pole should project about 2 or 3 feet below the bottom of the log or buoy and about 3 or 4 feet above its top. A flag about 1 foot square is fastened to the upper end of the pole as shown. The anchor rope is passed through a hole in the pole near its lower end and tied as described in the preceding article.

FIG. 24

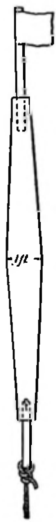

FIG. 25

35. Buoys for Tidal Waters.—The two forms of buoys just described are suitable for use in non-tidal waters and in rivers and streams where the current is not sufficiently strong to drag the top of the buoy under or level with the surface of the water.

Buoys for use in tidal waters should be of sufficient length to be visible at all stages of the tide. The best length for a buoy in a given tidal water will depend on the range of the tide, and is usually greater than that required for buoys in non-tidal waters.

A good form of buoy for use in tidal waters is illustrated in Fig. 25. The length shown is 10 feet, but this may be varied to conform to the range of the tide. The buoy is

made from a log or round piece of light wood and trimmed so as to taper gradually from the middle to each end. It has a diameter of about 1 foot at the middle and tapers to a diameter of from 4 to 6 inches at each end. It is sawed square at the ends, and holes are bored at each end, into which poles are inserted for attaching the flag and anchor rope, as previously described. If preferred, a ring may be fastened to the lower end of the buoy by means of a staple or screw, and used for securing the anchor rope to the buoy.

A more simple form of buoy, consisting of a single stick or log of timber of suitable length and diameter, is often used in tidal waters; the shape is similar to that illustrated in Fig. 25, but the buoy is more uniform in size from top to bottom. Such buoys are commonly called **spar buoys.**

METHODS OF LOCATING SOUNDINGS

36. Methods Employed.—In order to plat soundings correctly on a map or chart, the position of each sounding must be located; that is, its relative position with respect to known points on shore must be determined. Soundings can be located by various methods, depending on local conditions, the object of the survey, and the degree of accuracy required. The following list comprises the best known and most frequently used methods of locating soundings: (1) by time intervals; (2) by one angle measured on shore; (3) by two angles measured simultaneously on shore; (4) by two angles measured in the sounding boat; (5) by transit and stadia; (6) by a fixed line marked by a wire or rope; (7) by the intersection of fixed ranges. These methods will be described in order.

37. By Time Intervals.—When this method is employed, the soundings are made at stated intervals of time while the sounding boat moves at uniform speed along a range or on a course not marked by range signals. The soundings may be made under two conditions, namely: (*a*) The first and last soundings on a range or course are located by observation and all the intermediate soundings are

located by interpolation or time intervals. (*b*) Alternate soundings or those made at convenient intervals are located by observation, and such intermediate soundings as are not observed are located by interpolation. In either case the method of interpolating the intermediate soundings is the same. Knowing the distance between the two end soundings or between two adjacent observed soundings, the time interval between them, and the time interval between consecutive soundings, the position of each intermediate sounding can be determined by proportion as follows:

Let T = time elapsed between two observed soundings on a range or course;

D = distance between these observed soundings;

t = time interval between consecutive intermediate soundings;

d = distance between the intermediate soundings.

Then, since the boat moves at a uniform speed,

$$D : T = d : t$$

from which

$$d = \frac{D t}{T}$$

EXAMPLE.—A sounding boat moving at a uniform speed traverses a range 1,800 feet long in 20 minutes, and a sounding is made at each end of the range and at intervals of 1 minute; what is the distance between consecutive soundings?

SOLUTION.—The two end soundings are 1,800 ft. apart = D; the elapsed time between them is 20 min. = T; and the time interval t between consecutive soundings is 1 min. Substituting these values in the formula gives,

$$d = \frac{1,800 \times 1}{20} = 90 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. A range 500 feet long is traversed at uniform speed in 10 minutes by a sounding boat from which soundings are made at intervals of ½ minute; find the distance between any two consecutive soundings.

Ans. 25 ft.

2. In the preceding example, if the soundings are numbered consecutively 1, 2, 3, etc., from beginning to end of the range, what is the distance between soundings Nos. 5 and 12? Ans. 175 ft.

3. Soundings are made at intervals of 15 seconds or at the rate of 4 per minute, from a boat moving along a course at a uniform speed; the soundings made at the end of each minute are located by observations and the intermediate soundings are interpolated. The observed soundings, when located, are found to be at intervals of 204 feet apart; what is the distance between consecutive soundings? Ans. 51 ft.

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38. By One Angle Measured on Shore.—When this method is used, the boat containing the sounding party traverses a fixed range while an observer on shore measures the angle between a base line and the line of sight to the leadsman at the time a given sounding is made. The ranges are usually parallel to each other and are preferably at right angles to the base line. They should be at known distances apart and the distance from the observation station to each range should be determined by careful measurement along the base line. The base line may have an observation station at each extremity, in which case the observer is stationed at either end, as may be most convenient, and orients his instrument

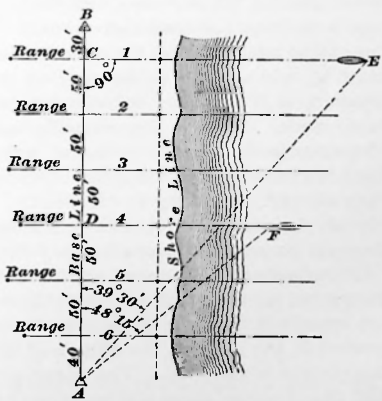

FIG. 26

by sighting toward the station at the other end. In Fig. 26 is shown a base line at the ends of which are two observation stations A and B, whose positions have been determined. The ranges numbered *1*, *2*, *3*, etc. are parallel to each other and at right angles to the base line. Each range is designated by front and back range signals whose positions are shown by the small dots in the figure. The

distances along the base line between the observation stations and the nearest ranges and the distances between adjacent ranges are as shown by the figures.

The field party usually consists of the observer on shore, and in the boat the recorder, the leadsman, the signalman, and the boat crew. In tidal waters, a tide-gauge reader should be added to the shore party; his duties consist in reading the tide gauge every 5 minutes during sounding operations and recording the times and gauge readings. The watches of the observer on shore and the recorder in the boat are set accurately to the same time. The sounding boat traverses the ranges in succession and soundings are made at regular intervals of time, which depend on the depth of the water and on the accuracy required. Usually, from one to four soundings are made per minute. A few seconds before the end of each interval the recorder has the signalman raise his flag, and exactly at the end of the interval the flag is lowered, as a signal to the observer that the sounding has been made. The leadsman calls out the observed depth of each sounding to the recorder, who enters it in his notebook together with the number of the sounding. The exact time of each sounding is recorded. The character of the bottom is observed by the leadsman and noted by the recorder at such intervals as may be required.

The observer is stationed at *A* with his transit set up over the transit point at that end of the base line. The vernier of the transit is first set at zero and the telescope is directed toward *B*, the point at the other end of the base line, and the instrument is clamped. The upper plate is then unclamped and the observer turns the instrument in azimuth toward the sounding boat. When making the observations, the observer usually keeps his watch open and lying face uppermost on the upper plate of the transit, for convenience in noting the time. When the signal flag is raised in the boat the observer sights through the telescope toward it, and by turning the upper plate slowly and carefully, keeps the line of sight fixed on the flag. At the instant the flag is lowered the observer ceases to turn the plate. He then looks at his

watch and notes the exact time, then reads the angle on the vernier plate and enters the time and angle in his notebook. The observations are usually made on the flag, as it can be seen more distinctly and at greater distances than a sounding line or pole. On this account the signalman should be stationed near the leadsman, so that the difference between the observed and the true position of each sounding will be small. The upper plate of the transit should remain unclamped while observations are being made on the sounding boat, since there is usually not sufficient time between observations to permit its being clamped, and frequent clamping and unclamping would tend to disturb the position of the transit.

In some instances, there is by each transit, in addition to tne observer, a recorder whose duties consist in recording the observed angle for each sounding, and the number of the sounding. The recorder enters in his notebook the observed angle and the number of each sounding, and he also notices that every fifth sounding is a "red" sounding. It is customary for the signalman to raise a red flag for every fifth sounding and a white flag for the intermediate soundings. Whenever the observer sees the red flag displayed by the signalman he calls out "red" to the recorder, who then notes the number of the sounding, which should be a multiple of five, in order to correspond with the number entered by the recorder in the boat. In this way a check is obtained on the numbering of the soundings, and any difference in the numbering by the recorder in the sounding boat and the recorder on shore can be readily detected.

Example.—The ranges shown in Fig. 26 are at right angles to the base line; a sounding is made while the sounding boat is at *E* on range *1*. The observed angle *E A B* is 39° 30′ and the distance *A C* along the base line from *A* to the intersection *C* of range *1* is 290 feet; find: (*a*) the distance *A E*; (*b*) the distance along the range line from *C*, its intersection with the base line to the sounding at *E*.

Solution.—(*a*) Since the ranges are at right angles to the base line, *A E* is the hypotenuse of the right triangle *A C E*. From trigonometry,

$$AE = \frac{AC}{\cos EAC} = \frac{290}{\cos 39° \, 30'} = 375.8 \text{ ft.} \quad \text{Ans.}$$

(*b*) The distance from E along range *1* to its intersection C with the base line is $EC = AC \tan EAC = 290 \tan 39° 30' = 239.1$ ft., nearly. Ans.

EXAMPLES FOR PRACTICE

1. The distance AD along the base line from A to the intersection of range *4*, Fig. 26, is 140 feet; the angle FAB is 48° 15′; find: (*a*) the distance AF; (*b*) the distance along the range from F to its intersection D with the base line. Ans. $\begin{cases} (a) & 210.2 \text{ ft.} \\ (b) & 156.9 \text{ ft.} \end{cases}$

2. A sounding boat is on a range perpendicular to the base line. The angle measured at an observation station between the base line and the line of sight to the boat at the time a given sounding is made is 30° 30′; the distance from the station to the intersection of the base line and range is 230 feet; find: (*a*) the distance from the station to the position of the sounding; (*b*) the distance along the range line from the base line to the position of the sounding. Ans. $\begin{cases} (a) & 266.9 \text{ ft.} \\ (b) & 135.5 \text{ ft.} \end{cases}$

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39. By Two Angles Measured Simultaneously on Shore.—This is one of the commonest methods, and if the work is carefully done it is both convenient and accurate. Two observers are required, each occupying a station whose position with respect to the shore survey has been determined. The observation stations should be so situated as to afford a clear field of view over the area to be surveyed, and when possible should be visible one from another. They may be at the extremities of a base line, whose length has been carefully measured, or at two points whose positions and distance apart have been determined by triangulation. The vernier of each instrument is set to read zero when the telescope is directed toward the other instrument point or toward some common point whose position is known. The field work is entirely similar to that described for the preceding method, and the field party and equipment is the same with the addition of an observer and transit. This method differs from the preceding method, however, in that two angles, instead of one, are measured for each location, the observations being made simultaneously, and the position of the observed sounding is determined by the intersection of the two lines of sight from the two observation stations,

instead of by the intersection of one line of sight with a range line. In using this method it is not necessary to have the sounding ranges parallel or to lay them out at right angles to the base line, although such an arrangement is advantageous in affording a means of checking the accuracy of the angular measurement. For locating soundings over a limited area by this method, the arrangement shown in Fig. 27 is convenient and gives good results. In this figure, *A* and *B* are the observation stations at the extremities of a base line *A B*. The ranges are shown by the parallel lines passing through the small dots indicating the range signals. One position of the sounding boat is shown at *C*, and the lines of sight to this position from the two observation stations are represented by the dotted lines *A C* and *B C*.

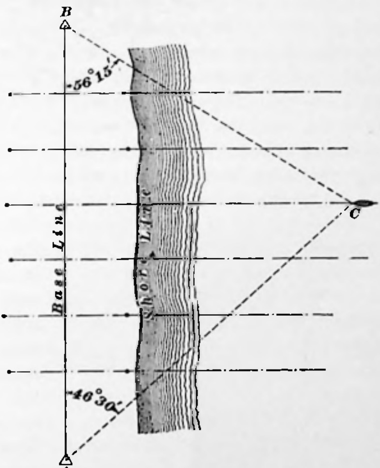


FIG. 27

EXAMPLE.—A sounding is made while the sounding boat is at the position *C*, Fig. 27. The angles observed from *A* and *B* are 46° 30′ and 56° 15′, respectively, as shown, and the length of the base line *A B* is 785 feet; find the distances *A C* and *B C*, giving values to the nearest foot.

SOLUTION.—The angle $BAC = 46° 30'$, and $ABC = 56° 15'$; hence, $ACB = 180° - (46° 30' + 56° 15') = 77° 15'$, and, from trigonometry,

$$AC = \frac{AB \sin ABC}{\sin ACB} = \frac{785 \sin 56° 15'}{\sin 77° 15'} = 669 \text{ ft.} \quad \text{Ans.}$$

$$BC = \frac{AB \sin BAC}{\sin ACB} = \frac{785 \sin 46° 30'}{\sin 77° 15'} = 584 \text{ ft.} \quad \text{Ans.}$$

40. By Two Angles Measured in the Sounding Boat.—This method, which is used extensively in harbor work, is one of the best general methods of locating soundings. The field party usually consists of two observers, a recorder, a leadsman, and boat crew. In tidal waters a tide-gauge reader is required. The observers, each with a sextant, occupy places in the sounding boat as close to the leadsman as practicable, in order that the observed position of each sounding may be very nearly the same as its true position. At the time a sounding is made, the two observers measure simultaneously the two angles between the lines of sight to three shore objects whose positions have been determined by the shore survey, one line of sight in each observation being directed toward the same object. The objects sighted to should be well defined and prominent, and so located with respect to the area to be sounded as to be readily visible from the sounding boat in all required positions. They should preferably be natural objects, such as church spires, windmills, lighthouses, cupolas, etc., but in case natural objects are not available, signal poles, such as are used for marking ranges, may be used. In making the observations, one observer measures the angular distance between the middle object and the object on the right; and the other observer measures the angular distance between the middle object and the object on the left. This is illustrated in Fig. 28, in which F represents the position of the boat at the time of a given sounding, and A and B are the two angles measured by the two observers between the lines of sight to the three shore objects E, C, and D. The sounding work is conducted as follows:

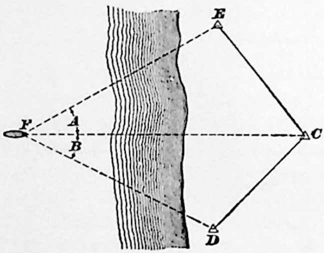


FIG. 28

The sounding boat moves slowly along a range or course, the leadsman making the soundings at the required intervals,

usually two or three per minute. A few seconds before a sounding is to be made the leadsman calls out "ready," when the observers hold their sextants in position to make an observation on the shore objects, each observer moving the vernier arm of his sextant so as to keep the two images in coincidence as the boat moves. At the moment the sounding is made, the leadsman calls out "sound," when each observer reads the angle on his sextant and calls it out to the recorder, who records each angle in its proper column in his notebook. The leadsman calls out the observed depth to the recorder, who also enters it in his notebook, together with the number of the sounding and the time the sounding is made. At required intervals, the leadsman observes the character of the bottom and informs the recorder, who enters it in the proper place in his notebook.

In some cases both angles are measured by one observer with a double sextant; they can also be measured successively by one observer with an ordinary sextant, if the boat is brought to a stop for each sounding. But they are usually measured by two observers, each using an ordinary sextant, in the manner just described. When sextants are used and the angles are so recorded by the recorder that the observer has only to observe and read them, four angles per minute can be observed under ordinary favorable conditions.

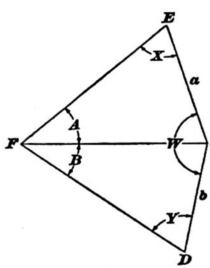


Fig. 29

41. The accurate location of soundings by two sextants involves what is known as the **three-point problem.** This problem can be solved trigonometrically as follows: Let F, Fig. 29, be the position of the boat when a given sounding is made; E, C, and D the three shore objects, whose positions are determined by the angle W and the sides $E\,C$ and $C\,D$, which are designated by a and b, respectively. The angles A and B are the two sextant angles measured in the boat. The problem is to determine the distances $F\,E$ and $F\,D$.

In the triangles $E F C$ and $C F D$,

$$C F = \frac{a \sin X}{\sin A} = \frac{b \sin Y}{\sin B} \quad (a)$$

Also, $X + Y + W + A + B = 2 \times 180° = 360°$

whence, $X + Y = 360° - (W + A + B) = S$, say $\quad (b)$

Therefore, $Y = S - X \quad (c)$

and $\sin Y = \sin (S - X) = \sin S \cos X - \cos S \sin X$

Substituting this value of $\sin Y$ in (a),

$$\frac{a \sin X}{\sin A} = \frac{b (\sin S \cos X - \cos S \sin X)}{\sin B}$$

Clearing of fractions,

$$a \sin X \sin B = b \sin S \cos X \sin A - b \cos S \sin X \sin A$$

Dividing by $\sin X$, replacing $\frac{\cos X}{\sin X}$ by $\cot X$, and solving for $\cot X$,

$$\cot X = \frac{a \sin B + b \cos S \sin A}{b \sin S \sin A} = \frac{a \sin B}{b \sin S \sin A} + \frac{\cos S}{\sin S}$$

or, writing $\cot S$ for $\frac{\cos S}{\sin S}$,

$$\cot X = \frac{a \sin B}{b \sin S \sin A} + \cot S$$

The value of $\cot Y$ can be determined in a similar manner, or by substituting in equation (c) the value previously found for X.

After determining the values of X and Y, the distances $E F$ and $F D$ can be determined by trigonometry as follows:

$$E C F = 180° - (A + X)$$

$$E F = \frac{a \sin E C F}{\sin A}$$

Similarly, in the triangle $F C D$,

$$D C F = 180° - (B + Y)$$

$$D F = \frac{b \sin D C F}{\sin B}$$

Having calculated the distances $E F$ and $D F$ for a given sounding, the position of the sounding can be located by drawing arcs with a pair of compasses, with E and D as centers and with radii of lengths $E F$ and $D F$, respectively. The intersection of the two arcs will be the point F, or the

position of the sounding. In practice, however, it is seldom necessary to calculate the position of soundings located by sextant angles to three fixed points on shore, since the positions of soundings so located can be platted easily without calculation, as will be described further on. But when an important location is made by this method, such as the position of a rock, a reef, a buoy, or a sunken wreck, the location as platted should be checked by calculation.

EXAMPLE.—Given $a = 850$ feet; $b = 760$ feet; $W = 150°$; $A = 41° 30'$; $B = 35° 30'$ (Fig. 29); what are the values of the angles X and Y, and of the sides EF and DF?

SOLUTION.—Substituting the given values in equation (b), $X + Y = 360° - 227° = 133° = S$; $\cot S = -\cot(180° - S) = -\cot 47°$.

Substituting known values in the formula,

$$\cot X = \frac{850 \sin 35° 30'}{760 \sin 133° \sin 41° 30'} - \cot 47°$$

$$X = 67° 49'$$

whence, $$Y = 133° - 67° 49' = 65° 11'$$

In the triangle FCE,

$$ECF = 180° - (41° 30' + 67° 49') = 70° 41'$$

$$EF = \frac{850 \sin 70° 41'}{\sin 41° 30'} = 1{,}211 \text{ ft.} \quad \text{Ans.}$$

In the triangle DCF,

$$DCF = 180° - (35° 30' + 65° 11') = 79° 19'$$

$$DF = \frac{760 \sin 79° 19'}{\sin 35° 30'} = 1{,}286 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In Fig. 30, in which F is the position of the sounding boat at the time a given sounding is made, and E, C, D are the three shore points, let $a = 1{,}200$; $b = 965$; $W = 146° 30'$; $A = 28° 15'$; and $B = 22° 30'$; find the angles X and Y and the distances EF and DF.

$$\text{Ans.} \begin{cases} X = 80° 21' \\ Y = 82° 24' \\ EF = 2{,}403 \\ DF = 2{,}437 \end{cases}$$

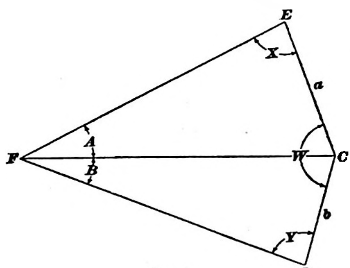


FIG. 30

2. With the same values for a, b, and W, let $A = 30°$

30′ and $B = 24° 28′$; find the angles X and Y and the distances EF and DF.

$$\text{Ans.}\begin{cases} X = 77° 3′ \\ Y = 81° 29′ \\ EF = 2{,}254 \\ DF = 2{,}240 \end{cases}$$

---

42. By Transit and Stadia.—This is a rapid and efficient method of locating soundings in bodies of calm, smooth water and at distances that do not exceed the limit of good practice for stadia readings. In this method, the positions of the soundings are located with a transit on shore by means of observations taken on the stadia rod held in the sounding boat. Since the stadia rod should be without vertical motion when a reading is taken, it is evident that this method can be used satisfactorily only on smooth water. When this method for locating soundings is used, a complete hydrographic party comprises the observer on shore, with a transit equipped with stadia wires, and the boat party, consisting of the recorder, the leadsman, the stadiaman, and the boat crew. No signalman is required; the recorder acts as signalman. The soundings are not identified by time intervals, but by means of differently colored flags. A red flag is shown for every fifth sounding and a white flag for the intermediate soundings. Two general cases may occur under this method.

1. The transit station may be a point on the sounding range; in this case the azimuth of the range is known and each sounding is located by the distance, corresponding to the observed interval on the stadia rod, as measured along the range.

2. The transit station may be a point whose position has been determined but which is not on a sounding range; in this case the reading of the azimuth angle, as well as the stadia interval, must be observed and recorded for each sounding.

In either case the field work is conducted in the following manner: The sounding boat moves slowly along the range or course while the soundings are being made. The leadsman stands in the bow of the boat and makes the soundings,

calling out the observed depth of each sounding and also the character of the bottom at required intervals. The recorder enters in his notebook the number and the observed depth of each sounding, and the character of the bottom when noted by the leadsman. If the soundings are in tidal waters, the time should also be noted in order to make reductions for the tide heights as given in the notebook of the tide-gauge reader. During the sounding operations, the stadiaman holds the stadia rod vertical and facing the observer. He should be stationed close to the leadsman, in order that the observed positions of the soundings will coincide nearly with their true positions.

The observer keeps the vertical wire of the transit telescope directed to the stadia rod in the boat. If the transit station is on the range on which the soundings are made, the observer merely reads the stadia interval for each sounding and enters it in his notebook, also noting the time and the number of the sounding. If the transit station is not on the sounding range, but is off to one side, the vernier of the transit is first set at zero, the telescope is sighted on some object whose position has been determined, and the instrument is clamped. The upper plate is then unclamped, and the instrument turned in azimuth toward the sounding boat as explained for the second method. Then, for each sounding, the horizontal angle is read and recorded, in addition to the stadia interval, and the number and time of the sounding.

43. By a Fixed Line Marked by a Wire or Rope. This is an accurate method, but it is adapted only to narrow channels. It is often used for measuring cross-sectional areas in a canal or a small stream in connection with the determination of discharge or the measurement of material removed from the channel by dredging or other means. In such cases the wire or rope is stretched from bank to bank between fixed points, as illustrated in Fig. 31, and the soundings are taken at regular intervals along the wire or rope. The points where the soundings are taken are marked

by tin tags or by bits of cloth tied to the wire or rope. When this method is employed in connection with the measurement of dredged material, the stakes $O\ O'$ are carefully located and their positions noted, in order that they may be replaced if disturbed. Soundings are made at

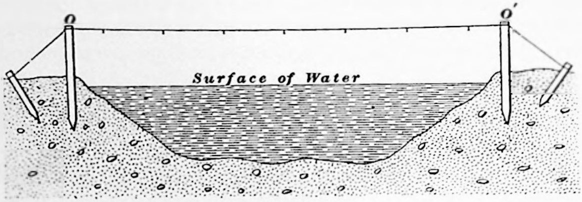


FIG. 31

known intervals along a wire or rope stretched from stake to stake, before and after the dredging operations.

44. By the Intersection of Fixed Ranges.—If a fixed range or section of considerable length is to be sounded a number of times, and the soundings are to be made at the same points each time, the soundings can be located by the

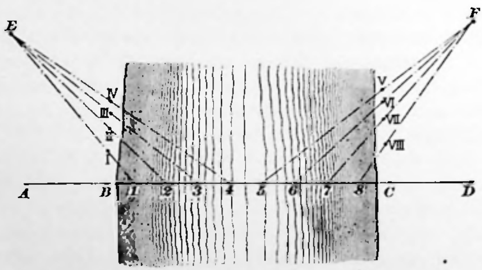


FIG. 32

intersection of a series of ranges with the fixed range or section. Let *A D*, Fig. 32, represent a range or section across a river, and *1*, *2*, *3*, *4*, *5*, *6*, *7*, *8* the points where soundings are to be made at successive periods. Range poles are set at *A*, *B*, *C*, and *D*, fixing the position of the range *A D*, unless

those points are marked by natural objects, and poles are also set at *E* and *F*; also at *I*, *II*, *III*, *IV*, etc. The ranges *E–I*, *E–II*, etc. and *F–V*, *F–VI*, etc., produced to their intersection with the range *A D*, will locate the soundings *1*, *2*, etc., and *5*, *6*, etc. The range signals *A*, *B*, *C*, *D*, and the back signals *E* and *F* for the numbered ranges may consist of ordinary range poles, whitewashed, as described in Art. **31**. The front range signals *I*, *II*, *III*, *IV*, *V*, etc., on the numbered ranges, should be designated by Roman numerals indicating the number of each range, as described in Art. **31**.

When this method is used, the sounding party consists of a recorder, a leadsman, and boat crew. The steersman keeps the boat on the fixed range *A D*, and each sounding is made when the leadsman, by sighting toward a numbered signal, finds himself in range with it, and the back signal at the same time in line with the signals designating the fixed range. Thus, when at the position *5*, Fig. 32, the leadsman is in line with the signals *C D*, designating the range *A D*, and also with the signals *V F*, designating range *V*. The boat can be stopped for each sounding if there is little or no current; or if the current is strong, the boat can move slowly, preferably against the current.

45. The Plane of Reference.—In making soundings, the leadsman notes the depth of each sounding below the water surface at the time the sounding is made. Since the elevation of the water surface is constantly changing, especially in tidal waters, it is necessary to select some particular stage or height of the water surface to which the depths of all the soundings are referred. Such a height of the water surface is called a **plane of reference**, and all observed depths are reduced to correspond to depths below this surface. In order to determine the proper reductions to apply to the soundings for different stages of the tide, it is necessary to know the elevation of the water surface at any given time during the sounding work; for this reason it is customary to employ a tide-gauge reader. In tidal waters, the surface of the water at mean low tide is usually taken as the

plane of reference, and the tide gauge should be set with the zero mark at the elevation of mean low tide. In such cases tide-gauge readings should be taken and recorded every 5 minutes during sounding operations, as has been explained. In lakes or reservoirs, where the elevation of the water surface changes but little and very slowly, the lowest recorded stage of the water is usually selected as the plane of reference. In such cases it is customary to take gauge readings twice a day—once in the morning and once in the afternoon during the period of sounding work. In rivers of variable stage, the plane of reference is usually the low-water stage of the river at the locality where the sounding work is done. In some cases, the general datum of the survey is used as a plane of reference. If the river is rising or falling rapidly while the soundings are being made, gauge heights should be read and recorded at intervals of 30 minutes, or even oftener if necessary. If, however, the river is at its normal stage during this period, and is changing but slowly, gauge readings should be taken twice a day, as for lakes or reservoirs.

FORMS FOR SOUNDING NOTES

46. Sounding Book.—For keeping the field notes of sounding work, three forms of field books are used; these are called, respectively, **sounding book, tide book,** and **angle book.** The sounding book, the form for which is shown in Form 1, is used by the recorder in the sounding boat. The first three columns contain, respectively, the number, the time, and the observed depth for each sounding; this information is obtained and entered by the recorder while in the sounding boat. The next two columns contain the reduction for tide and the reduced depth for each sounding; these are filled out in the office from the data obtained from the tide book, if the soundings are made in tidal waters. If the soundings are not made in tidal waters, no reductions are needed and these two columns are left blank. In the column headed Remarks should be entered the information obtained by the leadsman relative to the

character of the bottom, and also such information about the sounding ranges, the intervals between successive soundings, etc., as may be desirable. If a lead line is used in making the soundings, any errors in its length should be noted in this column, in order that proper corrections may be made to the observed depths.

FORM 1—SOUNDING BOOK

Soundings off Cape Charles
June 10, 1903

Johnson, Recorder
Kennedy, Leadsman

No.	Time	Soundings Feet	Reduction for Tide	Reduced Soundings Feet	Remarks
1	10:30	4.2			Range 13.
2		5.2			Soundings num-
3	10:31	8.2			bered west from
4		7.2			shore and made at
5	10:32	3.7			$\frac{1}{2}$-minute intervals.
6		3.3			Flag dropped at
7	10:33	3.8			1-minute intervals.
8		4.3			Bottom from sound-
9	10:34	2.8			ing No. 1 to 5, sand;
10		3.3			No. 6 to 8, shells.

47. Tide Book.—The tide book, shown in Form 2, is used by the tide-gauge reader; it contains the readings of the tide gauge at regular intervals of time and the time that each reading is taken. The direction and force of the wind also are usually noted and entered. From these notes, the proper reduction for tide can be obtained for each sounding, and the soundings can all be reduced to the plane of reference.

48. Angle Books.—Forms 3 and 4 show the form for the angle book that is used by the transit or sextant observer to record the angular measurements made in locating the

soundings. In the field he enters in the angle book the time and the observed angle for each sounding to be located. These are entered in the second and third columns, respectively, the first column being left blank until the observer obtains from the sounding book the numbers corresponding to the times of the observed soundings. Each observer is provided with an angle book in which he enters the field notes in the manner described. When the soundings are

FORM 2—TIDE BOOK

Observations of Tides at Cape Charles Gauge
June 10, 1903 J. Mason, Observer

Mean Time of Observation		Reading of Staff Gauge	Wind		Remarks
Hours	Min.	Feet	Direction	Force	
10	30	1.2	N W	Moderate	Gauge fastened to
10	35	1.3			pile at S. E. corner
10	40	1.4			of lighthouse wharf.
10	45	1.5			Tide rising.
10	50	1.6	W		Zero of gauge at
10	55	1.7			mean low water.
11	00	1.8			

located by time intervals and no angular measurements are made, the sounding book constitutes a complete office record of the soundings after the tide-gauge readings are obtained from the tide book and the reductions for the soundings have been entered.

49. Office Record.—When soundings are located by transit or sextant angles, a complete office record is obtained by combining the field notes in the manner shown in Form 5. The notes there shown are those given in Forms 1, 2, 3, and 4.

FORM 3—ANGLE-BOOK

Survey of Channel off Cape Charles, June 10, 1903
Observer No. 1, R. Briggs
Young & Sons' Transit No. 1612

No.	Time	Angle	Object Observed	Station Occupied	Remarks
1	10:30	41° 18′	Signal flag on launch	Transit Sta. A, at S.	Instrument set with vernier at
3	10:31	45° 00′		end of base line	zero when telescope points
5	10:32	49° 00′		on Cape Charles.	to Sta. B. Angles read to
7	10:33	54° 33′			the left from A–B. Sound-
9	10:34	61° 05′			ings on range 13, beginning
					at shore and running west.

FORM 4—ANGLE BOOK

Survey of Channel off Cape Charles, June 10, 1903 Observer No. 2, J. Smith, Buff & Berger Transit No. 2840					
No.	Time	Angle	Object Observed	Station Occupied	Remarks
1	10:30	43° 05′	Signal flag on launch	Sta. B on E. side of	Vernier at zero when telescope
3	10:31	49° 27′		entrance to bay,	points to Sta. A. Angles
5	10:32	57° 15′		opposite light-	read to right. Soundings on
7	10:33	64° 42′		house.	range 13, beginning near
9	10:34	73° 05′			shore and running west.

Form 5—Office Record

Survey of Channel off Cape Charles, June 10, 1903 C. F. Johnson, Recorder T. Kennedy, Leadsman						Observer No. 1, Briggs, on Sta A, zeros on Sta B Observer No. 2, Smith, on Sta. B, zeros on Sta. A		
No.	Time	Soundings Feet	Reduced for Tide	Reduced Soundings Feet	Character of Bottom	Angles and Ranges		Remarks
						No. 1, Range 13	No. 2, Range 13	
1	10:30	4.2	1.2	3	Sand	41° 18′	43° 05′	
2		5.2	1.2	4				
3	10:31	8.2	1.2	7		45° 00′	49° 27′	
4		7.2	1.2	6				
5	10:32	3.7	1.2	$2\frac{1}{2}$	Sand	49° 00′	57° 15′	
6		3.3	1.3	2	Shells			
7	10:33	3.8	1.3	$2\frac{1}{2}$		54° 33′	64° 42′	
8		4.3	1.3	3	Shells			
9	10:34	2.8	1.3	$1\frac{1}{2}$		61° 05′	73° 05′	
10		3.3	1.3	2				

Form 6

Survey of Midland Canal May 2, 1901				Johnson, Inst. Williams, Rod.			Jones Taylor } Assistants				Gurley Level No. 2916	
Sta.	Eleva.	Elevation of Water Surf	Remarks	Soundings								
128	52.0	50.0	The water surface is used as the plane of reference.	$\frac{0.0}{2.0}$	$\frac{3.0}{5.0}$	$\frac{6.0}{10.0}$	$\frac{6.5}{13.0}$	$\frac{6.8}{18.0}$	$\frac{6.2}{28.0}$	$\frac{7.4}{38.0}$	$\frac{4.0}{44.0}$	$\frac{0.0}{48.0}$
129	52.5	50.0	Numerator equals the depth, in feet. Denominator equals the dist. from sta.	$\frac{0.8}{3.0}$	$\frac{3.0}{6.0}$	$\frac{6.5}{12.0}$	$\frac{7.0}{15.0}$	$\frac{7.4}{20.0}$	$\frac{6.3}{30.0}$	$\frac{7.0}{39.0}$	$\frac{3.0}{46.0}$	$\frac{0.0}{40.0}$
			Distances measured from left to right.									

50. Form 6.—When soundings are located by means of a wire or rope stretched across the stream, the field notes may be kept in the manner shown in Form 6. The notes there given represent sounding measurements across a canal at Stations 128 and 129 of the shore survey or traverse along the canal bank. The left-hand page of the notebook contains the number of the shore station in the first column, the ground elevation in the second column, and the elevation of the water surface in the third column. In the column headed Remarks is given all necessary information concerning the details of measurements, stage of water, etc. On the right-hand page of the notebook are given the sounding measurements. These are expressed in the form of fractions, the numerator designating the depth, in feet, and the denominator the distance from the shore station for each sounding. Thus, the fraction $\frac{6.0}{10.0}$ represents a depth of 6.0 feet at a distance of 10.0 feet from the shore station. The notes given in Form 6, when platted to scale on cross-section paper, show the cross-section of the canal for each station for which sounding notes are given.

PLATTING THE SOUNDINGS

51. General Methods.—Soundings are platted in various ways, according to the methods by which they are located. When located by ranges or courses, they are platted as follows: Each range or line on which soundings have been made is first platted to scale, in pencil, in the proper position on the map or chart. Then, the distances between the soundings on that range and the distance of each sounding from the end of the range being known, these distances are scaled on the pencil line and the position of each sounding as thus located is marked by a dot.

When the soundings are located by means of two transits on shore, as described in the third method, the base line is platted to scale in its proper position on the map and from each of its ends pencil lines are drawn making angles with the base equal to the angles measured in locating the soundings. The lines thus drawn represent the lines of

sight from the ends of the base to the positions of the soundings, and their directions are determined by the measured angles given in the notes. The intersection of corresponding lines drawn from the opposite ends of the base, representing the lines of sight of two simultaneous observations to a given sounding, will locate the sounding on the map.

In Fig. 33, *A B* represents a given base line as platted. The dotted lines *A–1*, *A–2*, *A–3*, and *B–1*, *B–2*, *B–3*, represent the directions of lines of sight to the soundings *1*, *2*, *3*. These lines are laid off at the observed angles as recorded in the notes. The intersections of the corresponding lines drawn from opposite ends of the base line give the locations of the respective soundings.

Soundings that have been located by this method can also be platted in the following manner: The distances from

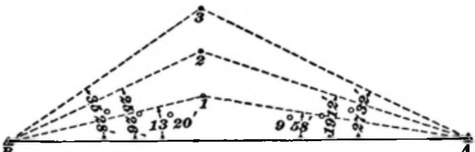


Fig. 33

each sounding to the respective extremities of the base line are calculated from the field notes. Then, the base line having been platted to scale in its proper position on the map, from its extremities as centers and with radii respectively equal to the two corresponding distances calculated for a given sounding, the arcs of two circles are drawn lightly in pencil with a pair of compasses. The intersection of the two arcs is the location of the sounding.

This method of platting is not nearly so expeditious as the method by the intersection of straight lines drawn from the instrument points, since the former method involves the calculation of two distances for each sounding, whereas the latter requires no calculation. It is sometimes valuable as a check, however, or to apply as a test in case of doubt regarding the position of a sounding.

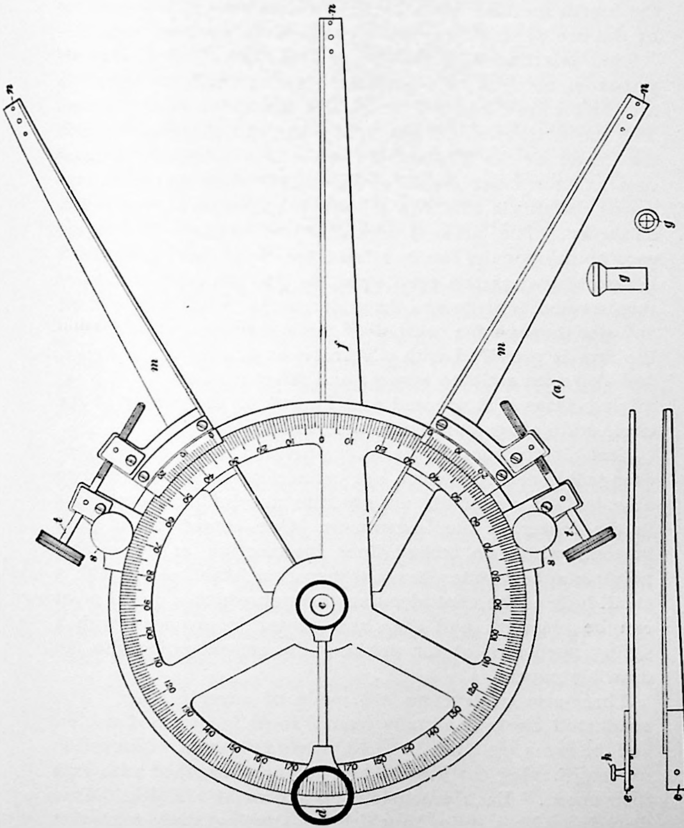


FIG. 34

52. The Three-Arm Protractor.—If the soundings have been located by sextant observations, as described in the fourth method, the most convenient way to plat them is by the use of what is called a **three-arm protractor**.

This instrument, which is sometimes called a **station pointer**, consists of a graduated metal circle, to which is attached a fixed arm *f*, Fig. 34, and two movable arms *m*, *m*. The movable arms revolve around a central point, which is the center of the graduated circle. One edge of the fixed arm and the inner edges of the movable arms extend outwards radially in line with the central point, and are beveled, as shown. The circle is divided into 360 degrees, with the zero point opposite the beveled edge of the fixed arm, which is also known as the **zero arm**, so that the prolongation of this beveled edge passes through the zero mark on the circle and also through the center *c* of the instrument. Each movable arm is provided with a vernier, as shown in the figure, and also with a clamp screw *s* and tangent screw *t*. A magnifying glass *d* is pivoted and hinged to the center of the circle and swings parallel to the graduations.

Each instrument is usually provided with three interchangeable centers, which are cylindrical in form, as shown at *g*. Each center fits snugly into a cylindrical opening *c* in the center of the instrument. One center has a glass bottom, with two etched lines intersecting at the central point; another center has a transparent horn bottom with a small hole at the central point, through which a pencil point can be inserted; and the third center is provided with a spring needle point for pricking the central point into the drawing paper.

Three-arm protractors are made of several sizes. The graduated circle is usually from 5 to 6¼ inches in diameter, and the arms are from 15 to 18 inches in length. Extensions for lengthening the protractor arms are furnished with each instrument. Each extension, as shown at *e*, *e*, Fig. 34, is provided with a splice to which are attached three studs that fit into corresponding holes at the end *n* of the protractor arm. After fitting the studs in place, the extension is secured

to the protractor arm by tightening the thumbscrew *h*. The extensions are used when soundings are to be platted that are beyond the reach of the regular protractor arms.

Before using a three-arm protractor, it is a good plan to carefully test the alinement and centering of the arms. To do this, place the protractor on the drawing board and draw lines along the straight edges of the three arms, then remove the protractor and prolong the lines inwards, noticing whether the three lines intersect in a common point. The operation should be repeated several times with the arms in different positions. If the three lines intersect in a common point for all positions of the arms, they may be considered to be truly centered.

53. The three-arm protractor is used almost exclusively for the purpose of platting soundings that have been located by sextant angles from the sounding boat. The way of using it is as follows: The movable arms of the protractor are set at the marks on the graduated circle designating the two sextant angles for any given sounding, and are firmly clamped. The instrument is then placed on the chart in such a position that the beveled edges of the three arms will pass through the platted positions of the three fixed points. This is done by placing the instrument on the paper with the beveled edge of the fixed or zero arm passing through the middle point, and sliding it around on the paper until the beveled edges of the two clamped arms also pass through the two respective outside points. The center of the instrument will then represent the position of the sounding. This point is marked by a pencil dot if a horn center is used, or pricked on the chart if a needle-point center is in the protractor at the time.

54. The Tracing-Paper Method.—When no three-arm protractor is available, soundings that have been located by sextant observations can be platted by means of a piece of tracing paper. Three lines are drawn on tracing paper in such positions as to intersect at a common point and include the two angles measured for any given sounding, the middle

line forming a side of each angle. Then, to locate the sounding, the tracing paper is placed on the map in such a position that the three lines will pass through the platted positions of the three fixed points. The intersection of the three lines will then be the position of the sounding, which is pricked through the tracing on the map or chart.

HYDROGRAPHIC MAPS AND CHARTS

55. Maps or charts of hydrographic surveys should be drawn in accordance with the principles stated in *Mapping*, Parts 1 and 2. An outline map of a river or lake survey should show the lines and angles of the outline survey, and the triangulation stations, if any. It should also show the shore line and such details of the adjacent topography as may be considered necessary.

A complete hydrographic map of a river, lake, or reservoir should show, in addition to the outline of the water surface and the adjacent topography, the form or contour of the river bed or of the submerged portion of the containing valley or basin. In order to do this effectively, lines of equal depth should be drawn; these lines show the contours of the submerged area and correspond to contour lines on a topographical map. They are located and drawn on a hydrographic map in the following manner: The soundings are platted, the position of each sounding is indicated on the map by a small dot, and the depth of each sounding is written directly over its location on the map. The lines of equal depth or the subaqueous contours are then located and drawn according to the method described in *Mapping*, Part 2, for platting contours. The contour interval will vary according to the importance of the survey, the frequency of the soundings, and the object for which the survey is made.

56. Navigation Charts.—A navigation chart of a river, lake, harbor, or other navigable body of water should show, in addition to the shore line and the adjacent

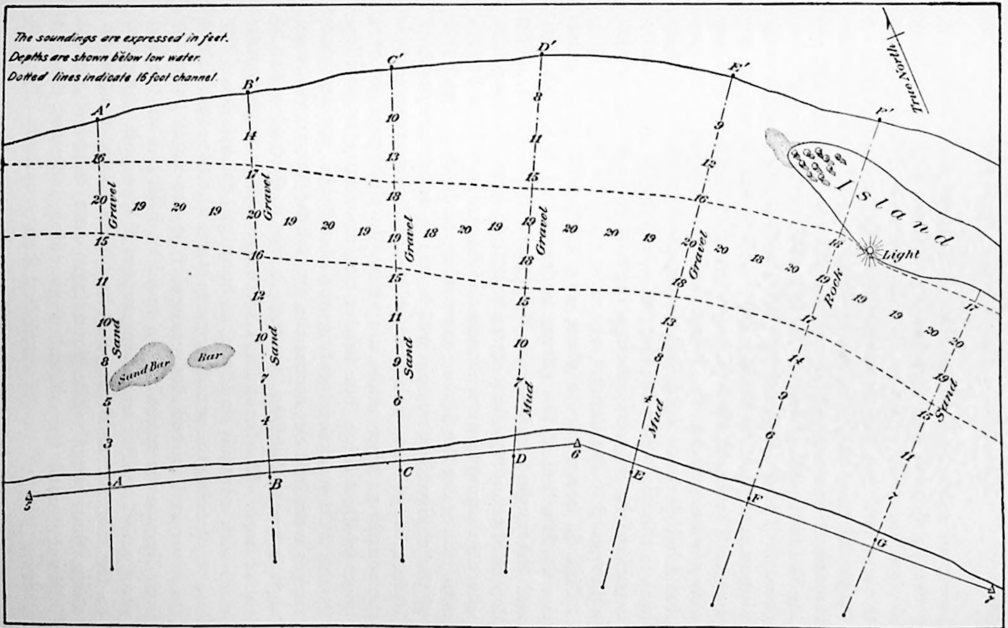

The soundings are expressed in feet.
Depths are shown below low water.
Dotted lines indicate 16 foot channel.
True North
Island
Light
Sand Bar
Bar
Gravel
Sand
Mud
Rock
A'
B'
C'
D'
E'
F'
A
B
C
D
E
F
G
5
6
7

FIG. 35

topography, the position of the navigable channel and of all rocks, sand bars, reefs, sunken wrecks, or other obstacles to navigation. In a chart of a navigable river, it is customary to show both banks and such of the adjacent topography as is desired; also the positions of islands and of such obstacles to navigation as may exist. Contour lines are drawn showing the navigable channel and the outlines of sand bars and shoals. The depths of other parts of the stream or body of water are written in the positions where the soundings are made. The depths are usually expressed in feet, and are always so expressed in shallow water. In some cases fractional half feet are used, but not smaller fractions. Soundings in the sea where the depths exceed 18 feet are usually expressed in fathoms, though there is no uniform practice with regard to the depth.

Fig. 35 represents a chart of a portion of a navigable river. The line of the survey is drawn in a light full line, and the angle stations are designated by numbers. The depths of soundings, in feet, are written in figures at the places where the soundings were made, and the character of the bottom is written under the figures expressing the depths. The limits of the navigable channel, which has a minimum depth of 16 feet, are shown by the dotted lines. The ranges on which the soundings were made are shown in the figure for purposes of illustration, but this is not customary in practice. In the field work of locating the soundings, the sounding ranges were laid out across the axis of the stream and range signals were established on both banks to fix the position of each range. The soundings were located by angles measured with a transit on shore, as described in Art. 38. The boat containing the sounding party started on the south side of the river and moved along range *A* to the north bank; the soundings were made at the required intervals as the boat progressed. After traversing range *A*, the boat proceeded to the north end of range *B*, and then moved southwards along that range while soundings were taken at regular intervals, until the depth of sounding indicated that the deepest part of the channel had been reached. Then, in

order to sound the deepest part of the channel, the boat was headed in a direction approximately at right angles to the range *B*, and propelled at a uniform speed toward the range *A*, soundings being made at regular intervals of time until the boat reached that range. The time taken to traverse the distance between the two ranges was noted, as was also the number of intermediate soundings and the time interval between soundings. Then the distance between the two ranges, the time taken by the boat to traverse this distance, the number of intermediate soundings, and the time interval between successive soundings all being known, the locations of the intermediate soundings were interpolated in the manner described in Art. **36.**

After the channel soundings between ranges *A* and *B* were made, the boat returned to range *B*, and sounding work on that range was resumed from the place indicated by signal from the transitman on shore. After range *B* was sounded, the boat proceeded to range *C* and moved northward along that range to the deepest part of the channel, then along the deep channel westwards to range *B*, soundings being made at regular intervals in the manner just described. The boat then returned to range *C* and proceeded northwards until the entire range had been sounded. It then proceeded to range *D* and sounded southwards along that range to the south bank, and so on back and forth across the river until all the ranges were sounded. Successive ranges were traversed in opposite directions and side trips were made for the channel soundings between each two adjacent ranges. For locating soundings made on ranges *A* and *B*, the observer was at the instrument point designated as Station 5, with the vernier of his transit at zero when the telescope was directed toward Station 6. After range *B* was sounded the transit was moved to Station 6 and set up over that station, from which soundings made on the remaining ranges were located. For locating soundings on ranges *C* and *D*, the vernier was set at zero when the telescope pointed to Station 5; and for locating soundings on ranges *E* and *F*, the vernier was set at zero when the telescope pointed to Station 7.

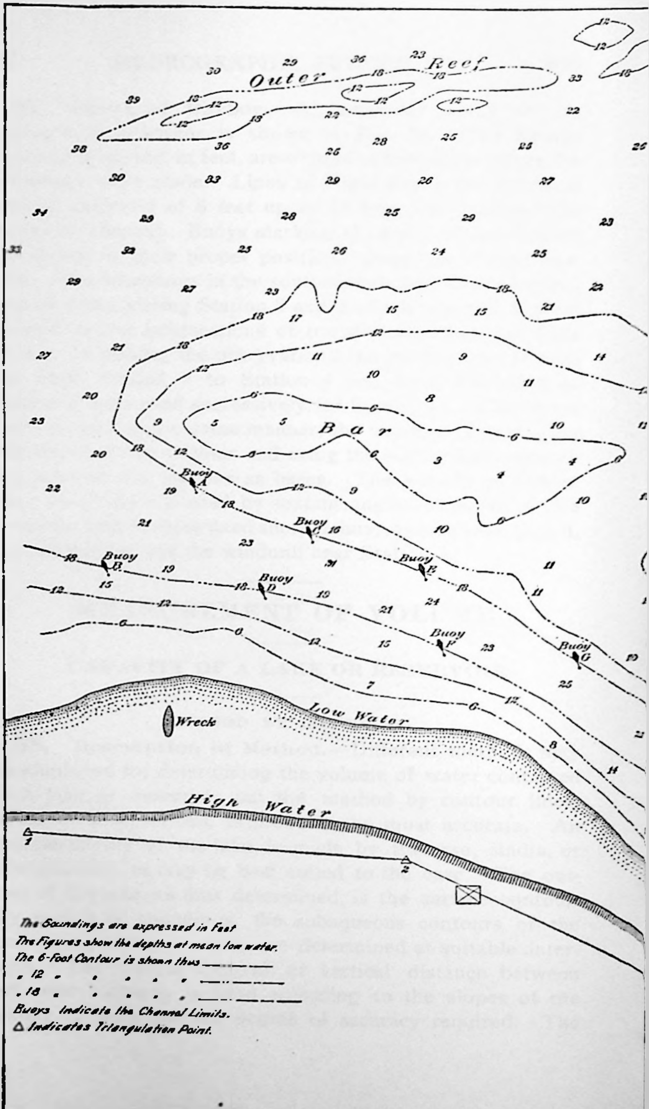
57. Chart of Harbor.—A navigation chart of the entrance to a harbor is shown in Fig. 36. The figures expressing depths, in feet, are written at the places where the soundings were made. Lines of equal depth are drawn at vertical intervals of 6 feet up to 18 feet, the depth of the navigable channel. Buoys marking the limits of the channel are shown in their proper positions along the 18-foot contour. The soundings in the southeastern part of the harbor, east of a line joining Station 3 and the buoy marked *H* were located by the intersections of transit lines observed from shore. In making the observations, the survey lines extending from Station 3 to Station 4 and from Station 4 to Station 5 were used successively for base lines. The buoys were located in the same manner, the transitmen occupying successive survey stations and using the survey lines extending between the stations as bases. The soundings farther from shore were located by sextant angles observed in the sounding boat to three fixed shore points; namely, the church, the lighthouse, and the windmill near Station 3.

MEASUREMENT OF VOLUME

CAPACITY OF A LAKE OR RESERVOIR

METHOD BY CONTOURS

58. Description of Method.—Different methods may be employed for determining the volume of water contained in a lake or reservoir, but the method by contour lines, about to be described, is probably the most accurate. An outline survey of the lake is made by traverse, stadia, or triangulation, as may be best suited to the case. The outline of the lake, as thus determined, is the surface contour. By means of soundings, the subaqueous contours of the containing valley or basin are determined at suitable intervals. The contour interval, or vertical distance between adjacent contours, is fixed according to the slopes of the valley or basin and the degree of accuracy required. The



Outer Reef
Bar
Buoy A
Buoy B
Buoy C
Buoy D
Buoy E
Buoy F
Buoy G
Wreck
Low Water
High Water
The Soundings are expressed in Feet
The Figures show the depths at mean low water.
The 6-Foot Contour is shown thus
" 12 " " " " "
" 18 " " " " "
Buoys Indicate the Channel Limits.
△ Indicates Triangulation Point.

Point of Rock
True North
Pinnacle Rock
Low Water
High Water
Church
Wharf
Buoy
Buoy
Lighthouse

notes thus obtained are then platted, and a map is made showing the outline of the water surface and the several contour lines.

The solid figure included between any two adjacent contours will resemble a prismoid, whose parallel end surfaces are the surfaces enclosed by the respective contour lines, and whose perpendicular length or height is the contour interval. The area of the water surface and the area enclosed by each contour line can be determined from the plat by any of the methods for finding the areas of irregular figures described in *Trigonometry*, Part 2. When the areas enclosed by the various contours, which form the end areas of the several prismoids, are known, the volume of each prismoid can be found approximately by multiplying one-half the sum of its end areas by its height. The sum of the volumes of the several prismoids will be the volume of water in the lake. This is what is known as the **end-area method** of calculating volumes.

59. Volume by End-Area Method.—Suppose that Fig. 37 represents the plat of a lake whose capacity is

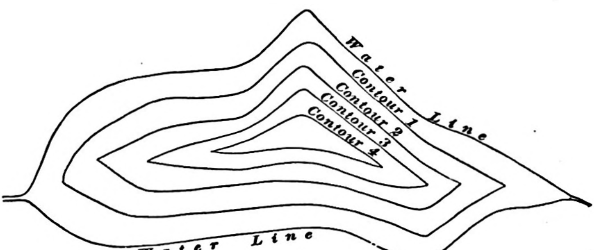


Fig. 37

required. Let A_0, A_1, A_2, A_3, and A_4 denote the areas bounded by the water-line and by the contours *1*, *2*, *3*, and *4*. respectively; let v_{0-1}, v_{1-2}, v_{2-3}, and v_{3-4} denote, respectively, the volumes of the prismoids included between the water surface and the first contour, and between the successive contours; also, let h denote the contour interval, and V the

total volume. By the method of average end areas, the approximate volumes of the several prismoids are

$$v_{0-1} = \frac{A_0 + A_1}{2} \times h$$

$$v_{1-2} = \frac{A_1 + A_2}{2} \times h$$

$$v_{2-3} = \frac{A_2 + A_3}{2} \times h$$

$$v_{3-4} = \frac{A_3 + A_4}{2} \times h$$

The total volume of the lake or reservoir is equal to the sum of the volumes of the several prismoids as expressed by the preceding equations, or

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4} = h\left(\frac{A_0}{2} + A_1 + A_2 + A_3 + \frac{A_4}{2}\right)$$

In order to express this as a general formula applicable to any number of contours, it may be written in the form

$$V = h\left(\frac{A_0}{2} + \Sigma A_m + \frac{A_n}{2}\right)$$

In this formula,

A_0 = area included by surface contour;
A_n = area included by lowest contour;
ΣA_m = sum of areas included by the intermediate contours.

EXAMPLE.—Suppose that in Fig. 37 the contour interval is 5 feet and that the areas enclosed by the several contours are as follows: A_0 = 13,350 square feet, A_1 = 8,100 square feet, A_2 = 4,280 square feet, A_3 = 1,925 square feet, and A_4 = 520 square feet; find the volume of water in the lake, in cubic feet, by the end-area method.

SOLUTION.—By substituting the given values in the formula, tha volume of water is found to be

$$V = 5 \times \left(\frac{13,350}{2} + 8,100 + 4,280 + 1,925 + \frac{520}{2}\right) = 106,200 \text{ cu. ft. Ans.}$$

60. Volume by Prismoidal Formula.—If the volumes of the prismoids are calculated by the prismoidal formula, two adjacent prismoids are taken as one prismoid whose height is equal to twice the contour interval. The area included by the contour that lies between the two prismoids

taken is considered the middle area of the prismoid and so used in the formula. By thus combining the first two prismoids of Fig. 37 and applying the prismoidal formula given in *Geometry*, Part 2, the expression for their volume is

$$v_{0-1} + v_{1-2} = \frac{2h}{6}(A_0 + 4A_1 + A_2)$$

The volume of the last two prismoids is

$$v_{2-3} + v_{3-4} = \frac{2h}{6}(A_2 + 4A_3 + A_4)$$

By adding these two expressions, the total volume of the lake is found to be

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4}$$
$$= \frac{h}{3}(A_0 + 4A_1 + 2A_2 + 4A_3 + A_4)$$

In order to express this as a general formula applicable to any number of contours, it may be written in the form

$$V = \frac{h}{3}(A_0 + 4\Sigma A_1 + 2\Sigma A_2 + A_n)$$

In this formula,

A_0 = area included by surface contour;
A_n = area included by lowest contour;
ΣA_1 = sum of areas included by intermediate contours whose subscripts are odd numbers;
ΣA_2 = sum of areas included by intermediate contours whose subscripts are even numbers.

EXAMPLE.—Suppose that in Fig. 37 all values are the same as in the example solved in Art. 59; namely, h = 5 feet, A_0 = 13,350 square feet, A_1 = 8,100 square feet, A_2 = 4,280 square feet, A_3 = 1,925 square feet, and A_4 = 520 square feet; what is the volume of water in the lake, in cubic feet, as determined by the prismoidal formula?

SOLUTION.—By substituting the given values in the formula,

$$V = \frac{5}{3}(13{,}350 + 4 \times 8{,}100 + 2 \times 4{,}280 + 4 \times 1{,}925 + 520)$$
$$= 104{,}217 \text{ cu. ft.} \quad \text{Ans.}$$

61. Construction for Interpolating Contour.—It is evident that the prismoidal formula can be applied to the prismoids in pairs, as just described, only when there is an even number of prismoids. When there is an odd number

of prismoids, the last prismoid may be computed separately by the method of average end areas, or by interpolating a middle contour on the contour map, calculating the area included by it, and then applying the prismoidal formula. The middle contour can be interpolated as follows: The two end contours are platted to the same scale, preferably on cross-section paper, the smaller inside the larger, making the two figures concentric as nearly as practicable. A third contour is then drawn in such position that each point will be midway between the corresponding points of the inner and outer figures. This interpolated contour can be sketched in largely by the eye, aided to such an extent as may be desired by measurements on lines drawn radially from a point approximately in the center of the figure. The area of the surface included by this interpolated contour may be taken as the middle area of the prismoid. This area can be easily determined by means of the cross-section paper or by the planimeter.

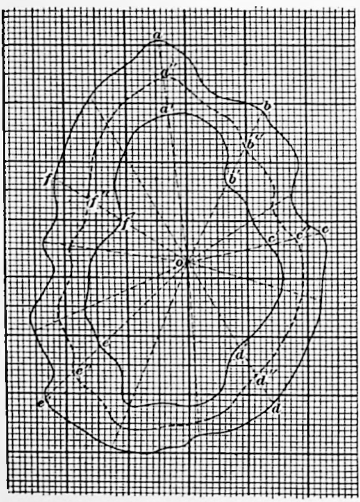


Fig. 38

Thus, in Fig. 38, *a b c d e f* and *a' b' c' d' e' f'* represent the contours including, respectively, the upper and the lower base of a prismoid, as platted on cross-section paper. The contour represented by the dotted line *a'' b'' c'' d'' e'' f''* lies midway between the boundaries of the two bases. This contour

is constructed or interpolated in the following manner: The point o is chosen as the center and the radial lines oa, ob, oc, etc. drawn to the outer contour; a', b', c', etc. are the points where these lines cross the inner contour. By measurement, the point a'' is located on the radial line oa midway between a and a'; in like manner, b'' is located midway between b and b'; c'' is located midway between c and c', etc. The contour is then sketched through the points a'', b'', c'', d'', e'', and f'', as represented by the dotted line. The area of the surface enclosed by this interpolated contour is then determined by counting the squares of the cross-section paper, and is taken as the middle area of the prismoid.

EXAMPLES FOR PRACTICE

1. Suppose that the areas bounded by the water-line of a lake and by contours 1, 2, 3, 4, and 5 are as follows: A_0 = 15,450 square feet, A_1 = 10,240 square feet, A_2 = 8,360 square feet, A_3 = 7,730 square feet, A_4 = 6,890 square feet, and A_5 = 5,240 square feet. If the contour interval is 10 feet, calculate the volume of water in the lake, in cubic feet, by the end-area method. Ans. 435,650 cu. ft.

2. Suppose that the areas bounded by the water-line of a lake and by contours 1, 2, 3, 4, 5, and 6 are as follows: A_0 = 14,320 square feet, A_1 = 10,280 square feet, A_2 = 9,360 square feet, A_3 = 7,480 square feet, A_4 = 5,780 square feet, A_5 = 4,760 square feet, and A_6 = 3,250 square feet. If the contour interval is 5 feet, calculate the volume of water in the lake by the prismoidal formula. Ans. 229,880 cu. ft.

METHOD BY PARALLEL CROSS-SECTIONS

62. Description of Method.—The following is also a good method for determining approximately the capacity of a lake or reservoir: A survey is made to determine the outline of the water surface, which is platted accurately to scale. Then, at selected points, parallel ranges are laid out across the lake, dividing its surface into trapezoids, as illustrated in Fig. 39. If the shores of the lake are irregular, the ranges are so located that straight lines connecting the points where the adjacent ranges intersect the shore line will be as much inside as outside of the water-line. By the

aid of the plat this can usually be done with a reasonable degree of accuracy. The small irregular areas included between the straight line and the water-line will then approximately balance, and it will be sufficiently accurate to consider the lake boundary as straight between adjacent ranges. The ranges having been located, soundings are made along them and the cross-section of the lake is determined on each range. The cross-sections are platted, as shown in Fig. 39, and the area of each cross-section is computed.

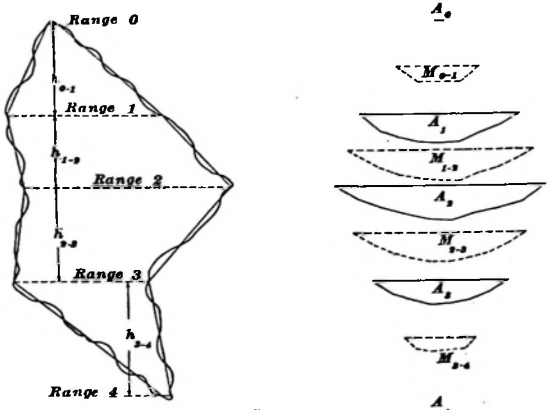


FIG. 39

The lake basin is thus divided into prismoids, whose bases are the cross-sections, and whose altitudes are the perpendicular distances between adjacent ranges, and the capacity of the lake is equal to the sum of the volumes of the prismoids.

63. Volume by End-Area Method.—The approximate capacity of the lake or reservoir can be calculated by the method of average end areas in the following manner: Let A_0, A_1, A_2, A_3, and A_4 denote, respectively, the areas of the cross-sections on the parallel ranges designated in Fig. 39 as *Range 0*, *Range 1*, *Range 2*, *Range 3*, and *Range 4*, respectively. Also, let h_{0-1}, h_{1-2}, h_{2-3}, and h_{3-4} denote, respectively,

the perpendicular distances between the adjacent parallel ranges *1*, *2*, *3*, and *4*, as shown in the figure, and let v_{0-1}, v_{1-2}, v_{2-3}, and v_{3-4} denote the volumes of the corresponding prismoids. The end range *0* is merely a short straight line that represents the end of the lake and corresponds somewhat to the cutting edge of a wedge in the prismoid included between it and the cross-section on the adjacent range *1*. The same is true of the end range *4* with respect to the prismoid included between it and the cross-section on range *3*. The cross-sections on *0* and *4* thus consist of a straight line merely, and each of the areas A_0 and A_4 is zero. These areas should therefore be taken at zero in computing the volumes of the two end prismoids. By the end-area method, the expressions for the volumes of the several prismoids are

$$v_{0-1} = \frac{A_0 + A_1}{2} \times h_{0-1}$$

$$v_{1-2} = \frac{A_1 + A_2}{2} \times h_{1-2}$$

$$v_{2-3} = \frac{A_2 + A_3}{2} \times h_{2-3}$$

$$v_{3-4} = \frac{A_3 + A_4}{2} \times h_{3-4}$$

and the total volume of the lake is

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4} = \tfrac{1}{2}\left[(A_0 + A_1)h_{0-1} + (A_1 + A_2)h_{1-2} + (A_2 + A_3)h_{2-3} + (A_3 + A_4)h_{3-4}\right]$$

This formula applies to Fig. 39 or to any lake or reservoir for which the measurements are made on five parallel ranges; its application to such a case will be clearly understood. In order to make the formula applicable to measurements made on any number of ranges, it may be written in the form

$$V = \tfrac{1}{2}\left[(A_0 + A_1)h_{0-1} + (A_1 + A_2)h_{1-2} \ldots (A_m + A_n)h_{m-n}\right]$$

In this formula, A_n denotes the area of the cross-section on the last range, and A_m that on the next to the last range.

EXAMPLE.—Suppose that the areas of the several cross-sections of the lake shown in Fig. 39, as measured on the ranges, are: $A_0 = 0$,

A_1 = 4,256 square feet, A_2 = 6,322 square feet, A_3 = 3,130 square feet, and A_4 = 0; also, that the perpendicular distances between ranges are: h_{0-1} = 250 feet, h_{1-2} = 192 feet, h_{2-3} = 256 feet, and h_{3-4} = 310 feet; what is the capacity of the lake, in cubic feet, as calculated by the end-area method?

SOLUTION.—By substituting the given values in the formula, the operations, in detail, are as follows:

$$\begin{aligned}
(A_0 + A_1)h_{0-1} &= (0 + 4{,}256) \times 250 \;.\;.\; = & 1\,064\,000 \\
(A_1 + A_2)h_{1-2} &= (4{,}256 + 6{,}322) \times 192 = & 2\,030\,976 \\
(A_2 + A_3)h_{2-3} &= (6{,}322 + 3{,}130) \times 256 = & 2\,419\,712 \\
(A_3 + A_4)h_{3-4} &= (3{,}130 + 0) \times 310 \;.\;.\; = & \underline{970\,300} \\
& & 2)\overline{6\,484\,988} \\
& V = & 3\,242\,494 \text{ cu. ft.} \quad \text{Ans.}
\end{aligned}$$

64. Interpolating Middle Cross-Section.—When the volume of a lake or reservoir is determined by measuring cross-sections on parallel ranges, the ranges cannot usually be located advantageously at uniform intervals, but must be located in such positions as will determine most accurately the form of the lake or reservoir. Consequently, two adjacent prismoids cannot be considered as one prismoid whose length is equal to the aggregate length of the two, and the intervening section considered as the middle section in applying the prismoidal formula, as in the preceding method. For, since the two prismoids are not of the same length, the intervening section is not the middle section. Hence, when the prismoidal formula is applied to this method, the middle area of each prismoid is determined by platting its two end cross-sections together on cross-section paper and interpolating a middle cross-section. The area of the interpolated cross-section is then determined and taken as the middle area of the prismoid. The method of interpolating the middle cross-section is similar to that for interpolating the middle contour explained in Art. **61**, but is even more simple.

Let LMN and OPQ, Fig. 40, represent the bottom profiles of the measured cross-sections on two adjacent ranges. A third line RST is drawn in such position that each point is midway between corresponding points in the other two lines. In most cases, it will be sufficiently accurate to locate

points on the interpolated line midway between corresponding points that are located by soundings on the other two lines. Thus, the point S is located midway between M and P, which are points that have been located by soundings. The point R is located in the horizontal surface line midway between O and L, and the point T in the surface line midway between N and Q. The surface lines of the three cross-sections coincide between L and N. The area of the interpolated cross-section can now be determined by counting the squares of the cross-section paper, and this is taken as the middle area of the prismoid whose end areas are the areas of the cross-sections $L\ M\ N$ and $O\ P\ Q$. The middle area of the two end prismoids is determined in a similar manner. In this case the end cross-section is a straight line, and points

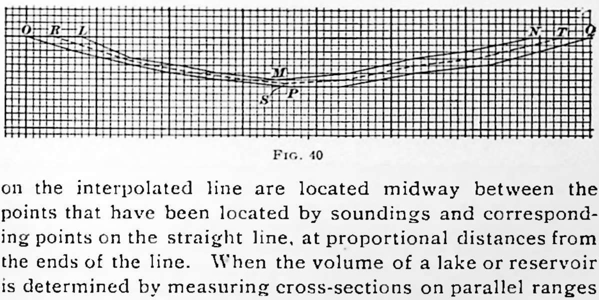


FIG. 40

on the interpolated line are located midway between the points that have been located by soundings and corresponding points on the straight line, at proportional distances from the ends of the line. When the volume of a lake or reservoir is determined by measuring cross-sections on parallel ranges and applying the prismoidal formula, the middle area of each prismoid is determined in the manner described.

65. Volume by Prismoidal Formula.—When the area of the middle cross-section has been determined, the volume of each prismoid can be determined by applying the prismoidal formula in the usual manner. The sum of the volumes of the several prismoids is the volume of water contained in the lake or reservoir.

In Fig. 39, let M_{0-1}, M_{1-2}, M_{2-3}, and M_{3-4} denote the middle areas of the prismoids whose altitudes are h_{0-1}, h_{1-2}, h_{2-3}, and h_{3-4}, respectively. Then, by applying the prismoidal

formula, the volumes of the several prismoids are found to be as follows:

$$v_{0-1} = \frac{h_{0-1}}{6}(A_0 + 4M_{0-1} + A_1)$$

$$v_{1-2} = \frac{h_{1-2}}{6}(A_1 + 4M_{1-2} + A_2)$$

$$v_{2-3} = \frac{h_{2-3}}{6}(A_2 + 4M_{2-3} + A_3)$$

$$v_{3-4} = \frac{h_{3-4}}{6}(A_3 + 4M_{3-4} + A_4)$$

Then, $V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4}$,

or $V = \frac{1}{6}[(A_0 + 4M_{0-1} + A_1)h_{0-1} + (A_1 + 4M_{1-2} + A_2)h_{1-2} + (A_2 + 4M_{2-3} + A_3)h_{2-3} + (A_3 + 4M_{3-4} + A_4)h_{3-4}]$

In order to express this as a general formula applicable to any number of cross-sections, it may be written in the form

$$V = \frac{1}{6}[(A_0 + 4M_{0-1} + A_1)h_{0-1} + (A_1 + 4M_{1-2} + A_2)h_{1-2} + \ldots (A_m + 4M_{m-n} + A_n)h_{m-n}]$$

In this formula,

A_n = area of last section;

A_m = area of next to last section;

M_{m-n} = area of middle section;

h_{m-n} = perpendicular distance between A_m and A_n.

EXAMPLE.—Suppose that all values are the same as in the example solved in Art. 63; namely, $A_0 = 0$, $A_1 = 4{,}256$ square feet, $A_2 = 6{,}322$ square feet, $A_3 = 3{,}130$ square feet, $A_4 = 0$, $h_{0-1} = 250$ feet, $h_{1-2} = 192$ feet, $h_{2-3} = 256$ feet, and $h_{3-4} = 310$ feet; and suppose, also, that the areas of the interpolated middle sections are: $M_{0-1} = 1{,}107$ square feet, $M_{1-2} = 5{,}498$ square feet, $M_{2-3} = 4{,}536$ square feet, and $M_{3-4} = 863$ square feet; what is the capacity of the lake, in cubic feet, as calculated by the prismoidal formula?

SOLUTION.—By substituting the given values in the formula, the operations, in detail, are as follows:

$(A_0 + 4M_{0-1} + A_1)h_{0-1} = (0 + 4 \times 1{,}107 + 4{,}256) \times 250$ . . . =	2 1 7 1 0 0 0
$(A_1 + 4M_{1-2} + A_2)h_{1-2} = (4{,}256 + 4 \times 5{,}498 + 6{,}322) \times 192$ . . . =	6 2 5 3 4 4 0
$(A_2 + 4M_{2-3} + A_3)h_{2-3} = (6{,}322 + 4 \times 4{,}536 + 3{,}130) \times 256$ . . . =	7 0 6 4 5 7 6
$(A_3 + 4M_{3-4} + A_4)h_{3-4} = (3{,}130 + 4 \times 863 + 0) \times 310$ . . . . . . =	2 0 4 0 4 2 0
6)	1 7 5 2 9 4 3 6
$V =$	2 9 2 1 5 7 3 cu. ft. Ans.

EXAMPLES FOR PRACTICE

1. In Fig. 41, which represents a lake, BC, DE, and FG are parallel ranges on which soundings have been taken. The depths of

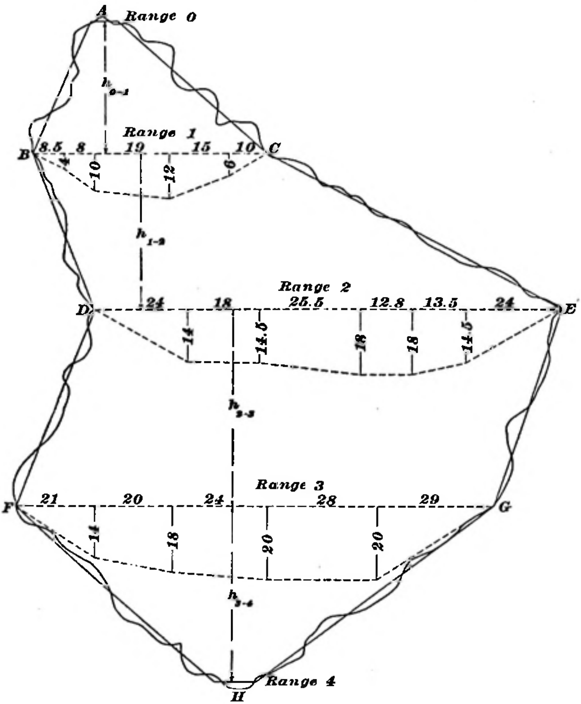


Fig. 41

the soundings and the distances between soundings are indicated in the figure. Using these values, calculate the areas A_1, A_2, and A_3 of the cross-sections on the ranges.

$$\text{Ans.}\begin{cases} A_1 = 447.0 \text{ sq. ft.} \\ A_2 = 1{,}462.7 \text{ sq. ft.} \\ A_3 = 1{,}773.0 \text{ sq. ft.} \end{cases}$$

2. If, in Fig. 41, the distances between adjacent ranges are $h_{0-1} = 35$ feet, $h_{1-2} = 42$ feet, $h_{2-3} = 54$ feet, and $h_{3-4} = 48$ feet, what is the capacity of the lake as determined by the method of average end areas? Ans. 177,840 cu. ft.

3. Calling Range *0*, 8 feet long and Range *4*, 10 feet long, plat the areas of the cross-sections in Fig. 41 on cross-section paper and determine the interpolated middle areas.

$$\text{Ans.} \begin{cases} M_{0-1} = 131.4 \text{ sq. ft.} \\ M_{1-2} = 856.3 \text{ sq. ft.} \\ M_{2-3} = 1{,}626.7 \text{ sq. ft.} \\ M_{3-4} = 487.3 \text{ sq. ft.} \end{cases}$$

4. As determined by the prismoidal formula, what is the capacity of the lake represented in Fig. 41? Ans. 160,480 cu. ft.

CAPACITY OF A VALLEY OR BASIN FOR WATER STORAGE

66. By Contours.—If close results are desired, it is best to make a complete topographical survey of the area to be flooded and construct a contour map of the area, employing the methods described in *Topographic Surveying*. The general method is as follows: The location of the dam for impounding the water having been selected, the elevation of the spillway or overflow is decided on; this determines the height of the water in the basin. The **spillway** is that part of the dam over which the waste water is allowed to flow, and is usually somewhat lower than the crest of the dam. The impounded water will rise to a height corresponding to the elevation of the spillway and will form a pond or lake whose boundary will be a contour line extending around the border of the basin. This line, whose position thus defines the limits of the area overflowed by the water, is called the **flow line**. In Fig. 42 the flow line, which is at the elevation of the spillway, is one contour interval lower than the crest of the dam.

After the survey has been made, the flow line, the successive contours, and the outline of the projected dam are platted, as shown in Fig. 42. The planes of the contours, including that of the flow line, will intersect the face of the dam in a series of horizontal lines, as shown. A line joining the ends of these horizontal lines on the inner face of the dam, at the points where they meet the sides of the valley,

will indicate the inner outline of the base of the dam, or the inner toe of the slope, as shown in Fig. 42. A similar line will indicate the outer toe of the slope. Only the contour lines and flow lines need be considered, however, in determining the capacity of the basin. The areas enclosed by the flow line and the several contour lines are either calculated or measured with a planimeter, and the capacity of the reservoir is determined by the method of average end areas or by the prismoidal formula, as explained in preceding articles.

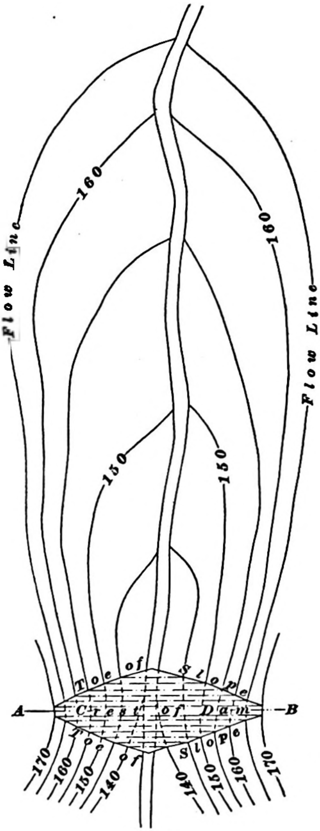


Fig. 42

Example.—Suppose that in Fig. 42 the contour interval is 5 feet and that the areas enclosed by the several contours are as follows: $A_0 = 9,475$ square feet, $A_1 = 7,415$ square feet, $A_2 = 4,175$ square feet, $A_3 = 1,810$ square feet, and $A_4 = 685$ square feet; what is the capacity of the reservoir, in cubic feet, as determined: (*a*) by the end-area method? (*b*) by the prismoidal formula?

Solution.—(*a*) Substituting the given values in the formula in Art. 59,

$$V = 5\left(\frac{9,475}{2} + 7,415 + 4,175 + 1,810 + \frac{685}{2}\right) = 92,400 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) Substituting the given values in the formula in Art. 60,

$$V = \tfrac{5}{3}[9,475 + 4(7,415 + 1,810) + 2 \times 4,175 + 685] = 92,350 \text{ cu. ft.}$$

EXAMPLES FOR PRACTICE

1. From a survey made of a reservoir to determine its capacity, the areas enclosed by the flow line and the four successive contours are found to be as follows: $A_0 = 4{,}095$ square feet, $A_1 = 3{,}156$ square feet, $A_2 = 2{,}369$ square feet, $A_3 = 1{,}854$ square feet, and $A_4 = 1{,}044$ square feet; if the contour interval is 4 feet, what is the capacity of the reservoir as determined by the method of average end areas?

Ans. 39,794 cu. ft.

2. What is the capacity of the reservoir referred to in the preceding example, as determined by the prismoidal formula? Ans. 39,889 cu. ft.

---

67. By Parallel Cross-Sections.—The capacity of a valley or basin for water storage can also be determined approximately as follows: A site having been selected for a dam, the elevation of the spillway is fixed and a survey is made to determine the location of the flow line on the ground. A plat of the survey is made showing the flow line and the outline of the projected dam. Suitable locations are then selected for a series of parallel cross-lines joining points on the flow line, situated on opposite sides of the valley. These cross-lines are located in such positions as to divide the area enclosed by the flow line into trapezoids. The cross-lines are located in such positions that straight lines joining the ends of adjacent cross-lines will either coincide with the flow line or lie equally on both sides of it. This can usually be done easily by the aid of the plat even when the flow line is quite irregular.

The locations for the parallel cross-lines having been determined from the plat, the lines are located and measured on the ground, and levels are taken over them, thus determining the cross-section of the valley within the flow line on each of the parallel cross-lines. The profile of this cross-section is platted, preferably on cross-section paper, and a straight line is drawn joining the points where the profile intersects the flow line. This straight line is horizontal and corresponds to what will be the water surface when the basin or reservoir is full of water to the flow line, and the cross-section thus formed represents what will be the cross-section

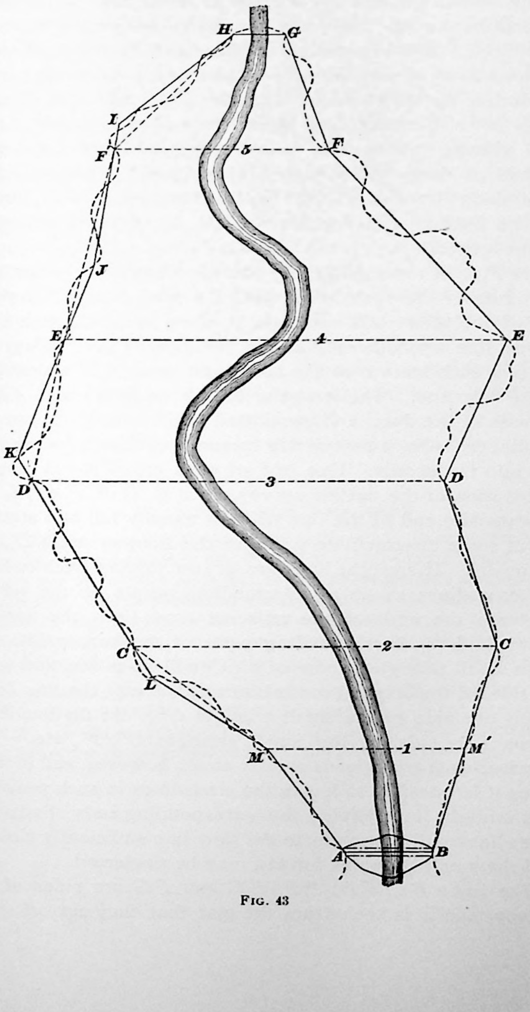


FIG. 43

of the water. The several cross-sections thus determined correspond to the cross-sections of a lake as obtained by soundings. The perpendicular distances between adjacent cross-sections are measured on the ground, calculated from the outline survey or scaled from the plat. The area of each cross-section is found, and the volume of each prismoid and total volume of the lake are determined by the end-area method as described in Art. 63, or by the prismoidal formula, as in Art. 65. In the latter case, the middle section may be interpolated, as described in Art. 64, or measured on the ground.

Let Fig. 43 represent the plat of a closed survey around the limits of the reservoir, following the approximate position of the flow line. Such a position is obtained for the survey line by following along the side of the valley and locating each station of the survey at or near the elevation of the flow line. The irregular line is the flow line. AB is the axis of the dam, and the dotted lines *1*, *2*, *3*, etc. are the parallel cross-lines that divide the area enclosed by the flow line into trapezoids. One end of each cross-line is located at a station of the outline survey, as at C, D, E, F, or M, but the opposite end of the line will not usually fall at a station, but at some intermediate point on the line, as at C', D', E', F', or M'. If straight lines are drawn joining such ends of the cross-lines as lie at intermediate points on the survey line with the ends of the adjacent cross-lines, the lines so drawn and the corresponding parts of the survey line will form small triangles, some of which will lie inside, and some outside, of the original outline survey. Thus, the line $D'E'$ forms one side of the small triangle $D'E'K$; the line $E'F'$ forms one side of the small triangle $E'JF'$, etc. The effect of such triangles is usually small, however, and in most cases it is possible to locate the cross-lines in such positions that straight lines joining the corresponding ends of adjacent cross-lines will approximate the flow line sufficiently closely, and the small triangles formed may be neglected.

The lines BC, CD, DE, EF, and FG are sides of the survey, and it is seen from the plat that they cut off small

irregular areas that lie between the survey line and the flow line on both sides of the latter, and that these areas will approximately balance; that is, the areas lying on the outside of the survey line will be approximately equal to those lying inside of it. Similarly, if the points H and F', F' and E', E' and D', etc. are joined by straight lines, these lines will cut off approximately equal areas on both sides of the flow line. The trapezoid $F'HGF$ will then contain approximately the same area as the irregular figure $F'IHGF$ bounded by the flow line; and similarly, each trapezoid included between any two adjacent parallel cross-lines will be approximately equal to the figure included between the same two cross-lines and limited by the flow line. The valley or basin is thus divided into prismoids whose bases are the cross-sections measured on the parallel cross-lines, and whose altitudes are the perpendicular distances between adjacent cross-lines. From these the volume of each prismoid can be calculated as explained in Art. **65**. The capacity of the valley or basin is the sum of the volumes of the several prismoids.

EXAMPLES FOR PRACTICE

1. The areas of the several cross-sections of a valley intended for water storage are found to be: $A_0 = 0$, $A_1 = 396$ square feet, $A_2 = 678$ square feet, $A_3 = 910$ square feet, $A_4 = 720$ square feet, and $A_5 = 586$ square feet. The perpendicular distances between the cross-sections are $h_{0-1} = 40$ feet, $h_{1-2} = 45$ feet, $h_{2-3} = 50$ feet, $h_{3-4} = 32$ feet, and $h_{4-5} = 36$ feet. What will be the capacity of the reservoir, in cubic feet, as determined by the end-area method? Ans. 121,373 cu. ft.

2. Assuming the middle areas to be: $M_{0-1} = 280$ square feet, $M_{1-2} = 410$ square feet, $M_{2-3} = 802$ square feet, $M_{3-4} = 830$ square feet, and $M_{4-5} = 670$ square feet, calculate, by the prismoidal formula, the capacity of the reservoir referred to in the preceding example.
Ans. 120,744 cu. ft.

UNITED STATES LAND SURVEYS

(PART 1)

SYSTEM OF ORIGINAL SURVEYS

PRELIMINARY

1. Land surveying consists in locating, measuring, and marking on the ground the boundary lines of tracts of land. This is done for the purpose of determining the position, form, and extent, and usually, also, the area, of each tract surveyed; the operation as a whole is called a **land survey.** Land surveys are of two classes; namely, *original surveys,* and *resurveys.*

2. Original surveys are made to determine the boundaries and areas of tracts that have not previously been surveyed. When this is done for the purposes of sale, and the land has been transferred in accordance therewith, the boundary lines thus fixed are unchangeable, except by the mutual consent of the interested parties, even though errors were made in the survey. The chief requirements in original surveys are:

(*a*) *Careful, accurate work* with the proper instruments, and correct computations to determine where the boundaries ought to be and the areas included by them.

(*b*) *Monuments* to mark the boundaries on the ground in the most permanent manner practicable.

(*c*) *A full and complete record* of the survey, giving the lengths and directions of the boundary lines and accurate descriptions of the location and character of the monuments

and other marks that define the position of the boundary lines on the ground. This record is known as the **field notes**, and is of especial value and importance as evidence by which to find or to relocate the boundary lines.

When original surveys are made by private owners, as, for example, the surveys for the sale of acreage tracts or town lots, they are made on any system to suit the circumstances of the case or the ideas and plans of the owners. When they are made by government authority, as, for example, the United States land surveys, they are based on a definite system provided by law.

3. Resurveys.—The determination of the true bearings and distances between the successive corners along the line of an original survey is commonly called a **retracement.** A retracement and survey made for the purpose of relocating and remarking the corners and lines of the original survey, when such corners and lines are wholly or partly lost or in dispute, is called a **resurvey.** Though their meanings are not exactly the same, the terms *resurvey* and *retracement* are often used interchangeably.

GENERAL OUTLINE OF SYSTEM

4. Authority for the Public-Land Surveys.—The surveys of the public lands of the United States are authorized by various acts of Congress. All executive duties appertaining to the surveying and sale of such public lands, or in any wise relating thereto, are performed by the Commissioner of the General Land Office, under the direction of the Secretary of the Interior. The General Land Office is connected with and under the authority of the Department of the Interior. Each large tract or region that by reason of its natural features or political boundaries is more or less separate from the other portions of the country, or that for any reason it is desirable or convenient to survey separately, is set apart as a separate **surveying district.** In many cases, a surveying district comprises one state, though in some cases it consists of parts of different states. The

surveys of the public lands in each surveying district are in charge of a surveyor general, who is appointed by the President of the United States, but the surveys are performed by skilful surveyors engaged as deputies by the surveyor general, and who are known as **United States deputy surveyors.** Under the United States system of land surveys, the public lands are divided by the deputy surveyors into tracts, approximately 1 mile square, called **sections,** and their further subdivision into half sections, quarter sections, half-quarter, and quarter-quarter sections is provided for by law.

5. Basis and Divisions of System.—All the surveys in each surveying district are referred to an **initial point.** This is located and marked accurately, and its true position in latitude and longitude is determined astronomically and recorded. From the initial point, a line, called a **base line,** is run east and west on a true parallel of latitude, and a true meridian, called the **principal meridian,** is run north and south. These two lines, perpendicular to each other, are taken as lines of reference for all the surveys of the region. Using them as a basis, the land is laid out into blocks approximately 24 miles square by means of **standard parallels,** which are lines running east and west, parallel to the base line and at intervals of 24 miles; and **guide meridians,** which are lines running north and south on true meridians at intervals of 24 miles.

Monuments are set along these lines at intervals of 1 mile, or 80 chains, for the corners of sections, and at the intermediate half miles for the corners of half and quarter sections. Such monuments are commonly known as **corners.** The blocks formed by the standard parallels and guide meridians are divided into **townships** of as nearly 6 miles square as the form of the earth will permit, by means of parallels of latitude and meridian lines, and each township is further divided into 36 **sections** containing as nearly as may be 640 acres each. A monument that is set to mark the corner of a section is called a **section corner,** and a monument set on the line between two section corners to mark

the corner of a half or quarter section is called a **quarter-section corner** or **quarter post**.

Corners set at the end of every mile and half mile on base lines and standard parallels, when those lines are run, are called **standard corners**. Other corners set on the same lines, where the subdivision lines close on them, are called **closing corners**. Thus, the corner of Sections 31 and 32, on a standard parallel, is a standard corner, while the corner of Sections 5 and 6, on the same line, is a closing corner. Previous to 1846, closing corners were set on the north and west boundaries of every township, but they are now restricted to base lines and standard parallels.

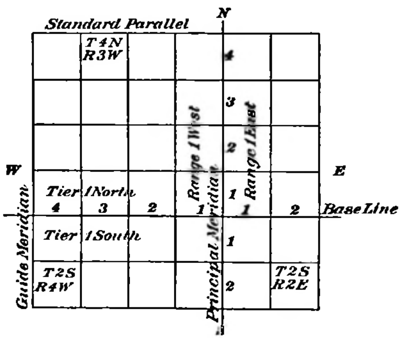


FIG. 1

6. Numbering of Townships.—According to the United States Manual of Surveying Instructions, any series of contiguous townships or sections lying north or south of each other constitutes a **range**, and the north-and-south line bounding a range of townships is called a **range line**. A series of contiguous townships or sections lying east and west of each other constitutes a **tier**. The east and west lines dividing a range into townships are called **township lines**.

Tiers of townships are numbered to the north and south from the base line, commencing with number 1 for the

townships adjacent to the base line, and ranges of townships are numbered to the east and west from the principal meridian, beginning with number 1 for the townships adjacent to the principal meridian, as shown in Fig. 1. By this means the location of a township is readily known from its description. Thus, Township 2 South, Range 4 West (usually abbreviated in the field notes to T. 2 S, R. 4 W) indicates that the township is in the second tier south of the base line and in the fourth range west of the principal meridian.

7. Numbering of Sections.—Sections are numbered successively from east to west and west to east across the township. Commencing with Section 1 in the northeast corner of the township, the sections are numbered in regular order westwards through the north tier of sections to Section 6 in the northwest corner, then through the next tier to the south they are numbered eastwards successively to Section 12 in the most eastern range of sections, and so on, alternately westwards and eastwards across the township, to Section 36 in the southeast corner of the township. This system of numbering is illustrated in Fig. 2.

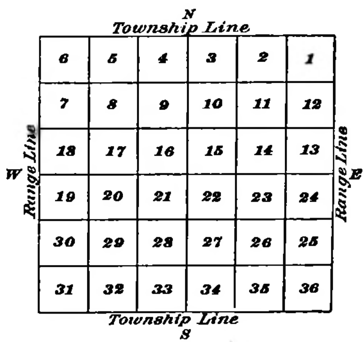


FIG. 2

Sections or legal subdivisions of sections that for any reason have not their regular legal complement of land, as when a portion of their area is taken up by waters or reservations, are termed **fractional sections.** In some cases, whole sections are occupied by lakes, rivers, etc. In such cases the remaining sections, whole or fractional, receive the same numbers that they would have received if all the sections in the township were complete.

8. Convergency of Meridians: Excess and Deficiency.—Since the range lines are required to be true meridians, the townships cannot be precisely 6 miles square, but are necessarily of a slightly trapezoidal form, owing to the fact that true meridians are not parallel to each other, but converge toward the poles of the earth. This is known as the **convergency of meridians.** This convergency of the meridians varies with the latitude. In a township 6 miles square, as measured on the parallels, that is, on the township lines, it is equal to 41.9 links in latitude 30°, while in latitude 49° it is equal to 83.5 links. Hence, the law provides that "in all cases where the exterior lines of the townships to be subdivided into sections and half sections shall exceed or shall not extend 6 miles, the excess or deficiency shall be added to or deducted from the western or northern ranges of sections or half sections in the township, according as the error may be in running lines from east to west or from south to north."

This produces fractional lots in those parts of Sections 1, 2, 3, 4, 5, and 6, adjoining the north boundaries of the townships, and in those parts of Sections 6, 7, 18, 19, 30, and 31 adjoining their west boundaries. The areas of these and all other fractional lots are marked on the official plats of the surveys, and the land is sold accordingly. The areas of the regular sections and lots in the township are not marked on the plats, but are sold as containing 640 acres, or aliquot parts thereof, although in a majority of the surveys it is the exception and not the rule that the sections do contain exactly 640 acres.

9. Initial Point.—In executing the United States land surveys in any surveying district, an initial point for the surveys of that district is first established. This is located astronomically under special instructions from the General Land Office; that is, in such position and by such methods as the General Land Office may direct, and its latitude and longitude are determined accurately.

An initial point should have a conspicuous location, visible from distant points on lines; it should be perpetuated

by an indestructible monument, preferably a copper bolt firmly set in a rock ledge; and it should be witnessed by rock bearings, without relying on anything perishable.

10. Classification of Lines.—The initial point having been established, the lines of public-land surveys are extended therefrom. They are classified as follows: (1) base lines and standard parallels; (2) principal and guide meridians; (3) township exteriors; (4) subdivision and meander lines. Only the base line and principal meridian can pass through the initial point.

11. Base Line.—From the initial point, the base line is extended east and west on a true parallel of latitude, by the use of transit or solar instruments. The transit is used for the alinement of all important lines.

The direction of a base line conforms to a parallel of latitude and is controlled by true meridians; consequently, the correct determination of true meridians by observations on Polaris at elongation is a matter of prime importance. Since the base line is at every point perpendicular to the true meridian at that point, it is not a straight line, but is necessarily slightly curved.

In laying out a base line or parallel of latitude, certain reference lines, called tangents and secants, having a known position and relation to the required parallel of latitude, are prolonged as straight lines, and the positions of points on the parallel of latitude are determined by offset measurements from these reference lines.

The proper township, section, and quarter-section corners are established at lawful intervals along the base line, and meander corners are established at the intersection of the line with all meanderable streams, lakes, or bayous.

12. Method of Prolonging Lines.—In prolonging these reference lines, two backsights and two foresights are taken at each setting of the instrument. In one method, the horizontal limb is revolved 180° in azimuth between the observations, taking the mean of observations. Another method, called *double backsights and foresights*, is still more

exact, and therefore preferable, since it not only insures a straight line, if the transit is leveled, but also detects the least error of collimation. In this method, the transit points are placed at short intervals along the line, and each backsight is taken by setting the vertical cross-wire of the telescope on two transit points at some distance apart back on the line. Then, if the transit point over which the instrument is set has not been fixed truly in line, the two transit points observed cannot be brought in line with the cross-wire at the same time, and the error will be detected at once. If the two transit points observed can be bisected with the cross-wire at the same time, the telescope is plunged forwards and one or two new points are set in advance. The transit is then moved forwards and set up over one of the new points and again backsighted on two or more transit points, in order to check its position. Thus, suppose that the instrument is set up at I, Fig. 3, and backsighted on the transit points A and B. If the point I is not truly in line and the cross-wire is made to bisect the point B, the line of sight will not bisect the point A, but will pass to one side of it, as indicated by the dotted line BA'. The error can be corrected by taking the instrument back to the point B and testing the forward sight, or by taking it to the point A and prolonging the line AB by a foresight. Or, when at the point I, the transit can be moved over until by trial it is found to be accurately in line with the points A and B, as indicated by the line BI'. The line can then be prolonged forwards as before.

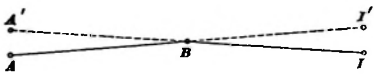


FIG. 3

Where a solar apparatus is used in connection with a transit, the deputy surveyor tests the instrument, whenever practicable, by comparing its indications with a meridian determined by observations of Polaris, and in all cases where error is discovered he makes the necessary corrections of his line before proceeding with the survey. All operations must be fully described in the field notes.

In order to detect errors and insure accuracy in measurement, two sets of chainmen are employed; one to note distances to intermediate points and to locate topographical features, the other to act as a check. Each measures 40 chains, and in case the difference is inconsiderable, the corner is placed midway between the ending points of the two measurements; but if the discrepancy exceeds 8 links on even ground, or 25 links on mountainous ground, the true distance is found by careful rechaining by one party or both.

13. Principal Meridian.—This line must be a true meridian and be extended from the initial point, either north or south, or in both directions, as the conditions may require, by the use of transit or solar instruments. The methods used for determination of directions, and the precautions to be observed to secure accuracy in measurement, are the same as for the base line.

14. Standard Parallels.—These were formerly called **correction lines.** The standard parallels are extended east and west from the principal meridian, at intervals of 24 miles north and south of the base line, in the manner prescribed for running said base line. In the earlier surveys, standard parallels as well as guide meridians were placed at various distances up to 60 or more miles apart. This was done under special instructions given by the surveyor general of the district at the time. It is no longer permitted.

15. Guide Meridians.—These lines are extended as true meridians north from the base line or standard parallels, at intervals of 24 miles east and west from the principal meridian, as measured on the base line or standard parallel, in the manner prescribed for running the principal meridian.

When conditions require that such guide meridians be run south from the base or correction lines, they are initiated at established corners on such lines, marked as closing corners.

16. Township Exteriors.—Whenever practicable, the township exteriors, in a block of land 24 miles square bounded by standard lines, are surveyed successively

through the block. Beginning with those of the southwestern township, the exterior boundaries of townships belonging to the west range are first surveyed in succession through the range, from south to north, and in a similar manner the other three ranges are surveyed in regular sequence. The township boundaries extending north and south formed by range lines, are called **meridional boundaries**, and those extending east and west, that is, the township lines, are designated as **latitudinal boundaries.**

The meridional boundaries of townships have precedence in the order of survey and are run from south to north on true meridians, with permanent corners at lawful distances.

The latitudinal boundaries are run from east to west on trial lines, called **random lines**, and are corrected back on true lines. When a random line intersects the line to which it is run at some point other than the true corner, the distance from the point of intersection to the true corner is called the **falling** of the random line. The falling of a random line north or south of the township corner to be closed on is measured, and, with the resulting true return course, recorded in the field notes.

When running random lines from east to west, temporary corners are set at intervals of 40 chains, and permanent corners established on the true line as corrected back, thereby throwing the excess or deficiency against the west boundary of the township.

SUBDIVISION OF TOWNSHIPS

17. Meridional Section Lines.—The exterior boundaries of a full township having been established, the lines subdividing the township into sections are run from the corners that have been set on the boundary lines of the township. The north-and-south section lines extending through the township are called **meridional section lines**, and the east-and-west section lines are called **latitudinal section lines.**

The meridional section lines are made parallel to the range line forming the east boundary of the township by applying

to the bearing of the latter a small correction, dependent on the latitude, taken from Table I.* This table gives, to the nearest whole minute, the convergency of two meridians 6 miles long and from 1 to 5 miles apart.

TABLE I

CORRECTIONS FOR CONVERGENCY WITHIN A TOWNSHIP

Latitude Degrees	Corrections, in Minutes of Arc, to be Applied to Bearing of Range Lines at a Distance of				
	1 Mile	2 Miles	3 Miles	4 Miles	5 Miles
30 to 35 . . .	1	1	2	2	3
35 to 40 . . .	1	1	2	3	3
40 to 45 . . .	1	2	2	3	4
45 to 50 . . .	1	2	3	4	5
50 to 55 . . .	1	2	3	5	6
55 to 60 . . .	1	3	4	5	7
60 to 65 . . .	2	3	5	7	8
65 to 70 . . .	2	4	6	8	10

By applying the corrections given in Table I according to the following rules, the bearing of each of the five meridional section lines in a township can easily be determined from the bearing of the range line forming the east boundary of the township:

Rule I.—*If the bearing of the range line is due north, a tabular correction will be equal to the bearing of the corresponding meridional section line west of north.*

Rule II.—*If the bearing of the range line is west of north, the sum of this and a tabular correction will be the bearing of the corresponding meridional section line.*

*Tables I, II, and III, and the list of abbreviations in Art. 32, descriptions of corners in Art. 33, specimen of field notes in Art. 38, and many authoritative statements throughout this Section are either wholly or partly from the United States Manual of Surveying Instructions for 1902.

Rule III.—*If the bearing of the range line is east of north, the difference between this and a tabular correction will be the bearing of the corresponding meridional section line; this bearing will be east of north when the tabular correction is less than the bearing of the range line, and west of north when the correction exceeds the bearing.*

Example.—In latitude 47°, the range line forming the east boundary of a township bears N 0° 2′ E; what should be the bearing of each of the five meridional section lines in the township?

Solution.—By reference to Table I, it is found that for a latitude between 45° and 50° the correction is 1 min. for each mile of distance from the range line. Hence, by applying rule III, the bearings of the five meridional section lines are found to be as follows:

Bearing of first line, from the corner for Sections 35 and 36, N 0° 1′ E. Ans.

Bearing of second line, from the corner for Sections 34 and 35, true north. Ans.

Bearing of third line, from the corner for Sections 33 and 34, N 0° 1′ W. Ans.

Bearing of fourth line, from the corner for Sections 32 and 33, N 0° 2′ W. Ans.

Bearing of fifth line, from the corner for Sections 31 and 32, N 0° 3′ W. Ans.

EXAMPLES FOR PRACTICE

1. In latitude 42° north, the range line forming the east boundary of a township bears N 0° 2′ W; what should be the bearing of: (*a*) the first meridional section line west of this range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans.
(*a*) N 0° 3′ W
(*b*) N 0° 4′ W
(*c*) N 0° 4′ W
(*d*) N 0° 5′ W
(*e*) N 0° 6′ W

2. In latitude 45° north, the bearing of the range line forming the east boundary of a township is due north; what should be the bearing of: (*a*) the first meridional section line west of the range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans.
(*a*) N 0° 1′ W
(*b*) N 0° 2′ W
(*c*) N 0° 3′ W
(*d*) N 0° 4′ W
(*e*) N 0° 5′ W

3. In latitude 35° north, the bearing of the range line forming the east boundary of a township is N 0° 1′ E; what should be the bearing

of: (*a*) the first meridional section line west of this range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans. (*a*) North
(*b*) North
(*c*) N 0° 1′ W
(*d*) N 0° 2′ W
(*e*) N 0° 2′ W

4. In latitude 40° north, the bearing of the range line forming the east boundary of a township is N 0° 1′ W; what should be the bearing of: (*a*) the first meridional section line west of this range line? (*b*) the second? (*c*) the third? (*d*) the fourth? and (*e*) the fifth?

Ans. (*a*) N 0° 2′ W
(*b*) N 0° 3′ W
(*c*) N 0° 3′ W
(*d*) N 0° 4′ W
(*e*) N 0° 5′ W

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18. Method of Subdividing Townships.—At or near the southeast corner of the township, a true meridian is determined by observing Polaris or by solar observations, and the deputy surveyor's instrument is tested thereon; then from said corner the first mile of the east and south boundaries are retraced, if the survey of the exteriors and the subdivisions have been provided for in separate contracts; but if the survey of the exterior and subdivisional lines are included in the same contract, these retracements are omitted. All disagreements of bearings or measurements are stated in the field notes.

After testing his instrument on the true meridian, the deputy surveyor commences at the corner to Sections 35 and 36, on the south boundary of the township, and runs a line parallel to the range line, establishing at 40 chains the quarter-section corner between Sections 35 and 36, and at 80 chains the corner for Sections 25, 26, 35, and 36.

From the last-named corner, a random line is run eastward, without blazing, parallel to the south boundary of Section 36, to an intersection with the east boundary of the township, placing a post for temporary quarter-section corner at a distance of 40 chains from the point of beginning. If the random line intersects the township boundary exactly at the corner for Sections 25 and 36, it is blazed back and established as the true line, the permanent quarter-section

corner being established thereon, midway between the initial and terminal section corners.

If the random line intersects the township boundary to the north or south of the corner, the falling, or distance of the point of intersection north or south of the corner, is measured, and from the data thus obtained, the true return course is calculated and the true line blazed and established, and on this line the position of the quarter-section corner is determined and established midway between the section corners. Then from the corner for Sections 25, 26, 35, and 36, the west and north boundaries of Sections 25, 24, 13, and 12 are established in the same manner as those of Section 36, with the exception that the random line for the north boundary of each of these sections is run parallel to the established south boundary of the same section, instead of the south boundary of Section 36; that is, the random line between Sections 24 and 25 is run parallel to the established south boundary of Section 25, etc.

Then, from the last established section corner, that is, the corner of Sections 1, 2, 11, and 12, the line between Sections 1 and 2 is projected northwards on a random line, parallel to the east boundary of the township, to its intersection with the north boundary of the township, setting a post for temporary quarter-section corner at 40 chains. If the random intersects said north boundary exactly at the section corner previously set on said boundary, it is blazed back and established permanently in its original position, allowing the fractional measurement to remain permanently in that portion of the line between said corner and the north boundary of the township.

If, however, the random line intersects the north boundary of the township to the east or west of the section corner previously set on that line, the consequent falling is measured, and from the data thus obtained the true return course is calculated and the true line established. The permanent quarter-section corner is placed on this line at 40 chains from the initial corner of the random line, thereby throwing the fractional measurement in that portion of the line lying

between the quarter-section corner and the north boundary of the township.

When the north boundary of a township is a base line or standard parallel, however, the line between Sections 1 and 2 is run parallel to the range line as a true line, the quarter-section corner is placed on this line at 40 chains from the initial corner, and a closing corner is established at the point of intersection with the base or standard line. The distance from the said closing corner, to the nearest standard corner on the base or standard line, is measured and noted as a connection line. This distance is the **closing distance.** The line that intersects the base line or standard parallel is a **closing line.**

Each successive range of sections progressing to the west is surveyed in a similar manner until the fifth range is attained; then, from the section corners established on the west boundary of the fifth range of sections, random lines are projected to their intersection with the west boundary of the township, and the true return lines established as in the survey of the first, or most eastern, range of sections, with the exception that on the true lines thus established the quarter-section corners are established at 40 chains from the initial corners of the random lines, the fractional measurements being thereby thrown into those portions of the lines situated between the quarter-section corners and the west boundary of the township.

On both meridional and latitudinal section lines, quarter-section corners are established at points equidistant from the corresponding section corners, except on the lines closing on the north and west boundaries of the township, and in those situations the quarter-section corners are established at precisely 40 chains to the north or west (as the case may be) of the respective section corners from which those lines respectively start, by which procedure the excess or deficiency in the measurements is thrown on the extreme tier or range of quarter sections, as the case may be.

19. An Exception—Witness Corners.—The deputy is not required to complete the survey of the first range of

sections from south to north before commencing the survey of the second or any subsequent range of sections, but the corner on which any random line closes must have been previously established by running the line that determines its position, except as follows: Where it is impracticable to establish such section corner in the regular manner, it is established by running the latitudinal section line as a true line, with a true bearing, determined as directed for random lines, setting the quarter-section corner at 40 chains and the section corner at 80 chains.

When the proper point for the establishment of a township or section corner is inaccessible, and a corner can be erected on each of the two lines that approach the township or section corner, at distances not exceeding 20 chains therefrom, such a corner is established on each line. Corners thus established are called **witness corners.** The witness corner is marked as conspicuously as a section corner, and bearing trees are used wherever possible.

20. Meandering.—At every point where a standard, township, or section line intersects the bank of a navigable stream or any meanderable shore, a corner, called a **meander corner,** is established at the time the line is run. The meander corners thus established on the survey lines that intersect a shore line are connected by a traverse line that follows the sinuosities of the shore. Such traverse lines are for the purpose of determining the form of the shore; they are called **meander lines,** and the process of running them is called **meandering.** All streams 3 chains or more wide are meandered on both banks. Navigable bayous, lakes, and ponds having an area of more than 25 acres are also meandered. In meandering, the deputy begins at a meander corner and, following the bank as nearly as practicable along the high water-line, takes the bearing and measures the length of each course, closing on the next meander corner.

A meander line is always a compass traverse. The courses are taken at the nearest quarter degree and are recorded as compass courses by their angle from the true

meridian, and not as the transit angles of a deflection traverse by the angular deviation of each course from the preceding course. The lengths of the courses are taken in full chains or multiples of 10 links for convenience of computation, using odd links only in the closing course. These meander lines are run merely for the purpose of defining the shore lines and ascertaining the approximate area of land in the subdivisions made fractional by the body or bodies of water. They follow the high water-line of the shore as nearly as practicable, but do not follow all the little sinuosities of the bank. A meander line is not a boundary line;* the high water-line of the bank is the boundary line.

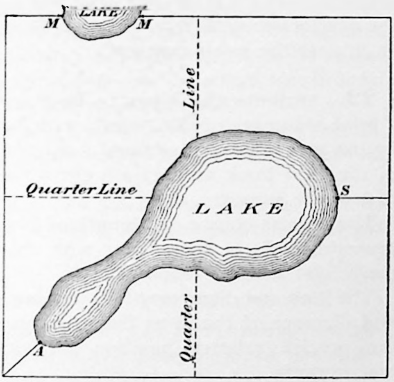

Fig. 4

21. Meander Corners.—The meander corners set on regular division lines of the government survey, that is, on standard, township, or section lines, are called **regular meander corners.** It is sometimes the case, however, that a lake or deep pond lies entirely within the boundaries of a section, so that its shore is not intersected by any regular division line of the government survey. In order to meander such a lake or pond, two lines are run to it from the two nearest survey corners on different sides of the lake, and the lengths and bearings of these lines are measured and recorded. Meander corners are established where the lines intersect the margin of the lake or pond, and the meander

*By the supreme court of Minnesota, however, a meander line has been held to be a boundary line in a case where a body of water never existed at substantially the place indicated on the official plat.

lines are then extended around the lake or pond from the points thus established. If the line thus run to the lake or pond from a survey corner coincides with a legal subdivision line of the section, that is, coincides with a quarter-section line, a half-quarter line, or a quarter-quarter line, the meander corner set on the line is called a **special meander corner**. But if the line run to the lake or pond does not coincide with a legal subdivision line of the section, the meander corner set on it is called an **auxiliary meander corner**. Fig. 4 shows a fractional section with regular meander corners at M and M, a special meander corner on the quarter-section line at S, and an auxiliary meander corner at A, near the section corner.

22. Objects and Data to Be Noted.—The following is a brief summary of the objects and data that, if intersected by the survey line or situated in its vicinity, are to be noted in the field book as they are encountered during the progress of the survey:

The precise course and length of every line run, noting all necessary offsets therefrom, with the reason for making them, and method employed.

The kind and diameter of all bearing trees, with the course and distance of the same from their respective corners; and the precise relative position of witness corners to the true corners.

The kind of materials of which corners are constructed.

Trees on line. The name, diameter, and distance on line to all trees that it intersects.

Intersections of the line with, and descriptions of, land objects and water objects, such as hills, ravines, settlers' claims, rivers, lakes, watercourses, swamps, etc.

Descriptions of the character of the land's surface, the soil, timber, prairie, bottom lands, springs, improvements, roads and trails, mineral deposits, rapids, cataracts, precipices, caves, etc.

Natural curiosities, fossils, ancient works of art, such as mounds, ditches, and objects of like nature.

The magnetic declination is incidentally noted at all points of the lines being surveyed, where any material change in the same indicates the probable presence of iron ores, and the position of such points is identified in the field notes.

23. Limits for Closing.—Every random line having a nominal length of 1 mile is required to close within 50 links of the objective point, and the length of the line must be within 50 links of its theoretical length. For random lines of greater length, the corresponding errors of closure must not exceed 50 links per mile of line run. Otherwise the line must be rerun, and the error found and corrected.

North-and-south section lines, except those of fractional sections next to the township boundary, must be 80 chains long. In those of fractional sections, the actual length must be within 150 links of the theoretical length.

The north boundary and the south boundary of any section, except in a fractional range, must be within 50 links of equal length.

Meanders between any two successive meander corners must close by traverse within $\frac{5}{8}$ link for each chain of the meander line.

CONVERGENCY OF MERIDIANS

24. Amount of Convergency of Meridians.—In latitude 30° north, the angle of convergency between meridians 1 mile apart is 30 seconds, very closely, so that in this latitude the meridians at the opposite sides of a township incline to each other at an angle of 3 minutes, very closely. In latitude 44° north, the convergency is very closely 50 seconds per longitudinal mile, or 5 minutes per township. The convergency of meridians can be determined approximately but very closely by means of the two following principles, which, though not rigidly exact,* are closely approximate:

Principle I.—*The angle of convergency between two meridians is equal to their difference in longitude multiplied by the sine of the latitude.*

*Because the earth is not a true sphere.

Principle II.—*The lengths of the parallels of latitude intercepted between any two meridians are proportional to the cosines of the latitudes.*

These principles are sufficiently accurate for the purposes of the surveyor when the meridians considered are not more than 24 miles apart. At the equator, the length of 1° of longitude is equal to 69.164 statute miles, and the cosine of the latitude is unity. Hence, the second principle can be stated in the following form, which for some purposes is more convenient:

The length of an arc of 1° of longitude at any given latitude, expressed in miles, is equal to 69.164 times the cosine of the latitude.

Let c° = angle of convergency, in degrees;
c' = angle of convergency, in minutes;
L = difference in longitude, in same unit as c;
l = latitude for which convergency is calculated;
m = distance, in miles, corresponding to difference in longitude.

Then, according to the principle first stated

$$c^\circ = L^\circ \sin l \qquad (a)$$

and according to the second principle,

$$m = 69.164\, L^\circ \cos l$$

$$L^\circ = \frac{m}{69.164 \cos l} \qquad (b)$$

Substituting this value of L° in (a),

$$c^\circ = \frac{m \sin l}{69.164 \cos l} = \frac{m \tan l}{69.164} \qquad (1)$$

The value of an angle expressed in minutes is 60 times its value expressed in degrees; hence, $c' = 60\, c^\circ$ and by multiplying both numbers of the preceding formula by 60 and substituting c' for $60\, c^\circ$, it becomes

$$c' = \frac{m \tan l}{1.1527} = .8675\, m \tan l \qquad (2)$$

Formulas 1 and 2 are solved easily by means of logarithms, and for this purpose formula 1 may be written in the form

$$\log c^{\circ} = \log m + \log \tan l + \bar{2}.16012 \qquad (3)$$

Likewise formula 2 may be written in the form

$$\log c' = \log m + \log \tan l + \bar{1}.93828 \qquad (4)$$

EXAMPLE 1.—What is the angle of convergency between two meridians 6 miles apart in latitude 30° north?

SOLUTION.—The natural tangent of 30° is .57735, and by substituting this and the value of m in formula 2, the angle of convergency is found to be

$$c' = .8675 \times 6 \times .57735 = 3.005' = 3', \text{ practically. Ans.}$$

EXAMPLE 2.—What is the angle of convergency between two meridians 6 miles apart in latitude 44° north?

SOLUTION.—Log 6 = .77815 and log tan 44° = $\bar{1}.98484$. The substitution of these values in formula 4 gives

$$\log c' = .77815 + \bar{1}.98484 + \bar{1}.93828 = .70127$$

which is the logarithm of 5.0265′ = 5′ 2″, nearly. Ans.

25. Difference in Length of Northern and Southern Township Boundaries.—Owing to the fact that the earth is not a true sphere, but is somewhat flattened at the poles, the difference in latitude subtended by an arc 6 miles in length extending north and south is not exactly the same at all latitudes, but varies from about 5.227 minutes in latitude 30° to 5.193 minutes in latitude 70°. For most purposes of the surveyor, however, it may be taken at 5.2 minutes for all ordinary latitudes. In what follows, this value will be used; the resulting error will be so small as not to affect the results materially.

Let l_s = latitude of southern boundary of township;
l_n = latitude of northern boundary of township;
d_s = length of southern boundary of township;
d_n = length of northern boundary of township.

Then, from principle II of the preceding article,

$$d_s : d_n = \cos l_s : \cos l_n$$

from which

$$d_n = d_s \frac{\cos l_n}{\cos l_s} \qquad (1)$$

For ordinary purposes it is sufficiently accurate to write $l_n = l_s + 5.2'$.

Formula 1 is easily solved by means of logarithms, for which purpose it may be written

$$\log d_n = \log d_s + \log \cos l_n - \log \cos l_s \qquad (2)$$

EXAMPLE.—The southern boundary of a township is in latitude 35° north and its length is just 480 chains. What should be the length of the northern boundary of the township?

SOLUTION.—Log 480 = 2.68124, log cos 35° = $\bar{1}.91336$, and log cos 35° 5.2′ = $\bar{1}.91290$. The substitution of these values in formula 2, gives

$$\log d_n = 2.68124 + \bar{1}.91290 - \bar{1}.91336 = 2.68078$$

whence $d_n = 479.49$. Ans.

EXAMPLES FOR PRACTICE

1. What is the angle of convergency between two meridians 6 miles apart in latitude 33° north? Ans. 3′ 23″

2. What is the angle of convergency between two meridians 6 miles apart in latitude 47° north? Use logarithms. Ans. 5′ 35″

3. The southern boundary of a township is on a correction line in latitude 45° north and its length is therefore 480 chains; what should be the length of the northern boundary of the township? Ans. 479.27 ch.

4. The southern boundary of a township is on a correction line in latitude 40° north; what should be the length of its northern boundary? Ans. 479.4 ch.

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26. Table of Convergency of Meridians.—Table II gives the amount of convergency of meridians for each degree of latitude between latitudes 30° and 70°, inclusive, and is used in determining the theoretical length of township and section lines.

The second column of the table contains the convergency of two meridians 6 miles long and 6 miles apart and having their south ends at the latitude stated in the first column. This convergency is the difference in the distances between the two meridians as measured on the parallels of latitude through their north and south extremities. When the latitude of the parallel that passes through the south end of the

TABLE II

CONVERGENCY OF MERIDIANS 6 MILES LONG AND 6 MILES APART, AND OTHER RELEVANT DATA, BETWEEN LATITUDES 30° AND 70° NORTH

Latitude Degrees	Convergency			Difference of Longitude per Range			Difference of Latitude, in Minutes of Arc, for	
	On the Parallel Links	Angle		Minutes of Arc	Seconds of Arc	Seconds of Time	1 Mile in Arc	1 Tp. in Arc
		Min.	Sec.					
30	41.9	3	0	6	0.36	24.02		
31	43.6	3	7	6	4.02	24.27		
32	45.4	3	15	6	7.93	24.53	.871′	5.225′
33	47.2	3	23	6	12.00	24.80		
34	49.1	3	30	6	16.31	25.09		
35	50.9	3	38	6	20.95	25.40		
36	52.7	3	46	6	25.60	25.71		
37	54.7	3	55	6	30.59	26.04	.870	5.221
38	56.8	4	4	6	35.81	26.39		
39	58.8	4	13	6	41.34	26.76		
40	60.9	4	22	6	47.13	27.14		
41	63.1	4	31	6	53.22	27.55		
42	65.4	4	41	6	59.62	27.97	.869	5.217
43	67.7	4	51	7	6.27	28.42		
44	70.1	5	1	7	13.44	28.90		
45	72.6	5	12	7	20.93	29.39		
46	75.2	5	23	7	28.81	29.92		
47	77.8	5	34	7	37.10	30.47	.869	5.212
48	80.6	5	46	7	45.79	31.05		
49	83.5	5	59	7	55.12	31.67		
50	86.4	6	12	8	4.83	32.32		
51	89.6	6	25	8	15.17	33.01		
52	92.8	6	39	8	26.13	33.74	.868	5.207
53	96.2	6	54	8	37.75	34.52		
54	99.8	7	9	8	50.07	35.34		
55	103.5	7	25	9	3.18	36.22		
56	107.5	7	42	9	17.12	37.14		
57	111.6	8	0	9	31.97	38.13	.867	5.202
58	116.0	8	19	9	47.83	39.19		
59	120.6	8	38	10	4.78	40.32		
60	125.5	8	59	10	22.94	41.52		
61	130.8	9	22	10	42.42	42.83		
62	136.3	9	46	11	3.38	44.22	.866	5.198
63	142.2	10	11	11	25.97	45.73		
64	148.6	10	38	11	50.37	47.36		
65	155.0	11	8	12	16.82	49.12		
66	162.8	11	39	12	45.55	51.04		
67	170.7	12	13	13	16.88	53.12	.866	5.195
68	179.3	12	51	13	51.15	55.41		
69	188.7	13	31	14	28.77	57.92		
70	199.1	14	15	15	10.26	60.68	.866	5.193

meridians and forms the south boundary of the township of which the meridians form the meridional boundaries is the same as a tabular latitude given in the first column, the corresponding convergency given in the second column will be the convergency required. For latitudes intermediate between the tabular latitudes, the convergency is obtained by interpolation.

The third column of the table contains the angle of convergency between meridians 6 miles apart and at the various latitudes.

For the various latitudes given in the first column, the difference in longitude between the east and west meridional boundary lines of a township is given in minutes and seconds of arc in the fourth column, and in the fifth column the same value is given in seconds of time.

The difference of latitude for 1 mile in arc, that is, for a distance of 1 mile measured along a true meridian, at the various latitudes, is given in the sixth column, and the difference in latitude for one township in arc, that is, for a distance of 6 miles measured along a true meridian at the various latitudes, is given in the last column.

For the purpose of computing the convergency of the section lines within boundaries of a regular township, the township may be regarded as a plane figure, generally a trapezoid, and its boundaries may be considered to be straight lines. The method of using the table in computing the convergency will be understood from the following example:

EXAMPLE.—In latitude 42°, the length of the south boundary of T. 3 S, R. 9 W is 5 miles 79 chains and 83 links (written 5 mi. 79 ch. 83 li., or 5 mi. 79.83 ch.). (*a*) What is the theoretical length of the north boundary? (*b*) What is the greatest and what the least length permitted for the random line for that boundary? (*c*) What is the theoretical length of the line between Sections 7 and 18? (*d*) What are the limiting lengths for the random of that line?

SOLUTION.—(*a*) By referring to the second column of Table II, we find that in latitude 42° the convergency of meridians 6 mi. long and 6 mi. apart, as measured on the parallel, is 65.4 li., or .654 ch. By subtracting this from the length of the south boundary, the theoretical

length of the north boundary is found to be equal to 5 mi. 79 ch. 83 li. − 65.4 li. = 5 mi. 79 ch. 17.6 li. = 5 mi. 79.176 ch. Ans.

(*b*) As the measured length of the random line must be within 3 ch. of its theoretical length, the greatest length permitted for the random line is 5 mi. 79 ch. 17.6 li. + 3 ch. = 6 mi. 2 ch. 17.6 li., and its least permitted length is 5 mi. 79 ch. 17.6 li. − 3 ch. = 5 mi. 76 ch. 17.6 li. Ans.

(*c*) According to Art. **8**, the excess or deficiency in the length of the latitudinal township boundaries is thrown into the west range of sections and, consequently, the length of the south boundary of Section 31 is equal to 5 mi. 79 ch. 83 li. − 5 mi. = 79 ch. 83 li. According to Art. **8**, the convergency is theoretically confined to the west range of sections, and since the line between Sections 7 and 18 is two-thirds of the distance from the south to the north boundary, its length will be equal to the length of the south line of Section 31 minus two-thirds of the total convergency, or 79 ch. 83 li. − $\frac{2}{3}$ × 65.4 li. = 79 ch. 39.4 li. Ans.

(*d*) Since, according to Art. **23**, the random for a section line must measure within 50 li. of the theoretical length of the line, its greatest permitted length is equal to 79 ch. 39.4 li. + 50 li. = 79 ch. 89.4 li., and its least permitted length is equal to 79 ch. 39.4 li. − 50 li. = 78 ch. 89.4 li. Ans.

NOTE.—It will be well to notice here that the least permitted length of a section line is just 1 chain less than its greatest permitted length. Hence, having found its greatest permitted length, it is merely necessary to subtract 1 chain from this in order to determine its least permitted length.

EXAMPLES FOR PRACTICE

1. The southern boundary of a township is on a correction line in latitude 40° north. (*a*) What should be the length of the northern boundary of the township? (*b*) What is the least length permitted for the random of the northern boundary of this township? (*c*) What is the greatest permitted length? (*d*) What is the theoretical length of the line between Sections 30 and 31?

Ans. { (*a*) 479.4 ch. (*b*) 476.4 ch. (*c*) 482.4 ch. (*d*) 79.9 ch.

2. The southern boundary of a township is in latitude 48° north and its length is 6 miles. (*a*) What should be the length of the northern boundary of the township? (*b*) What is the theoretical length of the line between Sections 19 and 30? (*c*) What is the least length permitted for the random of this line? (*d*) What is the greatest permitted length?

Ans. { (*a*) 5 m. 79 ch. 19 li. (*b*) 79.73 ch. (*c*) 79.23 ch. (*d*) 80.23 ch.

3. (*a*) What should be the length of the northern boundary of a township whose southern boundary is in latitude 38° north and has a length of 480 chains? (*b*) What are the limiting lengths permitted for the random of the northern boundary of this township? (*c*) What is the theoretical length of the line between Sections 18 and 19? (*d*) What are the limiting lengths for the random of this line?

Ans. (*a*) 479.43 ch.
(*b*) 476.43 and 482.43 ch.
(*c*) 79.72 ch.
(*d*) 79.22 and 80.22 ch.

4. (*a*) What should be the length of the northern boundary of a township whose southern boundary is in latitude 35° north and has a length of 480 chains? (*b*) What are the limiting lengths permitted for the random of the northern boundary of this township? (*c*) What is the theoretical length of the line between Sections 6 and 7? (*d*) What are the limiting lengths for the random of this line?

Ans. (*a*) 479.49 ch.
(*b*) 476.49 and 482.49 ch.
(*c*) 79.58 ch.
(*d*) 79.08 and 80.08 ch.

INSTRUMENTS USED

27. Instruments for Alinement.—For running lines of the United States land surveys, Burt's improved solar compass or a transit of approved construction, with or without solar attachment,* may be used. No other instrument is now permitted. The use of the magnetic needle for running these lines is strictly prohibited. The direction of all lines must be determined independently of the needle. Deputy surveyors using instruments with solar apparatus are required to make observations on Polaris at the beginning of every survey and whenever necessary to test the solar apparatus. Those using transits without solar attachment are required to obtain the meridian by observations of Polaris every clear night. Deputy surveyors are required to examine the adjustments of their instruments and take the latitudes daily, the weather permitting, while running all lines of the public surveys. All instruments used must be tested at least once a year, and oftener if necessary, on the true meridian established by the direction of the surveyor

* The solar attachment is an auxiliary telescope on a transit used for determining the meridian by observing the sun.

general of the district, and all defective instruments must be put in a proper condition for accurate work.

28. Instruments for Measuring Distances.—The instruments used for measuring the lengths of the lines of the United States land surveys are the surveyors' chain and marking pins, described in *Chain Surveying*. The use of the engineers' chain is not permitted in the United States land surveys. Distances of height or depth, however, may be given in feet and inches.

Each deputy surveyor must be provided with a standard steel chain or tape, precisely adjusted to the standard measures kept by the surveyor general. This is not used in the field work, but is kept for the purpose of frequently comparing and testing the field chains or tapes.

CORNERS OF THE PUBLIC-LAND SURVEYS

CORNER MONUMENTS

29. The establishment of corners is the consummation of the field work of land surveys. If this is not done in a distinct and permanent manner, the main object of the survey will not be attained. The United States Manual of Surveying Instructions is furnished to every deputy surveyor engaged in the public-land surveys. This gives complete and detailed instructions regarding how to mark corners under all circumstances likely to occur. The corners established in these surveys are township, section, quarter-section, meander, and witness corners. These corners are defined by planting monuments of the most enduring materials available and by marking and noting the directions and distances from the corners to the most permanent objects in the vicinity. A monument consists of what is called the *corner* and its *accessories*.

The **corner** may be an iron post, rod, or pipe, a cross cut on a rocky ledge, a marked stone of suitable size and form, or in case none of these can be obtained, it may be a post of durable timber.

30. The **accessories** to a corner are for the purpose of identifying or **witnessing** the corner, in order that its position can afterwards be determined definitely and unmistakably. They consist of prominent and durable objects, such as rocks, trees, etc., properly marked, whose bearings and distances from the corner are determined and recorded together with their descriptions. An object whose bearing and distance from a corner are determined and recorded is called a **bearing object**, and if a tree, it is called a **bearing tree**. The accessories to a corner may be any of the following, which are named in the order of their value and desirability:

Bearing objects, such as notable cliffs, rocks, boulders, etc., marked with a cross, the letters B. O. and a section number.

Memorials buried 12 to 24 inches deep at the corner, such as glass or stoneware, marked stones, cast iron, charcoal, or a charred stake.

Pits of proper size and arrangement.

Mound of stones at proper position and distance from the corner.

Bearing trees, blazed and marked as required.

Stake in pit, with letters and figures necessary.

Mounds of earth, which in many regions are the least durable and least useful of all accessories.

In establishing corners, the first preference is given to durable stones, then posts, and lastly mounds with stake in pit. The selection of the particular construction to be adopted for any corner is left to the deputy surveyor, who, in the selection of the corner, is instructed to assign the greatest weight to the durability of the materials and permanency of the monuments. Posts are not to extend more than 12 inches above ground, and if more than 3 feet long, the extra length is to be put below the surface.

31. Marking Lines Between Corners.—The marking of trees and brush along the lines of the public-land surveys is required by law. Trees that are intersected by the line

are marked by two chops or notches cut on the sides facing the line, without any other marks. Such trees are called **sight trees** or **line trees**. Fig. 5 illustrates the manner of marking line trees. The chops or blazes are made on each side of the tree where the line intersects it. A sufficient number of other trees are marked, if found within 50 links of the line, so that the line can be readily traced. Where a tree thus blazed stands very near the line, the blazes are opposite each other, coinciding in direction with the line, and where the tree is farther from the line, the blazes are placed nearer each other on the side of the tree toward the line; the farther the tree is from the line the

FIG. 5 FIG. 6 FIG. 7

nearer the blazes approach each other. Fig. 6 illustrates the method of marking a tree that stands near the line, as seen from the line at a point opposite the tree. Fig. 7 illustrates the method of marking a tree that stands at some distance from the line, as seen from a point in the line opposite the tree.

Trees are blazed through the bark into the wood, so as to make a permanent mark. Such a blaze endures and is recognizable as long as the tree stands. The blazing of trees is not omitted where trees 2 inches or more in diameter are found within 50 links of the line. Lines are also marked by cutting away such small brush as interferes

with correct sighting of instruments. Random lines are not blazed, but bushes and limbs may be lopped and stakes may be set on the random line at every 10 chains to enable the surveyor on his return to establish the true line. These stakes are removed when the true line is marked.

DESCRIPTIONS OF CORNERS

32. Abbreviations Allowed.—In the descriptions of corners in the field notes and records, the dimensions of stones, posts, and pits are expressed in the order of their length, breadth, and thickness, as for instance "a stone $23 \times 10 \times 8$ in." To describe a mound, the material, diameter of base, and altitude are given, as "mound of earth 4 ft. base, 2½ ft. high." "The following contractions are authorized to be used in the preparation of field notes, transcripts, inspection reports, and similar records, and no others should be introduced:*

A.	acres	elong.	elongation
a. m.	forenoon	frac.	fractional
A. M. C.	aux. meander corner	ft.	foot, feet
asc.	ascend	G. M.	guide meridian
astron.	astronomical·	h., hrs.	hour, hours
bdy.	boundary	ins.	inches
bdrs.	boundaries	lat.	latitude
bet.	between	L. C.	lower culmination
B. O.	bearing object	lks.	links
B. T.	bearing tree	l. m. t.	local mean time
C. C.	closing corner	long.	longitude
chs.	chains	m.	minutes
cor., cors.	corner, corners	mag.	magnetic
corr.	correction	M. C.	meander corner
decl.	declination	mer.	meridian
dep.	departure	mkd.	marked
desc.	descend	N.	north
dia.	diameter	NE.	northeast
diff.	difference	NW.	northwest
dist.	distance	obs.	observe
D. S.	deputy surveyor	obsn.	observation
E.	east	p. m.	afternoon

*United States Manual of Surveying Instructions.

Pol. . . .	Polaris
Pr. Mer. .	principal meridian
Pt. of Tr.	point of triangulation
¼ sec. . .	quarter section
R., Rs. .	range, ranges
red. . . .	reduce, reduction
S.	south
S. C. . .	standard corner
SE. . . .	southeast
sec., secs.	section, sections
S. M. C. .	special meander corner
sq.	square
St. Par. .	standard parallel
SW. . . .	southwest
T., or Tp.	township
Ts., or Tps.	townships
temp. . .	temporary
U. C. . .	upper culmination
var. . . .	variation
W. . . .	west
W. C. . .	witness corner
w. corr. .	watch correction
W. P. . .	witness point
w. t. . . .	watch time”

The abbreviations given in this list are required to be used in the field notes and similar records of an official character relating to the surveys of the public lands of the United States. The same abbreviations are not necessarily used in the field notes and records of private surveys, though it is well for the private surveyor to use them in order to be familiar with them. These abbreviations will be often used in this text, except for the words *chains*, *hours*, *inches*, and *links*, for which the abbreviations *ch.*, *hr.*, *in.*, and *li.*, respectively, will be used, in order to be uniform with the forms of these abbreviations used in other parts of this Course.

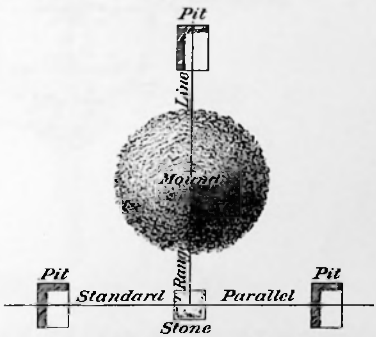


FIG. 8

33. Authorized Forms and Descriptions of Corners.—There are thirteen classes of corners with eight variations of construction and markings in each class. The following are examples of the descriptions and arrangements of corners—one in each class:

1. *Standard Township Corner: Stone, With Pits and Mound of Earth.*—Set a — stone, —×—×— in., — in. in the ground, for standard corner of Tps. 13 N, Rs. 21 and 22 E, marked S. C. on N; with six grooves on N, E, and W faces; dig pits, 30 × 24 × 12 in., crosswise on each line, E and W, 4 ft., and N of stone, 8 ft. distance; and raise a mound of earth, 5 ft. base, 2½ ft. high, N of corner.

Fig. 8 illustrates the standard township corner here described.

2. *Closing Township Corner: Stone, With Mound of Stone.* Set a — stone, —×—×— in., — in. in the ground, for

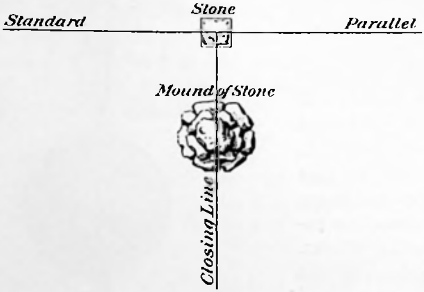


FIG. 9

closing corner of Tps. 4 N, Rs. 2 and 3 W, marked C. C. on S; with six grooves on S, E, and W faces; and raise a mound of stone, 2 ft. base, 1½ ft. high, S of corner. Pits impracticable.

Mound of stone will consist of not less than four stones, and will be at least 1½ ft. high, with 2 ft. base. Fig. 9 illustrates the closing township corner just described.

3. *Corner Common to Four Townships: Stone, With Bearing Trees.*—Set a — stone, —×—×— in., — in. in the ground, for corner of Tps. 2 and 3 N, Rs. 2 and 3 W, marked with six notches on each edge, from which

A —, — in. diameter, bears N —° E, — li. distance, marked T 3 N R 2 W S 31 B T.

A —, — in. diameter, bears S —° E, — li. distance, marked T 2 N R 2 W S 6 B T.

A —, — in. diameter, bears S —° W, — li. distance, marked T 2 N R 3 W S 1 B T.

A —, — in. diameter, bears N —° W, — li. distance, marked T 3 N R 3 W S 36 B T.

All bearing trees will be marked with the township, range, and section in which they stand. Fig. 10 illustrates the corner common to four townships as described above.

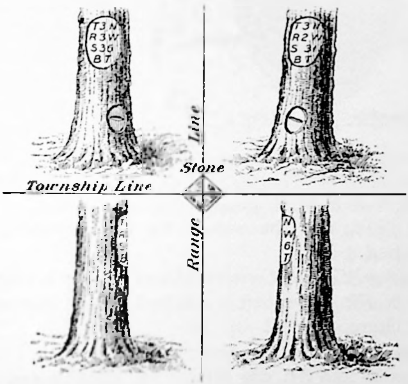


Fig. 10

4. *Corner Common to Two Townships Only: Post, With Pits and Mound of Earth.*—Set a — post, 3 ft. long, 4 in. sq., with marked stone (charred stake or quart of charcoal), 24 in. in the ground, for corner of Tp. 2 N, R. 5 W, and Tp. 3 N, R. 6 W, on N boundary Tp. 2 N, R. 6 W, marked

T 2 N R 5 W S 6 on SE, and

T 3 N R 6 W S 36 on NW face, with six notches on N and W edges; dig pits 30 × 24 × 12 in., on each line, E and W, 4 ft., and N of post, 8 ft. distance; and raise a mound of earth, 5 ft. base, 2½ ft. high, N of corner.

Fig. 11 illustrates the corner common to two townships only, as just described.

5. *Corner for One Township Only: Post, With Bearing Tree.*—Set a — post, 3 ft. long, 4 in. sq., 24 in. in the ground, for SW corner of Tp. 3 N, R. 6 W, marked

T 3 N R 6 W S 31 on NE,

S 1 on SE,

T 2 N R 7 W S 1 on SW, and

S 1 on NW face, with six notches on N and E edges; from which

A —, — in. diameter, bears N —° E, — li. distance, marked

T 3 N R 6 W S 31 B T.

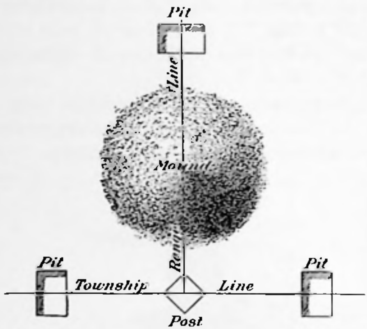


Fig. 11

Fig. 12 illustrates the corner for one township only, as just described.

6. *Standard Section Corner: Mound of Earth, With Deposit, and Stake in Pit.*—Deposit a marked stone (charred stake or quart of charcoal), 12 in. in the ground, for standard corner of Sections 33 and 34; dig pits, 24 × 18 × 12 in., crosswise on each line, N, E, and W of corner, 5 ft. distance; and raise a mound of earth, 4 ft. base, 2 ft. high, over deposit.

In E pit drive a — stake, 2 ft. long, 2 in. sq., 12 in. in the ground, marked

S C T 13 N R 22 E on N,

S 34 on E, and

S 33 on W face; with three grooves on E and W faces.

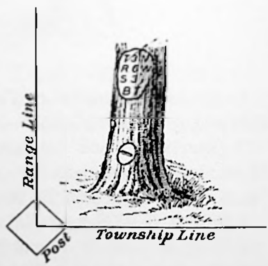


Fig. 12

Fig. 13 illustrates the standard section corner described in the preceding notes.

7. *Closing Section Corner: Tree Corner, With Pits and Mound of Earth.*—A —, — in. diameter, for closing corner of Sections 1 and 2, I mark

C C T 4 N R 3 W on S,

S 1 on E, and

S 2 on W side, with one notch on E. and five notches on W side, dig pits, 18 × 18 × 12 in. S, E, and W of corner, 5 ft. distance; and raise a mound of earth around tree.

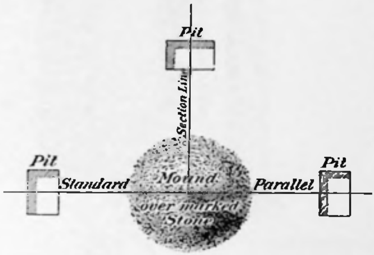


Fig. 13

Fig. 14 illustrates the closing corner just described.

8. *Corner Common to Four Sections: Tree Corner, With Bearing Trees.*—A —, — in. diameter, for corner of Sections 5, 6, 7, and 8, I mark

T 2 N S 5 on NE,

R 2 W S 8 on SE,

S 7 on SW, and

S 6 on NW side,

with five notches on S and E sides; from which

Fig. 14

A —, — in. diameter bears N —° E, — li. distance, marked

T 2 N R 2 W S 5 B T. A —, — in. diameter, bears S —° E, — li. distance marked

T 2 N R 2 W S 8 B T.

A —, — in. diameter, bears S —° W, — li. distance, marked

T 2 N R 2 W S 7 B T.

A —, — in. diameter, bears N —° W. — li. distance marked T 2 N R 2 W S 6 B T.

Fig. 15 represents the tree corner with four bearing trees,

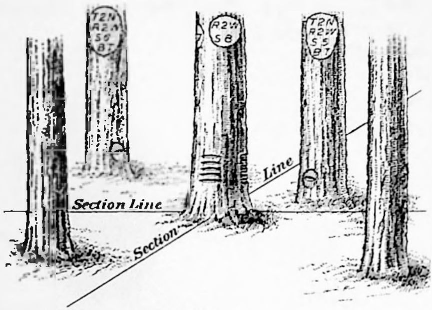


Fig. 15

the corner being common to four sections. Bearing trees are always marked on the side facing the corner, and, consequently, the marks are to be seen plainly on only two of the bearing trees in the figure.

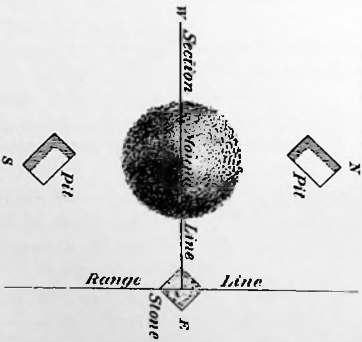


Fig. 16

9. *Section Corner Common to Two Sections Only: Stone, With Pits and Mound of Earth.*—Set a — stone, —×—×— in., — in. in the ground for corner of Sections 25 and 36 marked with five notches on N, and one notch on S edge; dig pits 24 × 24 × 12 in., in each section, 6 ft. distant; and raise a mound of earth, 4 ft. base, 2 ft. high, W of corner.

Fig. 16 shows the section corner common to two sections only, described in the preceding notes. Such a corner when on the range line has notches on its opposite edges, coinciding with the range line, to indicate the distances, in miles, to the NE and SE corners of the township.

10. *Section Corner Referring to One Section Only: Stone With Mound of Stone.* Set a — stone, —×—×— in. in the ground, for SW corner of Section 12, marked with one notch on E edge; and raise a mound of stone, 2 ft. base, 1½ ft. high, NE of corner.

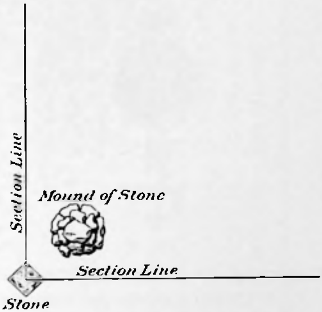


Fig. 17

Fig. 17 shows the stone corner to one section only as described in the preceding notes.

11. *Quarter Section Corner: Stone, With Bearing Trees.* Set a — stone, —×—×— in., — in. in the ground, for ¼ section corner marked ¼ on W face; from which

A —, — in. diameter, bears N —° E, — li. distance, marked ¼ S 16 B T.

A —, — in. diameter, bears N —° W, — li. distance, marked ¼ S 17 B T.

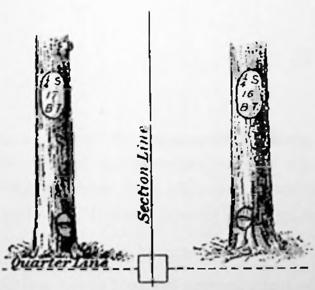


Fig. 18

Fig. 18 represents the quarter-section corner described in the notes.

12. *Meander Corners: Post, With Pit and Mound of Earth.*—Set a — post, 3 ft. long, 4 in. sq., with marked stone (charred stake or quart of charcoal), 24 in. in the

ground for meander corner of fractional Sections 19 and 20,

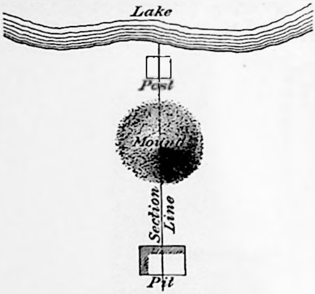


FIG. 19

marked
M C on N,
T 15 N on S,
R 20 E S 20 on E, and
S 19 on W face; dig a pit, 36 × 36 × 12 in., 8 ft. S of post; and raise a mound of earth, 4 ft. base, 2 ft. high, S of corner.

Fig. 19 represents the meander corner just described.

Corners on Reservation or Other Boundaries Not Conforming to the Rectangular System

Corner Monument of Stone, With Deposit

Deposit a marked stone (charred stake, quart of charcoal, or vial with record* enclosed), 12 in. in the ground, for the SW corner of the Nez Perces Indian Reservation; and build a monument of stone, 3 ft. sq. at base, 2 ft. sq. on top, 3 ft. high, over deposit; marked

SW cor N P I R on NE,†
P L ‡ — ‖ M — ‖ ch. on SE,
P L — § on SW, and
P L on NW face.

34. Size, Position, and Distance of Pits and Mounds.—The two following tables summarize briefly the rules relating to the requirements for pits and mounds with respect to their size, position, and distance from the corner.

* The record will consist of a brief description of the corner, with the date of its construction.

† The markings will be cut into large stones, inserted in the middle of the lowest course on each side of the monument.

‡ The letters P. L. indicate public land, unsurveyed.

‖ The proper number of miles and chains, from the initial point, will be stated.

§ The year in which the monument is established will be placed in the blank.

TABLE III

REQUIREMENTS AS TO SIZE AND POSITION OF PITS

Kind of Corner	Size at Tree Corner Inches	Size at Other Corners Inches	Position From Corner
Standard township corner	24×18×12	30×24×12	Across N, E, and W lines
Closing township corner	24×18×12	30×24×12	Across E, W, and S lines
Corner of 4 townships	24×18×12	24×24×12	On lines N, E, S, and W
Corner of 2 townships	24×18×12	30×24×12	On each line
Corner of 1 township	30×24×12	36×36×12	On each line
Standard section corner	18×18×12	24×18×12	Across E, W, and N lines
Closing section corner	18×18×12	24×18×12	Across E, W, and S lines
Corner of 4 sections	18×18×12	18×18×12	In each section NE, etc.
Corner of 2 sections	18×18×12	24×24×12	In both sections
Corner of 1 section	24×24×12	36×36×12	In the section
Quarter-section corner	18×18×12	18×18×12	On line each side
Meander corner	36×36×12	36×36×12	On line, rear of corner
On reservation line	36×36×12	36×36×12	See Manual

TABLE IV

DISTANCE OF PITS AND REQUIREMENTS AS TO MOUNDS

Kind of Corner	Distance of Pits at			Mounds		
	Post Corner	Mound of Earth Corner Feet	Tree Corner Feet	Size in Feet		Position From Corner
				Stone	Earth	
Standard township corner	E and W 4 feet, N 8 feet .	5	5	2×1½	5 ×2½	N
Closing township corner .	E and W 4 feet, S 8 feet .	5	5	2×1½	5 ×2½	S
Corner of 4 townships . .	N, E, and W 4 feet, S 8 feet	5	5	2×1½	5 ×2½	S
Corner of 2 townships . .	E and W 4 feet, N 8 feet .	5	5	2×1½	5 ×2½	Various
Corner of 1 township . .	8 feet	5	5	2×1½	5 ×2½	Various
Standard section corner .	E and W 3 feet, N 7 feet .	5	4	2×1½	4 ×2	N
Closing section corner . .	E and W 3 feet, S 7 feet .	4	5	2×1½	4 ×2	S
Corner of 4 sections . . .	5½ feet	4	5	2×1½	4 ×2	W
Corner of 2 sections . . .	6 feet	4	5	2×1½	4 ×2	W
Corner of 1 section . . .	8 feet	5	5	2×1½	4 ×2	Various
Quarter-section corner . .	3 feet	4	4	2×1½	3½×1½	Various
Meander corner	8 feet	5	8	2×1½	4 ×2	With pit
On reservation line . . .	4 feet	5	5	3×2	5 ×2½	Various

35. Bearing Trees.—When used for standard corners, bearing trees are selected on the north side of the line; when used for closing corners, they are selected on the south side of the line. When the trees can be found within 300 links of the corner, one tree is marked for a corner that relates to only one section, two trees are marked for a corner of two sections only, or for a quarter-section corner or meander corner, and four trees are marked for every corner common to four townships or sections and on reservation lines.

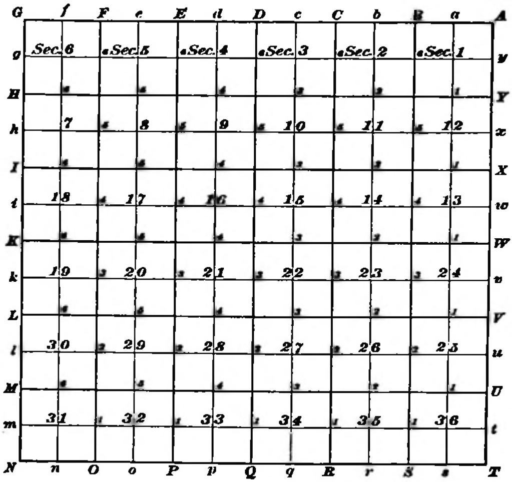


FIG. 20

When only a few bearing trees or rocks are accessible, they are used though even 10 or more chains distant. Bearing trees and objects must have their bearings given from the true meridian and the distance measured from the center of the corner to the center of the bearing tree. The following rule is now in force relative to marks on bearing trees:

Rule.—*Place all letters and figures on that part of the tree which would probably remain as the stump, and make one plain*

blaze high on the same side to attract notice in case of snow or dense undergrowth.

36. System of Designating Corners.—A system of notation that is used by the United States Land Department for designating the different corners in a township is shown in Fig. 20. By the use of this system, field notes can be abridged and rendered more concise and explicit, with correspondingly less liability to error both in recording and reading them. The diagram shown in the figure represents the plat of a township in which the different corners are designated by a conventional system of letters and figures, as shown.*

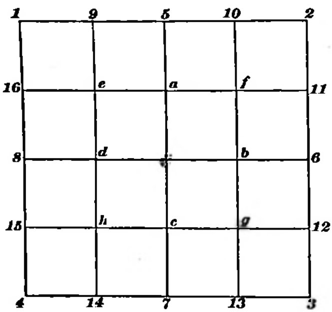


FIG. 21

All corners in the township boundaries are designated by letters, capital letters being used for the section corners and small letters for the quarter-section corners. Such corners are referred to merely by the proper letters, as *corner E* for the section corner in the north boundary between Sections 4 and 5, *corner m* for the quarter-section corner on the west boundary of Section 31. Section corners in the interior of the township are referred to by the numbers of the four sections to which the corner is common, as *corner of Sections 3, 4, 9, and 10.* Interior quarter-section corners are referred to by their positions on the lines, as *V to L at 4*, *O to F at 5*, etc.

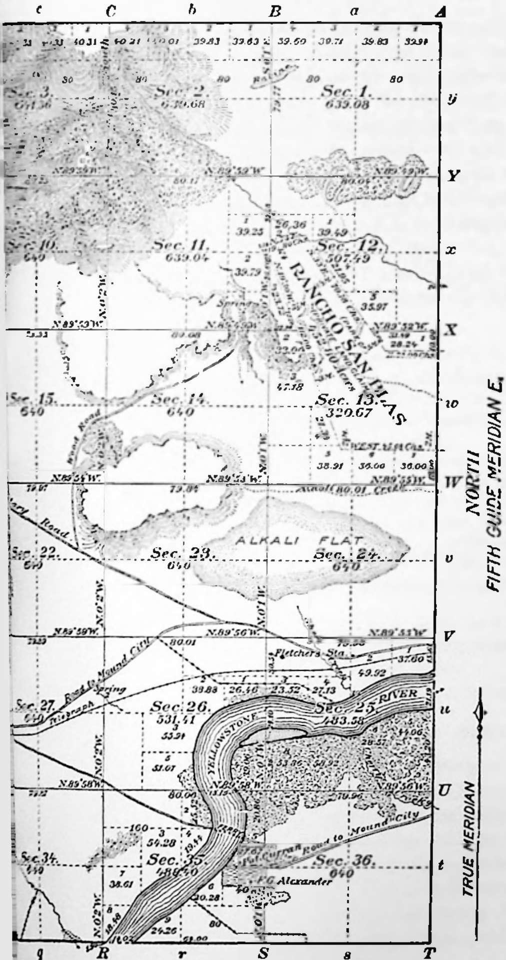
The system of notation† shown in Fig. 21 has been used by many private surveyors for designating the subdivision corners of sections. The corners on the exterior lines of the

*In the plats of the earlier surveys the letter *A* is placed in the upper left-hand corner and the letters pass around the plat in the opposite direction.

†From Hodgman's Manual of Land Surveying.

NORTH
Sec. 6. 622.07
Sec. 5. 590.95
Sec. 4. 628.21
Cemetery
Sec. 7. 386.28
LAKE CITY
Sec. 8. 640
Sec. 9. 640
Montana and Manitoba Railroad
W.C.60Lks.N. Wood Road
Limestone Quarry
Springs
Pond
Acequia
Sec. 16. 640
Sec. 17. 505.07
Sec. 18. 98.49
LIN'S LAKE
Diamond Rock
Ivy Island
Sec. 19. 431.63
Sec. 20. 556.53
Sec. 21. 640
Road to Lake City
Telegraph
Spring
Sec. 30. 626.24
Sec. 29. 640
Sec. 28. 640
James Parker's
Desert Land Claim
Sec. 31. 627.40
Sec. 32 640
Sec. 33. 368.59
CLEAR
Trail

1 L T 85, 419 § 21



Latitude 45°45′ N.
Longitude 107°54′ W.
Mean mag. decl. 18°10′ E.

section are numbered in definite order and according to their relative importance. Thus, the four section corners are designated by the first four numbers, the four quarter-section corners on the section lines by the next four numbers, etc. The corners in the interior of the section are designated by small letters, except the center of the section, which is designated by a capital *C*. The corners on the quarter-section lines are designated by the first four small letters, the other small letters being used to designate the corners at the centers of the quarter sections.

The following example of field notes illustrates the manner in which the notation is used in keeping notes when subdividing a section:

Chains	
	Began at 7. Found stake in place and both bearing trees standing. Planted stone 25 × 8 × 6 in. marked + for corner. Thence north on random, dec. 2° 30′ E, setting temporary stakes every 10 ch.
80.22	Intersected section line 26 li. west of 5.
	At 5 found rotten stake at correct point, S 28° W 66 li. from stump of W oak bearing tree of U. S. Survey.
	Drove stake for corner and put broken earthenware and glass around it and marked { Wh. oak 12 in. N 66° E 42 li. / Wh. oak 18 in. N 34 W 63 li.
	From 5 ran east on random, setting temporary stakes every 10 ch.
39.92	Intersected sec. line 12 li. north of 2. Found earthen post in correct position and bearing trees of resurvey standing. Thence west on corrected line.

OFFICIAL PLAT AND FIELD NOTES

37. Official Plat of Township.—Fig. 22 represents the official plat of the subdivision of a township, reduced to one-half size. The scale used for the official plats is 40 chains to an inch, making the outline of the township 12 inches square. Each official plat of the subdivision of a township is prepared in triplicate. One plat is retained in the office of the surveyor general of the surveying district, one is transmitted to the General Land Office, and one is filed in the

United States land office for the district. When the counties of any state become organized, copies of the official plats of the townships in each county are usually filed in the office of the recorder of deeds for the county, or in the office of the county surveyor, and are open to public inspection.

The township represented in Fig. 22 is township No. 15 north, range No. 20 east (T. 15 N, R. 20 E) of the principal meridian, state of Montana. The notes of the survey of the west and north boundaries of Section 36 and the west boundary of Section 25 of this township are given in Art. **38.** A careful study of this plat will be instructive.

On the plat are shown the bearings of all section lines, the lengths of all east-and-west section lines and of the north-and-south section lines in the north tier of sections, the bearings and lengths of all fractional subdivisions; all natural features, artificial features, and improvements, such as lakes, streams, mountains, wooded lands, swamps, roads, railroads, telegraph lines, irrigating ditches, irrigated areas, settlers' claims, ranches, and towns; and such other information regarding the country surveyed as may be of value. All section lines whose lengths are not given have the regular length of 80 chains.

The township is in range 20 east, and, consequently, its east boundary is formed by the fifth guide meridian east of the principal meridian. The respective bearings of the north-and-south section lines, passing to the westward through the township, vary by 1 minute, according to the principle explained in Art. **17.**

The north-and-south section line between Sections 21 and 22 and the east-and-west section line between Sections 21 and 16 are offset in order to pass around an impassable swamp. The method of offsetting the lines is clearly shown.

At about the center of Section 33 is a lake lying wholly within the section. The two positions of the north-and-south quarter line are extended from the quarter-section corners on the north and the south boundaries of the section to the respective shores of the lake, and special meander corners are set at the points where these lines intersect the shore of

the lake, and these corners are connected by meander lines delineating the form of the lake. The meander corner on Diamond Rock in Section 18 is an auxiliary meander corner.

It will be noticed that wherever a section line or other survey line crosses a stream, the width of the stream, in links, is marked on the plat.

The letters around the outer edge of the diagram are for the purpose of designating the different section and quarter-section corners on the township boundaries, according to a system that will be explained in the next article.

38. Field Notes.—The field notes of the public-land surveys are the record of the deputy surveyor's work. They are the evidence written at the time and place, as the work is performed, and which, if correctly recorded, furnish, after the monuments themselves, the best and most reliable evidence of the location of the boundaries. The field notes of the United States Surveys contain in a condensed form a minute record of everything done by the deputy surveyor and his assistants relative to running, measuring, and marking lines and establishing corners, and the observations made by them relative to the topography of the country surveyed, and such other data as are required to be recorded by them. These notes are usually spoken of as the *original field notes.* They are not necessarily the entries made in the field in the deputy's pocket notebooks or tablets, but the copy from such tablets fully written out in ink for the permanent record of the work. The following specimen of field notes shows the general style and manner of keeping the notes, and also the method of running the west and north boundaries of Section 36, and the west boundary of Section 25. The notes given are those of the respective boundaries of the two sections mentioned, in the township shown by the plat of Fig. 22.

SPECIMEN OF FIELD NOTES

Subdivision of T. 15 N, R. 20 E

Chains	
	I commence at the cor. of secs. 1, 2, 35, and 36, on the S bdy. of the Tp., which is a sandstone, 6 × 8 × 5 in. above ground, firmly set, and marked and witnessed as described by the surveyor general.
	Thence I run
	N 0° 01′ W, bet. secs. 35 and 36.
	Over level bottom land.
4.50	Wire fence, bears E and W.
20.00	Enter scattering cottonwood timber, bears E and W. F. G. Alexander's house bears N 28° W.
29.30	Leave scattering cottonwoods, bearing E and W; enter road, bears N.
30.00	SE cor. of F. G. Alexander's field; thence along west side of road.
39.50	To crossroads, bears E to Mound City; N to Lake City. F. G. Alexander's house bears S 40° W. The ¼ sec. cor. point will fall in road; therefore,
	Set a cedar post, 3 ft. long, 3 in. sq., with quart of charcoal, 24 in. in the ground, for witness cor. to ¼ sec. cor., marked W. C. ¼ S 35 on W and 36 on E face; dig pits, 18 × 18 × 12 in. N and S of post, 3 ft. dist.; and raise a mound of earth, 3½ ft. base, 1½ ft. high, W of cor.
40.00	Point for ¼ sec. cor. in road.
	Deposit a marked stone 24 in. in the ground, for ¼ sec. cor.
	The SE cor. of Pat. Curran's field bears W, 5 li. dist.
40.50	Set a limestone, 15 × 8 × 6 in., 10 in. in the ground, for witness cor. to ¼ sec. cor., marked W. C. ¼ S on W face; dig pits, 18×18×12 in. N and S of stone, 3 ft. dist.; and raise a mound of earth, 3½ ft. base, 1½ ft. high, W of cor.
	Thence along E side of field.
50.50	NE cor. of Pat. Curran's field, bears W 4 li. dist.
51.50	Leave road; which turns to N 70° W, leads to ferry on Yellowstone River; thence to Lake City.
57.50	Enter dense cottonwood and willow undergrowth, bears N 54° E and S 54° W.
72.50	Leave undergrowth, enter scattering timber, bears N 60° E and S 60° W.
80.00	Set a locust post, 3 ft. long, 4 in. sq., 24 in. in the ground, for cor. of secs. 25, 26, 35, and 36, marked T 15 N S 25 on NE, R 20 E S 36 on SE,

SPECIMEN OF FIELD NOTES—(Continued)

Subdivision of T. 15 N, R. 20 E

Chains	
	S 35 on SW, and
	S 26 on NW face; with 1 notch on S and E faces; from which
	An ash, 13 in. diam., bears N 22° E, 26 li. dist., marked T 15 N R 20 E S 25 B T.
	A sycamore, 23 in. diam., bears S 71½° E, 37 li. dist., marked T 15 N R 20 E S 36 B T.
	A walnut, 17 in. diam., bears S 64° W, 41 li. dist., marked T 15 N R 20 E S 35 B T.
	A cottonwood, 13 in. diam., bears N 21¼° W, 36 li. dist., marked T 15 N R 20 E S 26 B T.
	Last 20 ch. of this mile subject to overflow, 2 to 4 ft. deep.
	Land, level bottom.
	Soil, alluvial; 1st rate. No stones were obtainable.
	Timber, scattering cottonwood, sycamore, ash, and walnut; undergrowth, cottonwood and willow.
	Dense undergrowth, 15 ch.
	S 89° 57′ E, on a random line bet. secs. 25 and 36.
40.00	Set temp. ¼ sec. cor.
79.96	Intersect E bdy. of Tp. 3 li. N of cor. of secs. 25, 30, 31, and 36, which is a sandstone, 5 × 8 × 5 in. above ground, marked and witnessed as described by the surveyor general
	Thence I run
	N 89° 56′ W, on a true line bet. secs. 25 and 36.
	Over level bottom land, through scattering timber.
13.00	Leave scattering timber, bears N and S.
18.60	Cherry Creek, 12 li. wide; clear water, 1 ft. deep; gentle current, sandy bottom; course N.
20.50	Enter heavy timber, bears N and S.
32.50	Leave heavy timber, bears NW and SE.
39.98	Deposit a quart of charcoal, 12 in. in the ground, for ¼ sec. cor.; dig pits, 18 × 18 × 12 in. E and W of cor. 4 ft. dist.; and raise a mound of earth, 3½ ft. base, 1½ ft. high, over deposit. In E pit drive a cedar stake 2 ft. long, 2 in. sq., 12 in. in the ground, marked ¼ S 25 on N and 36 on S face.
46.50	Enter heavy timber, bears N and S.
76.00	Leave heavy, enter scattering timber, bears N 25° E and S 25° W.

Specimen of Field Notes—(Continued)

Subdivision of T. 15 N, R. 20 E

Chains	
79.96	The cor. of secs. 25, 26, 35, and 36. Land nearly level; mostly subject to overflow 2 to 5 ft. deep. Heavily timbered land, 41.5 ch. N 0° 1′ W, bet. secs. 25 and 26. Over level bottom land, through scattering timber.
25.36	Right bank of Yellowstone River. Set a locust post, 3 ft. long, 4 in. sq., 24 in. in the ground, for meander cor. of frac. secs. 25 and 26, marked M C on N, T 15 N on S, R 20 E, S 25 on E, and S 26 on W faces; from which A cottonwood, 12 in. diam., bears S 18¼° E, 16 li. dist., marked T 15 N, R 20 E, S 25, M C B T. A sycamore, 31 in. diam., bears S 74½° W, 25 li. dist., marked T 15 N, R 20 E, S 26, M C B T. Enter shallow channel, 1 to 2 ft. deep.
26.00	Across shallow channel, 64 li. wide, to sand bar parallel to river bank; thence on sand bar.
32.12	To right bank of main channel, course E; point for triangulation.
40.00	Point for ¼ sec. cor. falls in river. To determine the dist. across, I set a flag on line, on left bank; then measure a base, N 89° 59′ E, 20.00 ch. to a point, from which the flag bears N 49° 06′ W; from the flag the E end of base bears S 49° 6′ E; therefore, the dist. is tan. 40° 55′ × base, or .867 × 20 = 17.34 ch.; making the whole distance from meander cor., .64 + 6.12 + 17.34 = 24.1 ch., which added to 25.36, makes
49.46	To left bank of Yellowstone River; bank, 12 ft. high. Deposit a marked stone, 12 in. in the ground for meander cor. of frac. secs. 25 and 26, dig a pit, 36 × 36 × 12 in., 5 ft. N of cor. and raise a mound of earth, 4 ft. base, 2 ft. high, over deposit. In the pit drive a cedar stake, 2 ft. long, 2 in. sq., 12 in. in the ground, marked M C on S, T 15 N on N, R 20 E, S 26 on W, and S 25 on E faces.

SPECIMEN OF FIELD NOTES—(Continued)

Subdivision of T. 15 N, R. 20 E

Chains	
	Thence over level bottom land. Some small cottonwoods, none within limits suitable for bearing trees.
52.60	Leave bottom, begin ascent, bears E and W.
53.60	Top of ascent and edge of sandy plain, 40 ft. above river, bears E and W.
55.70	Wire fence, bears E and W.
62.80	Telegraph line, bears E and W.
80.00	Set a cedar post, 3 ft. long, 4 in. sq., with marked stone, 24 in. in the ground, for cor. of secs. 23, 24, 25, and 26, marked T 15 N, S 24 on N E, R 20 E, S 25 on S E, S 26 on S W, and S 23 on N W faces; with 2 notches on S and 1 notch on E edges; dig pits, 18 × 18 × 12 in. in each sec. 5½ ft. dist.; and raise a mound of earth, 4 ft. base, 2 ft. high, W of cor.
	Land, level.

39. Modifications and Changes.—There have been various modifications and changes in the methods of conducting the United States land surveys. In the surveys made previous to 1846, and in many of the surveys made since that time, closing corners were established on the north and west boundaries of every township, thus making a double set of corners on every township boundary. In these surveys, only two bearing trees were marked for any corner. Other trees were lettered for the township, range, and section, but their bearings and distances from the corners were not taken. These were spoken of as **witness trees** to distinguish them from bearing trees, but they do not appear in the field notes.

There has been much difference in the manner of marking bearing trees in the different surveying districts. One custom, quite prevalent in the earlier surveys, was to cut a blaze about a foot from the ground on the side of the tree facing the corner and then cut a notch in the middle of the blaze with two blows of the axe. The distance to the corner

was measured from this notch or witness mark. The tree was also blazed at the height of a man's head, and the required letters were cut in the tree within this blaze.

The early requirements for the surveys were not rigid with regard to either the directions or the lengths of the lines run, and such as they were, they were not enforced. Both base lines and meridians have in some instances been run by the magnetic needle in connection with, and as a part of, the survey of the township boundaries. The alinement was often bad and the chaining was no better. The surveyor who now retraces the lines of the United States land surveys will find in them every stage and condition of work from careful, well-executed surveys to works of the imagination made mainly on paper in the deputy surveyor's camp. Errors in direction, not of minutes merely, but of degrees, and errors of distance not of links merely, but of chains and tallies,* are not uncommon.

In the earlier surveys, the areas of fractional lots were computed by the deputy in the field, with the result that in many cases the areas thus given on the official plats cannot be verified from the field notes by any known method of computation. Indeed, they appear to have been guessed at without any computation whatever.

*Tally is the name applied to each distance of 10 chains, at the extremity of which the chainmen exchange pins and call *tally*.

UNITED STATES LAND SURVEYS
(PART 2)

SUBDIVISION AND RESURVEYS

SUBDIVISION OF SECTIONS

1. The general principles governing the subdivision of sections are to be found in the United States statute of February 11, 1805, of which the following are the leading provisions:

SECTION 100. The boundaries and contents of the several sections, half sections, and quarter sections of the public lands shall be ascertained in conformity with the following principles:

First.—All the corners marked in the surveys returned by the surveyor general shall be established as the proper corners of sections, or subdivisions of sections, which they were intended to designate, and the corners of half and quarter sections, not marked on the surveys, shall be placed as nearly as possible equidistant from two corners which stand on the same line.

Second.—The boundary lines, actually run and marked in the surveys returned by the surveyor general, shall be established as the proper boundary lines of the sections or subdivisions for which they were intended, and the length of such lines as returned shall be held and considered as the true length thereof. And the boundary lines which have not been actually run and marked shall be ascertained by running straight lines from the established corners to the opposite corresponding corners; but in those portions of the fractional townships, where no such opposite corresponding corners have been or can be fixed, the boundary lines shall be ascertained by running from the established corners due north-and-south or east-and-west lines, as the case may be, to the water course, Indian boundary line, or other external boundary of such fractional township.

Third.—Each section or subdivision of section the contents whereof have been returned by the surveyor general shall be held and considered

as containing the exact quantity expressed in such return; and the half sections and quarter sections the contents whereof shall not have been thus returned shall be held and considered as containing the one-half or the one-fourth part, respectively, of the returned contents of the section of which they may make part.

SECTION 101. In every case of the division of a quarter section the line for the division thereof shall run north and south, and the corners and contents of half-quarter sections which may thereafter be sold shall be ascertained in the manner and on the principles directed and prescribed by the section preceding, and fractional sections containing one hundred and sixty acres or upwards shall in like manner, as nearly as practicable, be subdivided into half-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Interior, and in every case of a division of a half-quarter section, the line for the division thereof shall run east and west, and the corners and contents of quarter-quarter sections, which may thereafter be sold, shall be ascertained, as nearly as may be, in the manner and on the principles directed and prescribed by the section preceding; and fractional sections containing fewer or more than one hundred and sixty acres shall in like manner, as nearly as may be practicable, be subdivided into quarter-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Interior.

It will be seen that the statute makes the corners and boundary lines of the United States land surveys unalterable, no matter what errors may have been committed in locating them. Although errors in the surveys have been numerous and some of them great, the surveys cannot be changed, and, however erroneous they may be, the boundaries and corners established by them are the true boundaries and corners. The statute also makes the length of the lines as returned by the surveyor general (which is the length given in the field notes) their true legal length. This defines and establishes a standard of measure for the line between every two corners of the government survey, which is the only legal standard for that line. All subsequent surveys under that system must make their measures conform to the standard of the measurement recorded in the field notes and marked on the ground by the corner monuments.

In subdividing a section, it may be necessary to first make a resurvey or retracement of the section in order to find or relocate any missing corner of the original survey. Supposing

all those corners to be known, there are four cases in subdividing sections not made fractional by waters or reservations. There is one general principle, however, that applies to all these cases, namely: *The section is divided into quarters by straight lines extending from the established quarter-section corners to the opposite corresponding corners, that is, from quarter post to opposite quarter post.*

The four cases in which the subdivision of a section, though the same with respect to the principle just stated, differs with respect to other conditions, are as follows:

2. Case I.—*Regular Sections.*—In the regular sections, such as Sections 8, 16, etc., all the corners on the exterior boundaries of the section are established by the original survey. The north-and-south lines or meridional boundaries, are 80 chains in length. The east-and-west lines, or latitudinal boundaries, though intended to be 80 chains in length, may or may not be that length, and usually are not. All the quarter-section corners are midway between the two section corners standing on the same section line. The section is divided into quarter sections by straight lines extending from each quarter-section corner to its opposite corresponding corner. The intersection of these lines establishes the legal center of the section; that is to say, the common corner of the four quarter sections.

To divide the quarter section, corners are first established on its north and south boundaries at points midway between the corners of the quarter section; that is, the corner on the section line is established midway between the section corner and the quarter-section corner, and the corner on the quarter-section line is established midway between the quarter-section corner on the section line and the corner at the center of the section. A north-and-south line running straight between the corners thus established divides the quarter into half-quarters. In a similar manner, the half-quarter is divided into quarter-quarters by an east-and-west line. This east-and-west dividing line may extend entirely across the quarter section, from a corner on the section line

midway between the section corner and quarter-section corner to a corner on the quarter-section line midway between the quarter-section corner on the section line and the corner at the center of the section, substantially the same as the north-and-south half-quarter line; or, in case it is desired to divide only one of the half-quarters, the line may extend from either of the above described corners to a corresponding corner established midway between the extremities of the north-and-south half-quarter line. It practically makes no difference whether the center of the quarter section is fixed at the intersection of the lines extending entirely through the quarter section, or at the center of the half-quarter line. The same result will be obtained in either case, but the latter method is usually the more expeditious when only one half-quarter is to be divided.

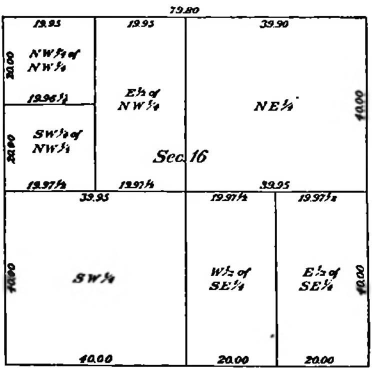


Fig. 1

Fig. 1 illustrates the manner of subdividing a regular non-fractional section, and the method of designating of the various subdivisions. The lengths of the latitudinal lines are merely assumed. In selling the public lands, the statute requires that these regular sections be "held and considered" as containing the exact 640 acres, and they are so sold, but as a matter of fact they seldom do contain exactly 640 acres, as is shown by the field notes of the United States survey. A complete subdivision would divide the section into quarter-quarters, or *forties*, as these lots are familiarly called. In the figure, the west half of the northwest quarter section is shown divided into quarter-quarters.

3. Case II.—*Fractional Sections Adjoining the North Boundary.*—This case relates to the subdivision of Sections 1, 2, 3, 4, and 5. These comprise all the sections adjoining the north boundary of the township except Section 6. The south half of each of these sections is regular and is subdivided in the manner described in Case I. Each quarter-section corner on a north-and-south section line through this tier of sections is at the regular distance of 40 chains north of the section corner on the section line forming the south boundary of the section. If the section adjoins a base line or standard parallel, such base line or standard parallel forms its north boundary, and its corners on such a line are not standard corners, but are merely closing corners, and do not affect the positions of any lines except those that close on them. In many of the older surveys, the same is true of these sections adjoining township lines that are not standard lines. These closing corners are the section corners for the sections adjoining the standard or township line on the south; that is, the sections now under consideration. *No corresponding quarter-section corners for these sections are marked in the original survey.* Quarter-section corners are established on the latitudinal township and standard lines when these lines are surveyed, but these quarter-section corners relate to the sections north of the lines; they do not in any way relate to the sections south of the lines or to the closing corners.

In subdividing the north half of the section under such conditions, the first thing necessary is to establish on the township line the quarter-section corner that relates to the section south of it. This is done by placing the quarter-section corner midway between the two corners of the section that stand on that line; that is, midway between the two closing corners.

The quarter sections adjoining the township line are divided into half-quarters by an east-and-west line that is 20 chains, *original measure*,* north of the corresponding

* By *original measure* is meant the measure actually laid down in the ground by the United States deputy surveyor when he ran the line and set the corners. A remeasure of the line is not to be expected to agree with this measure, but must be made to conform with it, as described elsewhere. See Art. 9.

quarter line, thereby throwing all of the fractions into the north tier of lots. The south half-quarters are thus of regular size. If either of the north or fractional half-quarters is further subdivided, this is done by a north-and-south line running midway between its end lines.

Fig. 2 illustrates the method of subdividing Sections 1, 2, 3, 4, and 5, and the method of describing the various lots. The fractional distances are of course merely assumed. The fractional lots are all in the north tier. The official plats give the area of these fractional lots for which they are sold.

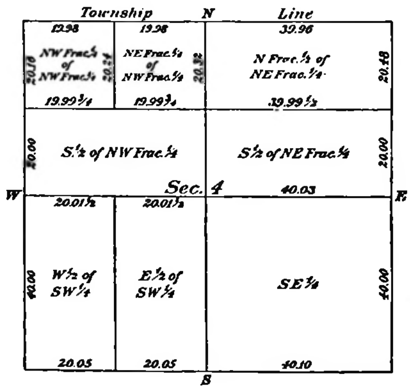


FIG. 2

The remaining lots are all sold as containing the exact 40, 80 or 160 acres, as the case may be. The fractional lots are also sold by number. The numbers commence with No. 1 for the northeast fractional quarter of the northeast fractional quarter, and run west to No. 4 for the northwest fractional quarter of the northwest fractional quarter. For the method of numbering all fractional lots, see the plat of a township in *United States Land Surveys*, Part 1.

4. Case III.—*Fractional Sections Adjoining the West Boundary.*—This case relates to the subdivision of Sections 7,

18, 19, 30, and 31. These comprise all the sections adjoining the west boundary of the township except Section 6. The east half of each section is regular and is subdivided as described in Case I. The quarter-section corner on each east-and-west section line through this range of sections is established at a distance of 40 chains west of the section corner on the section line forming the east boundary of the section, and consequently, the north-and-south quarter-section line through each section is parallel with and 40 chains distant from the east line of the section. In case the corners of the

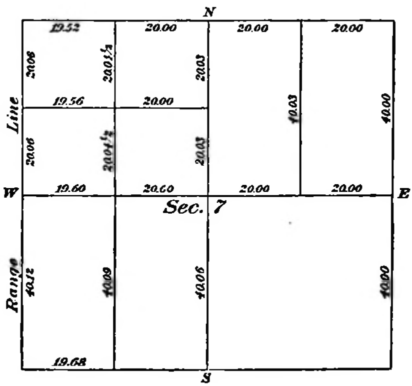


FIG. 3

section on the range line forming its west boundary are not standard corners but are merely closing corners, no quarter-section corner for the fractional section was planted by the original survey, and, consequently, the quarter-section corner for this section must be established on the range line midway between the two corners of the section that stand on that line. The fractional quarter sections adjoining the range line are divided into half-quarters by a north-and-south line 20 chains, by original measure, west from the corresponding quarter line, thus throwing the fractions all into

the west range of lots. The half-quarters are divided into quarter-quarters, the same as in Case I, by east and west lines. For method of numbering the fractional lots, see plat of township previously referred to.

Fig. 3 illustrates the method of subdividing Sections 7, 18, 19, 30, and 31. The lots adjoining the range line are sold as fractional, all others as containing the regular amount.

5. Case IV.—*Fractional Section in Northwest Corner.* Section 6, which is the fractional section in the northwest corner of the township, lying next to both the north and the west boundaries, is subject to the rules of subdividing applicable to both Case II and Case III. If the section corners at the northeast and southwest corners of the section are merely the closing corners for the section lines closing on the township and range lines, the quarter-section corners on the township and range lines forming the north and west boundaries of the section must first be established before the section can be divided into quarter sections. The quarter-section corner on the north boundary on the section should be placed in the township line at a distance of 40 chains, original measure, west from the closing section corner at the northeast corner of the section. The quarter-section corner on the west boundary of the section should be placed in the range line 40 chains, original measure, north from the section corner at the southwest corner of the section. The section is then divided into quarters by lines joining the opposite quarter-section corners, the same as in the other cases. The southeast quarter is regular and is subdivided as described in Case I. The northeast quarter is fractional and is subdivided in the same manner as described in Case II. The southwest quarter is fractional and is subdivided in the same manner as described in Case III. The northwest quarter is fractional in both its north-and-south and its east-and-west dimensions. It is divided into half-quarters by an east-and-west line parallel with and 20 chains, original measure, north from the east-and-west quarter line. The half-quarters are divided into quarter-quarters by a north-and-south line

parallel with and 20 chains, original measure, west from the north-and-south quarter line.

Fig. 4 illustrates the method of subdividing Section 6 and of numbering the fractional lots.

6. Quarter-Section Corners on Township Boundaries.—Under the present system of United States land surveys, no closing corners are allowed except on base lines and correction parallels. In all other cases, the section lines closing on township or range lines are run as random lines and corrected back so as to close on the true corners. As a

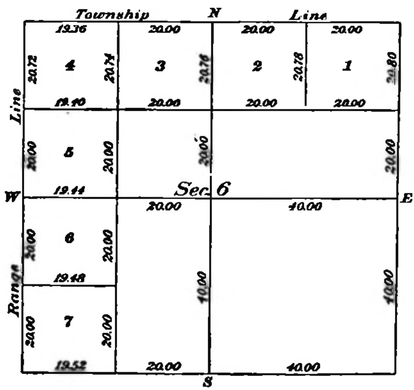


Fig. 4

consequence, the section and quarter-section corners on the township and range lines are common to the sections on both sides of the line, and form the basis of their subdivision. But in all the earlier surveys, closing corners were planted in the north and west boundaries of the township and no quarter-section corners were set on those boundaries for the fractional sections adjoining them on the south and east. These missing quarter-section corners are to be established by placing them midway between the two corners of the section; that is, the two closing corners that stand on

the same line. Section 6 is an exception to this rule. The quarter-section corners on the township and range lines are to be located at a distance of 40 chains, original measure, from the section corner at the northeast and southwest corners of the section, respectively, as described in the preceding article. This may readily be done by measuring along the township or range line from the standard quarter-section corner a distance equal to that between the standard and closing section corner at the northeast or southwest corner of the section, as the case may be, as found by actual measurement, and in the same direction that the closing section corner is from the standard section corner.

For example, suppose that it is stated in the field notes that the closing corner on the township line for the section line between Sections 5 and 6 is 135 links east of the standard corner for Sections 31 and 32 of the township north

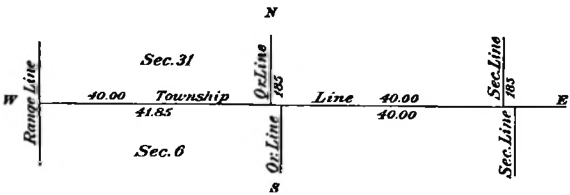


FIG. 5

of the township line, but by actual measurement the distance is found to be 185 links. In this case the quarter-section corner for Section 6 is placed on the township line at the same distance; namely, 185 links, actual measure, east of the standard quarter-section corner for Section 31 of the township north of the line. This is illustrated in Fig. 5, which shows the method of locating the north quarter post of Section 6.

7. Sections Made Fractional by Water, Etc.—The statute provides that "in those portions of fractional townships where no opposite or corresponding corners have been or can be fixed, the boundary lines not actually run and

marked shall be ascertained by running from the established corners due north-and-south or east-and-west lines, as the case may be, to the . . . external boundary of such fractional township." The section of the statute of 1805 from which the above is quoted evidently refers to surveys made previous to that time in which only the township boundaries were run and marked on the ground, and the subdivision into sections and the lesser tracts was made only on paper in the official plats. The general principle, however, has been applied to the location of the quarter-section lines in the subdivision of fractional sections.

The law presumes the section lines surveyed and marked in the field by the United States deputy surveyors to be due north-and-south or east-and-west lines, as they were intended to be, although as a matter of fact they are not actually so in all cases. The United States Land Office accordingly interprets the phrase "due north-and-south or east-and-west lines," as it occurs in the statute, to mean the lines surveyed and marked as such by the United States deputy surveyor, and holds that, in running the subdivision lines, mean courses must be adopted, or the subdivision lines must be run parallel to the existing section line when there is no section line on the opposite side of the subdivision line; that is, in case one of the quarter-section corners of a section was never set because of being inaccessible, so that the opposite quarter-section corner is the only corner that defines the position of the corresponding quarter-section line, this line must be run from the latter corner on a course that is a mean of the courses of the two section lines between which it is situated, and in case there is only one such section line, the course of the quarter-section line is made the same as

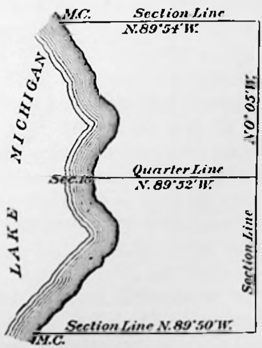


FIG. 6

that of this section line. Thus, suppose that in the fractional section represented in Fig. 6, it is found by trial that the course of the south line of the section is N 89° 50′ W, and that of the north line is N 89° 54′ W. Then the quarter line should be run westwards from the quarter-section corner on the mean course, which in this case is N 89° 52′ W. But if, for example, the lake extended eastwards entirely across the north portion of the section, so that there was no north section line for this section, the quarter-section line would be

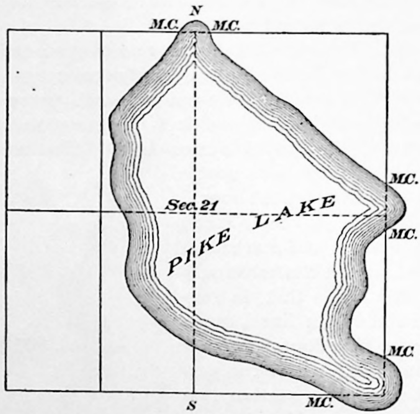


Fig. 7

run westwards parallel to the south section line, or on a course N 89° 50′ W in this case.

In case the missing corner can be established, as, for instance, by locating it on the ice in winter or by any other practical method, it should be located and the subdivision made in the manner described in Cases I, II, III, and IV. This is illustrated in Fig. 7, which represents a fractional section from which the north and the east quarter-section corners and the southeast section corner were omitted in the original survey by reason of being in a lake. When the

lake is frozen over, the positions of these corners can easily be located from the meander corners and the section can be subdivided in the regular way.

8. **Exceptions.**—There are many exceptions to the preceding methods for subdividing the fractional sections adjoining the township boundaries. The official plats in accordance with which the land was sold have been made up in various ways. In many cases, where the intersections of the section lines with the township boundaries were marked by closing corners, the north-and-south quarter-section lines of Sections 1, 2, 3, 4, 5, and 6 have been marked on the plats, and the areas of the fractional subdivisions computed, on the assumption that those lines were parallel with the east lines of the respective sections. In other cases, these quarter-section lines have been marked and the fractional areas computed on the assumption that the standard quarter-section corners on the township boundary were common to the sections on both sides of the line. This frequently made a wide discrepancy between the section lines and the intermediate quarter line in the sections bordering the township lines on the south. In Sections 6, 7, 18, 19, 30, and 31 the east-and-west quarter lines have sometimes been shown on the plats as being parallel with the south lines of the section.

In some cases, the areas of the fractions as laid down on the official plats can be accounted for only on the assumption that they were guessed at, as they undoubtedly were, without any computation whatever. As all sales of land are made by the United States Land Office in accordance with the official plats, the rule that has been adopted to meet all such cases is "to subdivide in such a way as to suit the calculation of the areas on the official plat." This cannot always be done.

Many of the surveys themselves have been made in a manner different from the present requirements. In some cases, the fractional lots have been thrown into the south and east tiers of sections in the township; and in other cases, where there were large streams or lakes, the surveys have

been made so as to throw all the fractions into the lots bordering on the water. In all exceptional cases, the official plat must be followed in making the subdivisions; and in cases where it is impossible to follow it literally and exactly, its intention must be interpreted as closely as possible.

9. Proportional Measures.—For each line of the public land survey, the length returned in the field notes is, by statute, made the true legal length of the line, as stated in Art. 1. But it is rare that, in remeasuring a line of the original survey, its length, as remeasured, is found to be the same as the length given in the field notes. The length of such a line, as returned in the field notes, is here called its length by **original measure**, and its length as actually determined by measurement is called its length by *actual measure* or its *remeasured* length. Thus, if according to the field notes the distance between a section corner and the adjacent quarter-section corner is 40 chains, a distance of 20 chains, original measure, would be just one-half the distance between the corners, although this distance may be 21 chains by actual measure.

In subdividing sections, the surveyor must use the original measure; that is, he must make his measures agree with the recorded lengths of the lines. This is practically done in the field by determining the difference between the recorded and remeasured lengths of any given line and distributing this difference throughout the several portions of the line in proportion to their respective lengths. By this means the subdivision corners on the line are placed in such positions that the distance between two corners is to the corresponding recorded distance as the remeasured length of the line is to its recorded length. This will be easily understood by an example.

EXAMPLE.—Suppose that the returned length of the line running north from the quarter-section corner between Sections 5 and 6 to the section corner on the township line is 42 chains, and that this distance, as remeasured, is found to be 42.63 chains. (*a*) What should be the remeasured length of the south part of the line or distance from the quarter-section corner to the half-quarter corner? (*b*) What should

be the remeasured length of the north part of the line or distance of the latter corner from the section corner?

FIRST SOLUTION.—(*a*) The excess of the remeasured length of the line over its recorded length is equal to $42.63 - 42 = .63$ ch. This excess is divided between the two parts of the line in proportion to their respective lengths. The south part of the line should have a length of 20 ch., original measure, and if x denotes the amount of excess to be given to this part of the line, its value may be expressed by the proportion

$$42 : 20 = .63 : x$$

from which $$x = \frac{20}{42} \times .63 = .3 \text{ ch.}$$

Hence, the length of this portion of the line by the standard of the remeasure should be equal to $20 + .3 = 20.3$ ch. Ans.

(*b*) By a similar process, the amount of excess that must be given to the north part of the line is found to be equal to $\frac{22}{42} \times .63 = .33$ ch., and the length of this part of the line according to the remeasure should be equal to $22 + .33 = 22.33$ ch. Ans.

SECOND SOLUTION.—(*a*) A rather more direct, though no more accurate, solution is obtained by letting x denote the required length of the part of the line under consideration. Then, for the length of the south part of the line, the proportion is

$$42 : 20 = 42.63 : x$$

from which $$x = \frac{20}{42} \times 42.63 = 20.3 \text{ ch.} \quad \text{Ans.}$$

(*b*) Similarly, for the north part of the line, the proportion is

$$42 : 22 = 42.63 : x$$

from which $$x = \frac{22}{42} \times 42.63 = 22.33 \text{ ch.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. If the returned length of the northern boundary of Section 6 was 77.75 chains, and this distance, as remeasured, is found to be 81 chains, what should be the remeasured length of that part of the line extending from the section corner on the township boundary between Sections 5 and 6 to the north quarter-section corner of Section 6? Ans. 41.67 ch.

2. Suppose that the distance from the section corner for Sections 9, 10, 15, and 16 to the section corner for Sections 3, 4, 9, and 10, as remeasured, is found to be 82.64 chains; at what distance from the nearer section corner should each quarter-quarter corner on this line be set in subdividing the section? Ans. 20.66 ch.

3. If the returned length of the line from the quarter-section corner between Sections 5 and 6 to the section corner on the township

boundary is 38.67 chains, and the length of the line as remeasured is found to be 39.72 chains: (*a*) what should be the remeasured length of the south part of this line or the distance from the quarter-section corner to the half-quarter corner? (*b*) what should be the remeasured length of the north part of the line or distance of the half-quarter corner from the section corner?

Ans. $\begin{cases} (a)\ 20.54\text{ ch.} \\ (b)\ 19.18\text{ ch.} \end{cases}$

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10. Accretions or Alluvium.—Some of the most difficult problems encountered by the land surveyor are those that result from the changing of the courses of streams or the shores of lakes. Owners of land bordering on navigable waters hold to high-water mark. On our non-navigable waters they usually hold to the center of the stream or lake, as the case may be, subject to the public rights of fishing, etc. When the shore line is changed by the recession of the waters or by additions gradually made by deposit from the water, it frequently becomes necessary to extend boundary lines over the land thus formed. Such newly formed land is called **alluvium, or accretions.** It is also necessary at times to extend these lines under the water for various purposes. The principle by which the lines are extended both over accretions and under water is based on the common law. **Common law** is law that is the outgrowth of customs, usages, and judicial decisions, in distinction from **statute law,** which consists of laws enacted by the law-making power of the country. The common law is recognized as binding wherever there is no statute law to the contrary.

11. Under the common law, it is taken for granted that the water front is an important part of the value of a lot. Hence, in extending the boundary lines to or under the water, they are so extended that each shore owner will retain his proportional share of the water frontage. This will frequently result in very complicated problems for the surveyor who is called on to locate the boundaries across the newly formed land. The lines will not be straight extensions or continuations of the established boundaries, but will run at

some angle therewith and with the shore or thread of the stream. The **thread of the stream** is defined as its center line, as measured between the shores, regardless of the current. The rule for extending these boundary lines has been laid down by the courts as follows:

"Measure the whole extent of the ancient line on the river and ascertain how many feet each proprietor owned on this line. Divide the newly formed river line into an equal number of parts, and appropriate to each owner as many of these parts as he owned feet on the old line. Then draw lines from the point at which the proprietors, respectively, bounded on the old to the points thus determined as points of division on the newly formed shore." "This rule is to be modified under some circumstances, as for instance, if the ancient margin has deep indentations or sharp projections, the general available line of the river ought to be taken and not the actual length of the margin as changed by the indentations or projections."

This rule was adopted for land bordering on rivers, but the same principle has been applied by the courts to land bounding on other waters. The surveyor will find many cases, especially on lands bordering on small lakes of irregular outline, where he cannot apply any mathematical rule to their solution, but will have to use his common sense and good judgment to determine what line will best preserve the equities of the different proprietors. In general, however, the boundary of lots fronting on lakes is extended under the water on a line running at right angles with the general course of the shore. Lots fronting on non-navigable streams have their boundaries extended at right angles with the thread of the stream.

An exception to this is found in the state of Indiana, in which state this common-law rule is not applied. The courts of that state hold that the boundary lines of subdivisions of land of the United States survey bordering on waters shall be extended the same as they would have been if no water had been there. As a result of this ruling of the courts in the state of Indiana, the owner of land fronting on water in

that state may be deprived of his entire water frontage by the shifting of a stream or the recession of a lake, and, at the same time, the water frontage of the adjacent land owner may be greatly increased.

12. The manner in which boundary lines are extended over accretions is illustrated in Fig. 8, which represents a fractional section containing a lake that lies chiefly in the northeast quarter of the section, though it extends into each of the other three quarter sections, and also into the adjacent section. The line *a b c d e* represents the original shore of the lake, *a* and *e* being regular meander corners. After a number of years, it is found that the surface of the water in

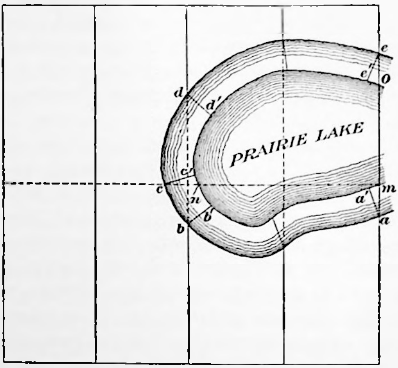

FIG. 8

the lake has become lowered, which has changed the shore line to the position represented by the line *a′ b′ c′ d′ e′*, and it is necessary to extend the boundary lines of the adjoining properties over the newly formed land to the present shore of the lake. According to the common-law rule for extending such boundary lines, they would be extended on the lines *a a′*, *b b′*, etc. in directions substantially at right angles to the shore line, thus giving each property owner his proportionate amount of the newly formed land and proportionate

length of the new shore line. If this fractional section were in the state of Indiana, however, the boundary lines would be extended the same as though no water had been there, that is, they would be continuations of the regular subdivision lines of the section, and would be in the same positions that these portions of the subdivision lines would have been in had the present shore $a' b' c' d' e'$ been the shore of the lake at the time of the original survey. As the effect of this, the shore line of the southeast fractional quarter section, which formerly extended from a to b, would now extend from m to n, and the shore of the northeast fractional quarter section, which formerly extended from d to e, would now extend from n to o, which would give this quarter section a much greater water frontage than it had originally. At the same time, the southwest and northwest quarter sections, which originally had the water frontages $b c$ and $c d$, respectively, would be entirely deprived of their water frontage. If each fractional quarter section is divided into fractional half-quarters, the injustice in this distribution of the water frontage is even more apparent.

RESURVEYS

FINDING CORNERS

13. General Principles.—In an old settled country, much of the work that the land surveyor has to do consists in finding old corners, restoring them when obliterated or destroyed, and retracing old boundary lines, either as a basis from which to locate other lines or for the purpose of settling disputed questions of boundary. In doing this, as well as in some original surveys, the surveyor is controlled by certain legal requirements that do not at all times harmonize with accurate instrumental work or correct mathematical computations. In an original survey, the problem is to locate the corner or line where it ought to be. In a resurvey, it is to find where the corner or line actually was located, and not where it ought to have been located.

It is a leading provision of the United States law relative to the surveys of the public lands that the corners and lines actually marked on the ground and returned in the field notes of those surveys must be established as the proper corners or lines that they were intended to designate. After the land has been sold in accordance with that survey and record, no one—not even the government itself—has any right or authority to correct any errors that may have been made in the original survey, or to change it in any way. By common law, the same is true of the lot corners and lines of village or city lots where they are sold by plat and the corners and lines have been marked on the ground previous to the execution of the plat. It is also true of all other boundaries of land that are marked on the ground at the time of, or previous to, the sale and transfer made in accordance therewith. Though neither the directions nor distances of boundaries, nor the areas enclosed, correspond with the description or plat, the original stakes, monuments, and marked lines are conclusive as to the boundaries.

In the United States land surveys, no special weight or preponderance is given to any corner above that of any other corner of the same survey. A quarter-section corner is entitled to just as much respect or authority as a section corner or a township corner. So long as it can be found and identified, there it must stay and control the location of the subdivision lines of the section. If lost or destroyed, the corner must be found or relocated. By a lost corner is here meant a corner that is still in existence and not destroyed, but whose location is lost and cannot be determined without a resort to surveys from distant corners.

14. The Search for a Corner.—In the directions heretofore given for subdividing sections, it was presupposed that the section and quarter-section corners of the original survey were known. When a number of years have passed since the original survey was made, however, it is usually the case that some of these corners have been lost or totally destroyed. In such cases, it is the surveyor's first work to

find the corners if lost, or to relocate them if destroyed or obliterated.

To find a lost corner is largely a matter of skill and evidence. The evidence may be of any kind that would tend to prove the location of the corner. The best evidence is the monument itself as planted by the original survey. When looking for the monument, the surveyor should know, to begin with, just what he is looking for—whether a stake, a stone, a hole in a rock, or whatever else it may be. This he will learn from the field notes, if it is the original monument; from any other available testimony, in case a new monument has been planted in place of the original monument. The field notes will also describe the witness or bearing trees or objects, if there are any. If more than one bearing tree or object can be found, the point where the corner post or monument should be found will be indicated by the intersection of arcs drawn from these objects with radii, equal, respectively, to the distances of the objects from the corner, as given in the field notes. If only one bearing tree or object remains, it is necessary to measure from it the recorded distance in a direction just opposite to its recorded bearing, and at the extremity of the distance, search for the monument.

15. Decayed Wooden Post.—If a wooden post is the object sought, it may have decayed so that nothing but the rotten wood is left, and in that case careful work is required. The surface dirt should be removed to the depth of a few inches or to a point where the subsoil has not been disturbed. It is very essential not to drive the spade down into the earth and throw it out by spadefuls, as one may thus destroy what one is looking for without knowing it. The surface should be pared, shaved, or scraped down a little at a time, and very carefully, by clean cuts of the spade. If the search is made properly and in the right place, and the earth has not been previously removed or disturbed, the remains of the post are very likely to be found, even though a great many years have passed since it was planted. If the soil is a stiff clay, packed hard or covered with a sward, a hole will

be found of the size and shape of the post that made it, and it will contain the decayed remains of the post. By carefully cutting down at the side of the hole, its size, shape, and direction can be seen. Fig. 9 represents the appearance of a decayed wooden post when the earth has been carefully removed from the surface, then an excavation made at one side of the post and the side of the excavation toward the post shaved away carefully by vertical cuts of the spade until the decayed remains of the post are seen in the position in which it was driven.

The position of a corner is often as well marked by the decayed remains of the post as it was by the sound post. Such corner monuments often outlast those of iron or stone, being less subject to removal by vandal hands. They are best seen in light-colored clay subsoils. In sand, the cavity made by the decaying post is gradually filled by the falling sand, but the decayed wood mingles with this loose sand and discolors it to such an extent that where it has not been disturbed, the position of the post can be easily traced. In black muck, it is not so easy to distinguish the remains of the post near the surface, but such soils are soft, so that the posts, being driven easily, are usually driven deep, and where it is also wet, the bottom of the post will be preserved in a sound condition for a great length of time.

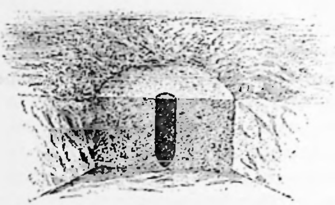

Fig. 9

16. Bearing Trees.—Both bearing trees and witness trees may usually be known by their kind and size, and by the scars on them. In some cases, the trees have grown to such an extent that it is necessary to cut away the wood that has grown over the marks, in order to be certain that they are the right trees, especially if there are other similar trees near by. As cutting away the wood injures the tree, it

should not be done unless absolutely necessary in order to establish the corner.

When the bearing trees themselves have entirely disappeared and no trace of them remains above the surface of the ground, unmistakable traces of the roots, sufficient to fully identify the trees, can often be found by digging into the undisturbed soil below the surface soil. Conditions much the same as those just stated regarding the traces and remains of decayed stakes also obtain with respect to the decayed roots of a tree. By carefully removing the surface soil to the depth to which it has been disturbed, which quite commonly is only to the depth to which the plow has penetrated, then shaving away the undisturbed soil by clean horizontal cuts of the spade, the exact form of the roots of the tree can usually be discerned unmistakably, especially in light-colored soil. In this way and by such means, the approximate size and position of the tree, and in many cases the variety of the tree, can be determined, if the surveyor is sufficiently skilled in work of this character. The roots of some varieties of trees penetrate much more deeply than those of other varieties. For example, a white oak tree or a hickory tree will have, besides the usual branching roots, what is called a tap root, that is, a large root that extends nearly straight downwards from beneath the center of the tree and usually penetrates to a great depth. Most other kinds of oak have only branching roots that are usually of considerable size and extend outwards at very great depths below the surface of the ground. Some varieties of trees, such as the elm, have large branching roots that extend outwards at slight depths below the surface. Other varieties, such as the tamarack, usually have a large number of small branching roots that extend outwards very near the surface. From such conditions, the surveyor who is familiar with the characteristics of the roots of different varieties of trees, can often determine the variety of a tree with sufficient certainty to identify it as a bearing tree. But this is a matter that requires personal observation and experience, and cannot be learned from books.

17. Errors in Field Notes.—It should be remembered that field notes are not infallible, and errors in them are not rare. A direction may be given as north instead of south, or as east instead of west, and vice versa; the bearing may have been wrongly read on the graduated circle of the instrument, as 64° instead of 56°; the figures denoting the bearing may have been transposed, as 53 for 35; the chain may have been wrongly read by counting the links from the wrong end, as 48 instead of 52; or they may have been counted from the wrong tally mark, as 42 instead of 32. In working from a bearing tree or object to find a corner, and finding no indications of a post at the place indicated in the notes, all these sources of error should be tested. For the post planted at the time of the original survey is the best evidence of the position of the corner it was intended to mark, provided that it has not been moved.

18. Evidences of Location of Corner.—An old fence will indicate in a rough way where to look for a corner, but the history of the fence with respect to whether it was originally built to the corner, should be carefully inquired into before accepting it as evidence of the position of the corner. Such a fence may be the best possible evidence of the location of a line, or it may be worse than worthless for such purpose. The evidence of persons who have been familiar with the location of the corner sought may be of great service in finding it, but there is a great difference in people in this regard. Some have an accurate sense of locality and can tell very closely where an object is located; others cannot tell anywhere near it. Their ability in this respect, as well as their means of knowledge, should be inquired into before placing implicit confidence in their testimony.

Measures and lines run from the nearest known corners of the same survey in each direction will assist in locating the point at which to look for a corner. But the measurements of the United States land survey have seldom been uniform in different sections and not always uniform throughout the same section, and, consequently, this method cannot be

depended on to relocate a corner in its original position. The surveyor should remember that the problem is to find where the corner of the original survey actually was, and not where it should have been. A pick and shovel, when used intelligently, are of great assistance in finding lost corners. especially where wooden posts were used to mark the corners. In searching for a lost corner, the surveyor should not give it up as destroyed until he has made every possible effort to find it and has failed.

TO REPLACE LOST AND OBLITERATED CORNERS

19. Rules for Restoring Obliterated Corners.—Some of the most troublesome matters encountered by the land surveyor are those relating to the restoration of corners that have become destroyed or obliterated. In many cases, it is impossible to decide whether a corner has become totally obliterated or is merely lost, until this has been determined by a careful and thorough search. Hence, the preliminary portion of the process for restoring an obliterated corner consists in the search for the corner, and is the same as the process of trying to find a lost corner. There is but one rule for restoring corners that have become destroyed or totally obliterated, and this rule has no exception.

The corners must be replaced in their original locations if possible, regardless of errors in the original survey.

Failing of nearer and better evidence by which to locate the missing points, resort is had to the nearest other corners and monuments in each direction; these may be monuments of the same survey, or of other surveys when their relative location is known. The following rules apply to the United States surveys:

(*a*) *On base lines, correction parallels, township and range lines*, the lost corners should be restored by placing them in line between the nearest known corners on the same line and at distances from the known corners proportional to those laid down in the field notes of the original survey.

This rule supposes the original line to have been a straight line, though it is frequently not so. If there is reason to

believe the line has angles in its course, measurements from known corners to the right and left of it will aid in determining its true position. It is sometimes found that the position of a closing corner cannot be located from the nearest standard corner, because in some cases the distance of the closing corner from the standard corner is not given in the field notes, and in other cases it is erroneously stated. In such cases, if a portion of the closing line is known, the rule is to prolong it to its intersection with the township boundary and there look for or locate the closing corner.

(*b*) *Lost interior section corners* should be restored at distances from the nearest known corners, north, south, east, and west, proportional to those distances as given in the field notes of the original survey.

This rule supposes that the measurements of the original survey were uniform on adjacent sections. But they are not generally so, and, consequently, before measuring any line, it is well for the surveyor to compare his chaining with that of the original survey by measuring between the known corners nearest to the missing one that were set by that survey.

(*c*) *Lost township corners*, when common to four townships, are restored in the same manner as interior section corners. When common to only two townships they are restored as in (*a*).

(*d*) *Lost quarter-section corners* are to be restored in line between the section corners and at distances from them proportional to those returned in the field notes of the original survey; that is,' midway between the section corners except in north and west tiers of sections. The same rule applies to meander corners wherever it can be applied.

20. Other Cases.—There are many cases in which other methods for restoring corners will be more satisfactory than the rules given. For instance, a half-quarter corner that was established when the adjacent section and quarter-section corners were known, may be used to restore either of them, when lost or destroyed, by prolonging the line over the known corners and doubling the distance, that is, by prolonging the

line of the two known corners to a point at a distance beyond the half-quarter corner equal to its distance from the section or quarter-section corner. If the half-quarter corner is the corner of a fractional half-quarter in the north or west tier of sections, the line is prolonged a distance that, instead of being equal to the distance between the corners, is in the same proportion to it that the corresponding returned distances are to each other. Any other intermediate corner whose location is definitely known may be used in a similar manner. Most of the difficulties encountered in finding lost or restoring obliterated corners arise from imperfections or errors in the original survey, or in the field notes. Hence, it is difficult, and sometimes impossible, to restore a corner to its original location by surveys from distant corners. After making such surveys, the ground should again be examined most carefully for the nearer and better evidence of the monument or post itself, as described in the directions for finding corners.

RESURVEY AND SUBDIVISION OF A SECTION

21. The example below, which is from actual practice, is given to illustrate the ordinary methods of finding corners, restoring those that are destroyed, and subdividing the section. (See Fig. 10.) The notes are kept on the general plan of those of the United States land survey. The following is a copy of such parts of the original field notes as were needed in the survey:

FIELD NOTES OF THE UNITED STATES SURVEY
Section 5, T. 3 S, R. 9 W

Chains		
	North between secs. 8 and 9.	
80.00	Set post cor. to secs. 4, 5, 8, and 9,	Y. oak 22 in. S 87 E 171 li. W. oak 36 in. N 1 E 140 li.
	North between secs. 4 and 5.	
6.00	Enter wet prairie.	
33.00	Left wet prairie.	
33.80	W. oak 8 in.	
40.00	Set post qr. sec. cor. secs. 4 and 5,	W. oak 18 in. S $60\frac{1}{2}$ E $47\frac{1}{2}$ li. W. oak 18 in. N 22 E 71 li.

FIELD NOTES OF THE UNITED STATES SURVEY—(Continued)
Section 5, T. 3 S, R. 9 W

Chains	
55.50	Enter wet prairie.
74.00	Brook 40 li. wide C. SW.*
79.80	Intersected N boundary 139 li. E of post. Set post at intersection cor. of secs. 4 and 5, {W. oak 14 in. S 83¼ E 451 li. / W. oak 14 in. S 86¾ W 720 li.
	North between secs. 7 and 8.
80.00	Set post cor. to secs. 5, 6, 7, and 8, {W. oak 8 in. N 8 E 29 li. / W. oak 12 in. S 44 E 43 li.
	West corrected line between secs. 5 and 8.
40.20½	Set post qr. sec. cor. {W. oak 22 in. S 22 W 50 li. / Y. oak 20 in. N 49 E 11 li.
80.41	Section corner.
	West between secs. 6 and 7.
17.71	W. oak 18 in.
40.00	Set. post qr. sec. cor. {W. oak 10 in. S 87 E 48½ li. / R. oak 13 in. S 87 W 15½ li.
	North between secs. 5 and 6.
25.17	W. oak 18 in.
40.00	Set post qr. sec. cor. {Aspen 14 in. S 41½ E 80 li. / W. oak 14 in. N 57 E 41 li.
66.47	Stream 40 li. wide, C. SW.
68.44	Same stream, C. SE.
69.40	Same stream, C. SW.
81.41	Intersected N boundary. (Post removed.) Set post at intersection cor. to secs. 5 and 6, {W. oak 10 in. S 53 E 268 li. / W. oak 14 in. S 81 W 166 li.
	North boundary of sec. 5.
	West on S boundary to sec. 33, T. 2 S, R. 9 W.
80.00	Set post cor. to secs. 32 and 33, {W. oak 12 in. N 74 E 299 li. / W. oak 10 in. N 88 W.

*Abbreviation for *course southwest*.

FIELD NOTES OF THE UNITED STATES SURVEY—(Continued)
Section 5, T. 3 S, R. 9 W

Chains	West on S boundary of sec. 32, T. 2 S, R. 9 W.
14.69	W. oak .9 in.
40.00	Set post qr. sec. cor. {W. oak 36 in. N 75 E 42 li. / W. oak 6 in. N $45\frac{1}{4}$ W. 33 li.
53.85	W. oak 12 in.
80.00	Set post cor. to secs. 31 and 32, {W. oak 15 in. N 43 W 91 li. / W. oak 16 in. N $30\frac{1}{4}$ E 255 li.

The following are the complete field notes of the resurvey of the section, made for the purpose of finding the corners, restoring those that were destroyed, and subdividing the section. These notes give a large amount of valuable and practical information, and should be studied carefully.

Resurvey and Subdivision of Section 5, T. 3 S, R. 9 W, May 23–26, 1883

Chains	
	Begin at the corner of secs. 4, 5, 8, and 9. Found bearing tree north standing dead and decayed. Ran thence S 1° W 140 li. where I found one foot beneath the surface of the road a stake hole one foot deep, with the decayed wood plain and distinct. Set a stone 18 × 10 × 6 in. in its place marked + on top and put broken brick around for cor. and mark {Maple 16 in. N 46 E 72 li. / Maple 12 in. N 61 W 89 li.
	Set up a flag 20 li. east of cor. From cor. measured north alongside of line fence.
33.90	W. oak line tree now 24 in. dia.
40.12	Qr. sec. cor. dug out in highway. Stumps of two bearing trees are standing, showing marks plainly. From these stumps I strike arcs of $47\frac{1}{2}$ li. and 71 li., respectively, and at their intersection plant an earthenware post 2 ft. long, 3 in. dia. and mark {W. oak 12 in. S 62 W 112 li. / W. oak 10 in. S 3 W 46 li.
	I then set up the transit 20 li. east of the qr. sec. cor., backsight on flag 20 li. east of sec. cor. and prolong line north.
74.20	Brook.
80.00	Set temporary stake 20 li. west of random line and look for cor. in wet prairie or marsh. Found stump of bearing tree, showing marks, on high ground SE. Run thence

Resurvey and Subdivision of Sections 5, T. 3 S, R. 9 W, May 23–26, 1883
(Continued)

Chains	
	N 83¼ W 451 li.; set temporary stake and run thence S 86¾ W 720 li.; set temporary stake and look for evidence of bearing tree.
	By digging away the surface earth I find, where a tree has stood, the decayed roots being still plainly visible. This is S 88 W 6 li. from temporary stake. I locate the center

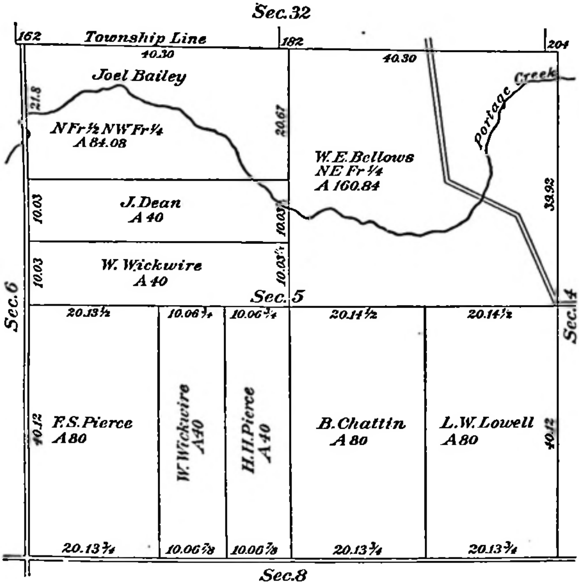

FIG. 10

	of the tree as nearly as possible and run thence N 86¾ E 720 li. Renew search for cor. and find the sound remains of a stake 2 ft. below surface of marsh. This stake is 80.04 ch. from cor. of secs. 4, 5, 8, and 9, and 2 li. west of line prolonged over sec. cor. and qr. post. I then go to

Resurvey and Subdivision of Sections 5, T. 3 S, R. 9 W, May, 23–26, 1883
(Continued)

Chains	
	the cor. of secs. 32 and 33 and find post standing in correct position as shown by stump of both bearing trees. I also find that the jog between the two corners is 204 li. instead of 139 li., as given in the field notes. Set new post with brick around for cor. of secs. 4 and 5. No tree or bearing object near. Left flag on the cor.
	I then return to qr. sec. cor. of secs. 4 and 5, set up transit 10 li. north of it, sight to flag on the sec. cor. north, which line I assume as a meridian, and run thence on random S 88° 50′ W along N side of line fence, setting temporary stakes every 10 ch. at an offset distance of 10 li. south of the random line.
80.00	Set temporary stake on east side of road. Cor. post and bearing trees both dug out in the highway and no trace of cor. left. I then go to the cor. of secs. 5, 6, 7, and 8. This is also dug out in the highway and no evidence of it to be found. I next go to the qr. sec. cor. of secs. 7 and 8, which I find and identify, and run thence north on a random line along the center of the road without noting the course.
40.00	Set temporary stake.
80.00	Set temporary stake.
121.78	Intersect north boundary 12 li. west of post, which is decayed and in its correct position, with both bearing trees standing. Set stone 18 × 10 × 5 in., marked +, for new corner post of secs. 5 and 6, and put broken glass and crockery around it. I then go to the cor. of secs. 31 and 32, T. 2 S, R. 9 W, and find a stone planted for cor., and stumps of both bearing trees standing. The jog between the cors. is 162 li. I then go to the qr. sec. cor. of secs. 6 and 7, which I find and identify, and measure east along the center of the road.
40.00	Set temporary stake 40.12 ch. north from the qr. sec. cor. of secs. 7 and 8 and set up flag.
	Thence measure east along center of road, setting temporary stakes every 10 chains.
40.00	All traces of qr. sec. cor. destroyed.
80.62	Cor. of secs. 4, 5, 8, and 9.
	West corrected line by single sight to flag near cor. of secs. 5, 6, 7, and 8.
20.13$\frac{3}{4}$	Set for $\frac{1}{4}$ qr. sec. cor. a stone 20 × 12 × 8 in. and put brick around it and mark { W. oak 16 in. S 26 E 46 li. W. oak 12 in. N 14 E 42 li.

Resurvey and Subdivision of Section 5, T. 3 S, R. 9 W, May 23–26, 1883
(Continued)

Chains	
$40.27\frac{1}{2}$	Set iron plow coulter 18 in. long with broken brick and crockery around for qr. sec. cor. of secs. 5 and 8, and mark $\left\{\begin{array}{l}\text{Cherry 12 in. S 47 W 143 li.}\\ \text{Blk. walnut 16 in. N 23 W 76 li.}\end{array}\right.$
$50.34\frac{3}{8}$	Set plow point with brick around for cor.
$60.41\frac{1}{4}$	Set black walnut post with broken glass around for $\frac{1}{4}$ qr. sec. cor. and mark $\left\{\begin{array}{l}\text{Blk. walnut 20 in. N 20 W 51 li.}\\ \text{Blk. walnut 18 in. N 45 E 76 li.}\end{array}\right.$
80.55	Cor. of secs. 5, 6, 7, and 8. By my measure the distance from the qr. cor. of secs. 7 and 8 north to the township boundary is 121.78 ch. as compared with the returned distance, which is $40 + 81.41 = 121.41$ ch., a surplus of 37 li. Hence, I place the corner at a distance of $\frac{40}{121.41} \times 121.78 = 40.12$ ch. from the qr. sec. cor. of secs. 7 and 8 and at a distance of $\frac{81.41}{121.41} \times 121.78 = 81.66$ ch. from the cor. of secs. 5 and 6 on the township boundary.
	I also find that the distance from the qr. sec. cor. of secs. 6 and 7 east to the corner of secs. 4, 5, 8, and 9 is 120.62 ch., a surplus of 21 li. as compared with the original measure of $40 + 80.41 = 120.41$ ch. Hence, I place the corner at 40.07 ch. from the qr. sec. cor. of secs. 6 and 7 and 80.55 ch. from the cor. of secs. 4, 5, 8, and 9. Put in a 2-ft. length of 3-in. sewer pipe for cor. of secs. 5, 6, 7, and 8 and mark $\left\{\begin{array}{l}\text{Hickory 12 in. S 17 W 193 li.}\\ \text{W. oak 18 in. N 42 W 73 li.}\end{array}\right.$
	I leave a flag on the cor. and go to the qr. sec. cor. of secs. 5 and 6 and put my transit in line between the flags at the sec. cors. and on the true line plant an iron plow beam $26 \times 4 \times 1$ in. for qr. sec. cor. of secs. 5 and 6, and mark $\left\{\begin{array}{l}\text{Hickory 10 E 42 li.}\\ \text{Cherry 16 S 26 E 104 li.}\end{array}\right.$
	This cor. is 40.12 li. from the sec. cor. south and 41.54 li. from the sec. cor. north. Thence
	North on true line from qr. sec. cor. of secs. 5 and 6.
10.03	Set stake with brick around for cor.
20.06	Punch hole with iron bar 3 ft. deep and 2 in. diam. in the ground and fill it with Portland cement mortar for $\frac{1}{4}$ qr. sec. cor. and mark $\left\{\begin{array}{l}\text{Maple 6 in. N 24 E 165 li.}\\ \text{Maple 8 in. N 72 W 213 li.}\end{array}\right.$

Resurvey and Subdivision of Section 5, T. 3 S, R. 9 W, May 23–26, 1883 (*Continued*)

Chains	
	I then return to temporary stake set near qr. sec. cor. of secs. 5 and 6 and continue the random line from the east on a course S 88° 50′ W.
80.56	Intersect the section line 14 li. north of the qr. sec. cor. Deducting the 10-link offset made to the north on this line leaves 4 li. correction to be prorated on the stakes of the random. Thence east corrected.
20.14	Set temporary stake 3 li. south of random to true line.
30.21	Set temporary stake 2½ li. south of random to true line.
40.28	Set temporary stake 2 li. south of random to true line.
	I set up transit over this last stake, backsight to flag on qr. sec. cor. of secs. 5 and 8 and prolong the line north.
10.00	Set temporary stake.
20.00	Set temporary stake.
40.73	Intersected township boundary 184 li. east of the qr. sec. cor. of sec. 32. I found the closing distance between cors. at NW cor. of sec. 5 to be 162 li. and at the NE cor. of sec. 5 to be 204 li., the half sum of which is 183 li. Hence, I plant a temporary stake in the township boundary 183 li. east of the qr. sec. cor. of sec. 32, and remeasure the north line of sec. 5 and find the middle point to be 182 li. east of the qr. sec. cor. of sec. 32, at which point I plant for qr. sec. cor. of sec. 5 a piece of 1½-in. gas pipe 3 ft. long and mark { W. oak 16 in. S 23 E 26 li. / Hickory 12 in. S 44 W 53 li. }
	By ruling of the general land office the distance from this point to the center of sec. 5 is $\frac{39.80 + 41.41}{2} = 40.60\frac{1}{2}$ ch.; hence, I have $40.73 - 40.60\frac{1}{2} = 12\frac{1}{2}$ li. surplus to be prorated. Therefore, I go south on corrected line.
20.66¾	Set stone 16 × 9 × 6 in. for ¼ qr. sec. cor. Put broken crockery around it and mark { Apple 10 in. N 46 W 176 li. / Maple 16 in. N 17 E 227 li. }
30.69¾	Set marked stone for cor.
40.73	At intersection of quarter lines set stone 20 × 12 × 4 in. with cross-mark on it for center of sec. 5.
	Thence west on true line from center of sec. 5.
10.06¾	Set marked stone 12 × 8 × 4 in. with brick around.
20.13½	For ¼ qr. sec. cor. set marked stone 20 × 9 × 5 in. with brick around.
	East on true line from center of sec. 5.

Resurvey and Subdivision of Section 5, T. 3 S, R. 9 W, May 23–26, 1883
(Continued)

Chains	
20.14½	For ¼ qr. sec. cor. set stake with broken glass around and mark { W. oak 6 in. S 87 E 46 li. / W. oak 18 in. S 5 W 23 li.

NOTE.—It is not expected that the surveyor will follow any special order in subdividing a section. The conditions are not likely to be the same in any two sections, and the subdivision of each section will be controlled by local circumstances, and should be made in such order as may be most expedient for that section.

LEGAL PRINCIPLES CONTROLLING SURVEYS

22. General Statement.—There are various statutes and common-law principles that control surveys. If in the endeavor to secure mathematical accuracy of his work, the surveyor fails to observe these principles, he is quite certain to be discredited if the case comes into court. For instance, the United States statute requires that quarter-section lines be straight lines across the section from corner to corner, although such lines will seldom divide the section into four equal parts. Most of the common-law principles that follow are universally accepted by the courts. On some points, the decisions vary in different states. Every surveyor should inform himself on the decisions of the supreme court of the state in which he surveys. They are to be found in the state reports, which are usually kept by the proper officials at the county seat, and by leading lawyers. The surveyor has to properly construe descriptions in deeds, and in resurveys he has to consider and weigh evidence of every kind that tends to show the location of corners and boundary lines.

The word *call* is commonly used in land law with reference to the descriptions in surveys or grants of land. When used in this sense, it means any fact, condition, or requirement called for or specified in the description, and that serves to define or identify the survey. For example, the description,

"Thence N 40° E 24 chains to a granite boulder at an angle in the south boundary of the Wellstown turnpike," contains six distinct calls, namely: (1) a call for the course, as *N 40° E*; (2) a call for the distance, as *24 chains;* (3) a call for the object at the extremity of the course, on which the course closes, as *a granite boulder;* (4) a call describing the position of this object, as *at an angle;* (5) a call further describing and identifying the object, as *in the south boundary;* (6) a call identifying the boundary and thus further identifying the object, as *of the Wellstown turnpike.*

23. Interpretation of Descriptions in Deeds.—In construing descriptions in deeds, the courts have laid down the following principles:

(*a*) The description is to be taken most strongly against the grantor. If it is capable of more than one meaning, that one should be adopted which will give the grantee the greatest amount of land.

(*b*) A deed must be construed according to the condition of things at the time it was made. The written descriptions are to be interpreted in the light of the facts known to, and in the minds of, the parties at the time, and with reference to any plats, facts, and monuments on the ground, which are referred to in the deed. If the descriptions are uncertain, the construction given by the parties, as manifested by their acts on the ground, is deemed the true one unless the contrary is clearly shown.

(*c*) Every call in the description must be answered if it can be done, and none is to be rejected if they can all stand consistently together. If one part of the description is false and impossible, but by rejecting that part a perfect description remains, the false part should be rejected and the deed held good. Those boundaries are to be retained which best subserve the prevailing intention manifested on the face of the deed. The certain description must prevail over the uncertain in the absence of controlling circumstances.

(*d*) Where the description calls for land "owned and occupied," the actual line of occupation is a material call to

be considered, but where land is conveyed "beginning at" and "bounding on" certain land, the point of beginning and boundary is on the true line of that land.

(*e*) Where land is described as running a certain distance by measure to a known line, the land will extend to that line whether the measure is correct or not. Not so if the line is not definitely marked, fixed, or known.

(*f*) Where land is conveyed as "beginning at and bounding land of B," the point of beginning and boundary is the true line of B's land.

(*g*) A conveyance by metes and bounds will carry all the land included in them, although it be more or less than is stated in the deed. The mention of quantity after a definite description, being the least certain, does not control, but if the boundaries are in doubt, quantity may become a controlling consideration.

(*h*) "Northward" or 'northerly" means due north when nothing is mentioned to show deflection to the east or west. A course from corner to corner means a straight line, but may be explained by other matters to be a marked line, as following a hedge or stream. The use of the word "about" indicates that exactness is not intended, but where nothing more certain can be found, the grant is limited to the exact courses* and distances given.

(*i*) Where lines are laid down on a map or plan, and are referred to in a conveyance of the land, all the particulars shown on the map or plan are considered as much the true description as though they were expressly recited in the deed. A reference in a description to the government patent makes that patent and the government survey a part of the deed. So where for greater certainty any survey is referred to in a deed, the survey referred to legally forms a part of the deed.

(*j*) A grant of land bounded by the highway takes to the center of the highway, unless the highway is excluded

* The use of the word *course* in the sense of *bearing* is universal with the courts and is common everywhere. Hence, in quoting the principles laid down by the courts, this usage has been retained.

in explicit terms. The monuments referred to in a deed, whether they are natural or artificial, control the courses and distances.

24. Descriptions of Land Bordering on Water.—In construing descriptions of land bordering on waters, the surveyor will need to inform himself regarding the laws of the locality, as they vary in different states. The following principles are stated as a general guide:

(*a*) It is a universal rule that grants of land bordering on navigable waters take only to high-water mark, while grants on non-navigable streams take to the center of the stream. The same rule applies to non-navigable lakes in some states. But what is a navigable stream? According to the common law, a navigable stream is one emptying into the ocean, and into which the tide ebbs and flows. By the federal law, it is one capable of being used as a highway of commerce. In some states one law prevails, and in some the other. Under the Michigan law, the Detroit river is not a navigable stream, and adjacent proprietors own to the center of the river, although it is a great highway of commerce. A similar contradiction exists in the case of the upper Mississippi; a state on one side holds it to be a navigable river, while a state on the opposite side holds the reverse. The center or thread of a stream is measured between the low-water marks, regardless of the main channel.

(*b*) A boundary on, or by, or to a stream includes flats at least to low-water mark and in some cases to the thread of the stream. A boundary on the bank referring to fixed monuments limits the grant to the bank. The words "along the bank" exclude the river and its bed. A bank is the continuous margin where vegetation ceases. The shore is the space between the bank and low-water mark. That only is the river bed which the river occupies long enough to wrest it from vegetation. When by action of the water the river bed changes, the ownership of adjoining land changes with it. Meander lines are not boundary lines. A patent from the United States, of land bounded by a lake or stream, conveys

land to the water edge, although the meander line does not coincide with the shore line.

25. Locating Corners and Boundaries.—The following principles govern and should be fully recognized in locating the corners and boundaries of surveys:

(*a*) In locating a deed on the ground, the surveyor is to rely: (1) on the actual lines originally surveyed; (2) on lines run from acknowledged calls and corners; (3) on lines run according to the courses and distances.

(*b*) When the boundaries are fixed by known monuments, though neither courses, distances, nor contents correspond, the monuments govern. Surplus lands do not vitiate a survey, nor does a deficiency of the area called for operate against it.

(*c*) Course and distance yield to monuments, but where the monuments are wanting and course and distance cannot be reconciled, there is no rule that compels any preference of one over the other. Local circumstances will usually determine which should be preferred. Where no bounds were established, the line must be run by aid of the measurements in the deeds, the oldest title receiving its full measure first.

(*d*) Boundary may be proved by any evidence that is admissible to establish any other fact. A long-established fence is better evidence of actual boundary settled by practical location than any survey made after the monuments of the original survey have disappeared.

(*e*) Where a survey is made previous to the plat and there is a difference between the survey and the plat regarding the location of the lines and monuments, the lines and monuments originally marked as such are to govern. Purchasers of town lots have a right to locate them according to the stakes that they find planted and recognized. If the stakes were planted by authority and the lots were purchased and taken possession of in reliance on them, no subsequent survey can be allowed to unsettle them. When surveys made many years apart disagree, and the original corners

and witness trees are gone at the time of the later survey, the probabilities favor the earlier survey, especially if the line has remained unquestioned for many years.

(*f*) Streets that are well defined and marked by natural or artificial monuments govern course and distance in fixing boundaries of lands. Ancient reputation and possession in regard to streets in a town are entitled to more respect in deciding on the boundaries of lots than any experimental survey afterwards made.

(*g*) Where lots are sold by numbers and a plat, any surplus or shortage in a block is to be divided pro rata between the lots.

(*h*) The original surveys by which the government sold and conveyed its land establish the rights of the parties as to boundaries. Land sold under the United States surveys pass according to the descriptions of the legal subdivisions, whether those subdivisions contain the legal quantity of land or not.

(*i*) Each section or subdivision of a section is independent of any other section or subdivision in the township and is governed by its own boundaries. The corners established by the original survey of the United States public lands are conclusive, and no error in placing them can be corrected by an individual or state surveyor. Quarter posts, however distant from the direct line, are to be as much respected as section or township corners.

(*j*) Field notes must yield to actual monuments erected by the original surveyor. They are only to be relied on as evidence to assist in finding the exact situation of the monuments. When there are conflicting monuments, that is to be considered the true one which most nearly conforms to the field notes.

(*k*) Any difference in the length of a line by actual measure, as compared with that indicated by the government survey, should be divided between the parts in proportion to their respective lengths, as shown by the survey. In the absence of evidence to the contrary, it is not permitted to presume that the variation arose from the defective survey of any part.

(*l*) A fence line between two properties that is agreed on by the owners of the properties to be the division line between them, or is openly recognized by them as such, and undisputed, for a term of years, becomes the legal division line between those properties, regardless of whether it is or is not in the true position of the original line. But such fence line does not in any way affect the position of any other boundary line, and in case it is supposed, or was intended, to be on a line of the public-land survey, as, for example, a section or a quarter-section line, but really is not on such line, the fact that the fence is the legal division line between the adjacent properties does not in any way affect the position of the survey line if the latter can be determined.

THE EARLIER SURVEYS

26. Surveys by Metes and Bounds.—As the present rectangular system of land surveys was not inaugurated until 1784, the land surveys made previously were not in conformity with this system. In the older states of the American Union, the original surveys of the public lands were made by what is commonly known as **metes and bounds.** Each tract, whatever its shape, was enclosed by a traverse line that started from some given point and, extending entirely around the tract, closed on the starting point. In many cases, and probably in most cases, these surveys were imperfectly made and were full of errors. This was due to two principal causes, namely, the cheapness of the lands and the lack of skill in the surveyors. The traverse was always a compass traverse, and usually the magnetic bearings only were taken, without noting the magnetic declination. In some cases, the starting point and angular points of the traverse were marked more or less permanently and described definitely, but in very many cases they were marked merely with a wooden stake driven into the ground, without anything else whatever to identify them, and sometimes they were not marked at all. It is often found that the traverse, when run accurately according to

the description, cannot be made to close on the starting point. Boundary lines described in deeds and shown on maps as straight are found to be crooked on the ground. Tracts commonly contain less or more land than called for in descriptions. Records of adjoining tracts often make one tract overlap another or leave an unclaimed gore between them. These discrepancies and blunders often render the work of the surveyor exceedingly difficult when retracing such old boundaries or locating their corners, and great tact and judgment are often necessary in making amicable and satisfactory adjustments of contending claims. In general, old boundaries, such as line trees, stone monuments, and fences, are sustained by the courts as holding; but, before retracing the lines, the surveyor should, if possible, secure the consent of adjacent owners to abide by such monuments and boundaries, irrespective of the lines or quantities called for in contracts or deeds.

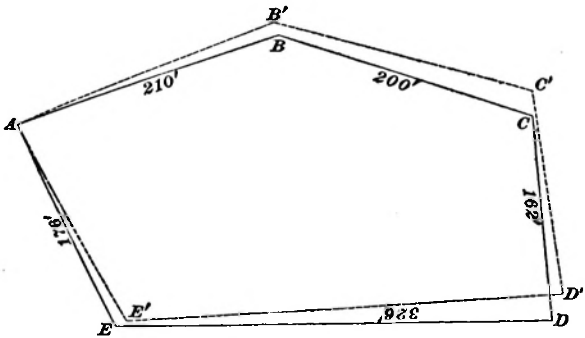

FIG. 11

27. Effect of Magnetic Variation.—It must be borne in mind that the bearings of lines are each year undergoing a slight change which, in a long period, amounts to several degrees, and if the lines were rerun according to original bearings as given in descriptions, they would enclose a tract quite different from that included in the original survey.

The surveyor must accordingly determine the amount of magnetic variation or change that has taken place between the date of the original survey and that of the survey about to be made, and having determined such change or variation, he must make his bearings conform to those of the original survey when making the resurvey. Before commencing the resurvey, the surveyor should correct all the bearings and write them out in their proper order together with the original bearings.

Fig. 11 illustrates the effect of magnetic variation in altering the direction of lines. The polygon *A B C D E* shows the outline of a tract according to the original survey, and *A′ B′ C′ D′ E′* the relative directions of the boundaries when resurveyed with the original bearings, there having been during the intervening time a west magnetic variation, or a westerly change in the magnetic declination. When the old bearings and the variation are known, the new bearings are determined in the same general way used for changing magnetic to true bearings, or vice versa, when the declination is known. (See *Compass Surveying.*)

Below are given the original and the corrected bearings of a survey, it having been ascertained that during the time elapsed between the original survey and the resurvey of the tract shown. the variation of the needle was 3° 00′ west. Let the student verify the values of the corrected bearings. In case of doubt, draw a diagram showing the new magnetic meridian 3° west of the original.

1 Courses	2 Bearings	3 Distances	4 Corrected Bearings
A B	N 68° 00′ E	210	N 71° 00′ E
B C	S 73° 00′ E	200	S 70° 00′ E
C D	S 8° 00′ E	162	S 5° 00′ E
D E	S 87° 00′ W	326	S 90° 00′ W
E A	N2 8° 00′ W	176	N 25° 00′ W

EXAMPLES FOR PRACTICE

1. If the variation in the magnetic declination is 3° 00′ east, what is the corrected bearing of a line whose original bearing was N 88° 30′ W? Ans. S 88° 30′ W

2. If during the period that has elapsed since a survey was made the variation in the magnetic declination is 4° 25′ west, what are the present bearings of the following lines, whose original bearings were: (*a*) S 82° 36′ E, (*b*) N 87° 35′ E, (*c*) S 42° 48′ W, and (*d*) N 32° 06′ W?

Ans. (*a*) S 78° 11′ E; (*b*) S 88° 00′ E; (*c*) S 47° 13′ W; (*d*) N 27° 41′ W

3. The original bearing of a certain line of a survey was S 25° 28′ E, and its present bearing is S 30° 40′ E; what should be the present bearings of certain other lines of the same survey whose original bearings were: (*a*) N 5° 30′ E, (*b*) S 89° 10′ E, and (*c*) S 75° 20′ W?

Ans. (*a*) N 0° 18′ E; (*b*) N 85° 38′ E; (*c*) S 70° 08′ W

4. A certain line of a survey is found to have a bearing N 27° 55′ E, and its bearing as recorded in a former survey was N 25° 00′ E; what should be the present bearings of certain other lines of the same survey whose original bearings, as recorded in the notes of the former survey, were: (*a*) N 89° 00′ E, (*b*) S 1° 30′ E, (*c*) S 88° 45′ W, and (*d*) N 45° 00′ W?

Ans. (*a*) S 88° 05′ E; (*b*) S 1° 25′ W; (*c*) N 88° 20′ W; (*d*) N 42° 05′ W

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28. How to Determine Magnetic Variation.—If the date of the original survey is known, the amount of variation may be determined approximately from a chart and tables published by the United States Coast and Geodetic Survey, which give the approximate yearly variation for different sections of the country. But no method that depends on computation for determining the amount of change in the declination can be anything better than a rough approximation. Moreover, the date of the survey is often omitted; the date of the deed must not be taken as the date of the survey.

If one of the original boundaries remains unobliterated and can still be traced, the magnetic variation can be determined

at once by taking the present bearing of the line. The difference between the present bearing and that of the original survey is the required correction. The corrections are then to be applied to the original bearings of the other lines and the resulting courses run out.

If any two corners of the original boundary can be identified, the true bearings and distances between them can be found. A traverse can be run as a random line between the two known corners, according to the notes of the original survey, the direction and distance found between the corresponding points of the original survey and the random traverse, and from them the corrected bearings and distances may be computed.

29. Straightening Boundaries.—Where the description and map show a boundary to be a straight line and the actual boundary is found to be crooked, it is sometimes good policy to establish a new and straight boundary by the principle of "give and take," provided that the owners of the adjoining lands will agree to the adjustment.

Fig. 12 illustrates the principle that is frequently employed in correcting such boundaries. Let A and E be two corners of a survey, and suppose that the boundary line joining them is described and shown on the map as a straight line, but that the irregular line $A\,B\,C\,D\,E$ represents the actual

FIG. 12

boundary. It is evident that the dotted straight line $A\,E$ may be substituted for the irregular line $A\,B\,C\,D\,E$, and equitably divide the adjoining properties. The principle of give and take is applied, the adjoining owners making exchanges of equal areas.

The location of the new boundary is determined by making a careful survey of the old boundary and platting it to a large scale; a fine thread is then stretched on the plat and a line of division made as closely as can be estimated by the eye.

The areas of the equalizing triangles are then calculated by scaling their dimensions, and if they do not balance, the dividing line can readily be shifted until the desired result is obtained. The line is then laid out and marked on the ground, and permanent corners established. Where the boundary is in woodland, careful search must be made for line and bearing trees. Blaze marks are very enduring, being easily recognized on most varieties of trees after a lapse of many years.

MAPPING

(PART 1)

PLATTING ANGLES AND LINES

INTRODUCTION

1. A **map** is a drawing representing the outlines, dimensions, and natural and artificial divisions and features of a part of the earth's surface by means of lines whose forms, lengths, and relative positions are determined by a survey.

2. Scale of Map.—Nearly all the straight lines shown on a map represent either boundaries or divisions of the given surface; they have only the properties of direction and length. Since the map is a representation of the surface of a given locality, the distance between any two points on the map bears a certain definite ratio to the distance between the two points represented by them. The ratio of a distance on the map to the corresponding actual distance is called the **scale** of the map. Thus, if 1 inch on the map represents 200 feet on the ground, the scale of the map is $\frac{1}{12} \div 200$, or $\frac{1}{2400}$. The scale is uniform for the entire map; consequently, all distances and areas shown on the map are proportional to their actual values. Hence, the distance between any two points shown on a map, or the area of any portion of the surface represented by it, can be measured directly on the map by means of the scale used in drawing it. The scale of a map depends on the size of the map and the purposes for which the map is to be used.

3. Measuring Scales.—A **measuring scale**, or simply a **scale**, is a graduated ruler used for making measurements on a drawing. Of the many kinds used by draftsmen, the one best adapted to the needs of the surveyor is the triangular scale divided decimally. A perspective view of it is shown in Fig. 1. The scale is 1 foot long, and contains six systems of graduations, one on each side of each edge; practically, therefore, it is a combination of six scales. Each scale is divided into inches and fractional parts of an inch, these parts being submultiples of 10—tenths, twentieths, fiftieths, etc. The number of parts into which each inch is divided is indicated by a large number in the center of the scale. Thus, the numbers *10* and *50* in Fig. 1 indicate that the upper scale there shown is divided into inches and tenths, and the lower into inches and fiftieths. Except in the 10-scale, the numbers opposite the graduations do not indicate inches; but, when multiplied by 10, they indicate the number of divisions from the zero of the scale. Thus, the number 12, opposite a division line on the 50-scale, denotes $\frac{12 \times 10}{50} = \frac{120}{50} = 2\frac{20}{50}$ inches measured from the zero of the scale. So, too, the number 22 on the 60-scale denotes $\frac{22 \times 10}{60} = \frac{220}{60} = 3\frac{40}{60}$ inches measured from the zero of the scale. In practice, it is not necessary to perform these operations. To determine, for instance, the value of the division marked 12 on the 50-scale (that is, the distance of that division from the zero mark), take on the scale the first number preceding 12 that is divisible by 5: this number is 10, which divided by 5 gives 2; to this add the number of divisions between 10 and 12, which is 20, thus obtaining $2\frac{20}{50}$ inches. Likewise, in using the 60-scale, the distance

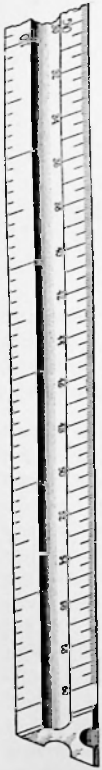


Fig. 1

that the division marked 22 is from the zero of the scale is obtained by dividing 18, which is the next lower multiple of 6, by 6, and adding the number of divisions between the 18-mark and the 22-mark, which in this case is 40; the result is, then, $\frac{18}{6} + \frac{40}{60} = 3\frac{40}{60}$ inches. Similarly for other divisions.

DRAWING THE PLATES

4. Drawing Plates.—The principles and methods of mapping described in *Mapping*, Parts 1 and 2, are fully illustrated by drawings. These comprise, besides the illustrations that appear in various parts of the text, seven drawing plates that are sent to the student, three with *Mapping*, Part 1, and four with *Mapping*, Part 2. The examples given in the plates are similar to those met in practical field and office work, and in each case the field notes of the survey are given in the text, the parts not required for the platting being omitted for brevity. From these field notes the student is required to draw the plates to the scale stated in each case.

5. Size of Plates.—As explained in *Geometrical Drawing*, the size of each finished plate is to be 14 by 18 inches. A border line is to be drawn ½ inch from each edge of the plate, thus making the size of the plate within the border line 13 by 17 inches. The sheet itself, when first placed on the board, must be somewhat larger than the finished plate, so that the holes made by the thumbtacks will not appear on the finished drawing. The extra margin is very convenient for testing the pen in order to see whether the ink flows freely and the lines have the proper width.

6. Titles of Mapping Plates.—The titles of all maps and plates drawn in connection with this part of the Course are to be in capital letters of the style shown in Fig. 2 (*b*). Such letters are commonly known as **Italic caps.**, and are used extensively for the titles of maps. The size of letters must be governed by the purpose for which they are to be used, but they should be of uniform size throughout each title, name, or statement. The letters in the title

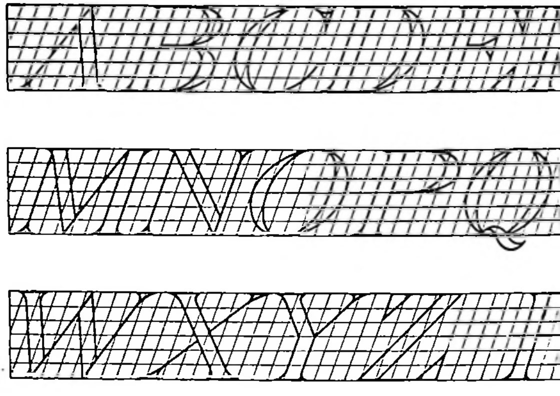
should be the largest letters on the drawing, but should harmonize with the rest of the lettering. Their size should depend somewhat on the size of the drawing. The letters in the titles to the mapping plates should be capitals and should have a uniform height of $\frac{3}{10}$ inch.

The student who has had no practice in making this style of letters will find it advantageous to construct the letters of the titles in the manner illustrated in Fig. 2 (*a*). The construction may be described as follows: The height of the letters, $\frac{3}{10}$ inch, is divided into six equal parts, and horizontal lines are drawn through the points of division. These horizontal lines are thus spaced $\frac{1}{20}$ inch apart. The lower horizontal line is then laid off in spaces of $\frac{1}{20}$ inch, and from the points of division parallel slanting lines are drawn to the upper horizontal line. The slanting lines are thus at the same distance apart horizontally that the horizontal lines are vertically. These parallel horizontal and slanting lines, which are to be drawn lightly with a fine-pointed pencil, form a series of construction lines on which to outline the letters. It is very important that all letters in the title have the same slant, though just what the slant is, within reasonable limits, is not important. In Fig. 2, each slanting line has a horizontal projection equal to two spaces, and this slant will be used for the letters of the titles to the mapping plates.

There is no rigid rule for the ratio of the width to the height in letters of this class, though, in general, this ratio should be less for large than for small letters. The horizontal widths of the letters shown in Fig. 2, expressed in terms of the spaces, are as follows:

Width of 1 space, letter I
Width of 4 spaces, letter J
Width of $4\frac{1}{2}$ spaces, letters L, N, U
Width of $4\frac{3}{4}$ spaces, letter F
Width of 5 spaces, letters A, B, D, E, H, K, P, S, V, Y, Z
Width of $5\frac{1}{2}$ spaces, letters C, G, R, T, X
Width of 6 spaces, letters M, O, Q
Width of 8 spaces, letter W

These widths are measured from outside to outside of the



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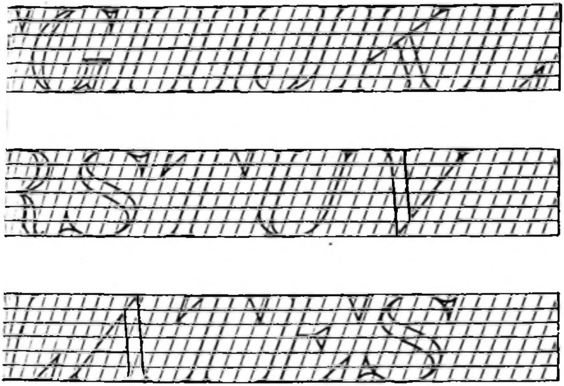


GHIJKL
RSTUV
LATES

slant lines, and do not include the projections of the short horizontal lines at the tops and bottoms of the letters. These horizontal lines should project one-half of one space at the top of the letter and three-fourths of one space at the bottom.

The letters should be spaced at regular intervals. For the letters here described, the spacing shown in Fig. 2 (*a*) gives a satisfactory appearance. The horizontal distance between the slant lines in adjacent letters should usually be uniform, except where *A* follows *P* or is adjacent to *T*, *V*, or *W*. In any of these cases, the extremity of the projecting horizontal line at the bottom of *A* should be in the same slant line with the top extremity of *P*, *T*, *V*, or *W*.

The spacing should be laid out and the construction lines drawn in pencil. The outlines of the letters should then be sketched carefully in pencil. In inking, the straight lines that compose a part of nearly every letter should usually be drawn with the ruling pen, giving all such lines the same slant and width, though after a draftsman becomes expert in lettering he may do this freehand. Each letter should then be finished carefully freehand, the shade parts inked black, and the construction lines erased. The letters should then have the same general appearance as the letters shown in Fig. 2 (*b*).

Expertness in lettering can be obtained only by constant practice. In practicing, the student should try to give each mark its proper position and form by a single stroke of the pen; then, if the result is reasonably satisfactory, it should not be changed, since trying to change it is not likely to improve it but is more likely to make it worse. Bold, sharp lines give the best appearance in lettering, and too fine lines detract from, rather than add to, the appearance of the letters. Attempting to make very fine lines is a common fault with beginners, and should be avoided.

Although the lettering of the title is the last work to be done on each mapping plate, the construction of the letters in the title is described here, since it is a matter that relates to all the mapping plates. Before the student begins to draw the title to the first mapping plate, he should study this article carefully and practice the construction of the letters.

DRAWING PLATE, TITLE: PLATTING ANGLES—I

NOTE.—In order to avoid confusion, a star (*) will be used to distinguish figures in the plates from those in the text. Thus, Fig.* 1 means Fig. 1 in the plate, while the usual notation Fig. 1 means Fig. 1 in the text.

7. Preliminary Explanations.—This plate shows six angle lines, three of which are grouped under Fig.* 1 and three under Fig.* 2. The three lines *a*, *b*, and *c*, under Fig.* 1, should be drawn to a scale of 200 feet to the inch, platting the angles by means of a protractor, the use of which is fully explained in *Geometrical Drawing*.

These lines are to be platted according to the directions given below. Care should be taken to locate each line approximately in the same relative position that it occupies on the plate. This statement also applies to all the plates to be drawn from the data given in this text. In some of these plates, distances are expressed in stations of 100 feet, in accordance with the common practice. The direction of each line is referred to that of the immediately preceding line prolonged, and the angle is recorded as being to the right or left, according as the line is to the right or left of the prolongation. In practical office work, the lines prolonged are drawn lightly in pencil, and erased as soon as the angles are laid off. In the lines *a* and *b*, Fig.* 1, the lines prolonged are dotted, and the angles marked by dotted arcs, in order that the method may be clearly understood. The dimensions of the plates and the directions for drawing the border lines are the same as for the plates in *Geometrical Drawing*. The notes for line *a* in Fig. *1 are given in the accompanying table.

NOTES FOR LINE *a*

Station	Angle
25 + 84	End of line
21 + 94	L. 32° 35′
15 + 53	R. 44° 10′
11 + 72	L. 60° 30′
5 + 25	L. 25° 15′
0	

8. Platting Angle Lines by Protractor.—The starting point *A* of line *a*, Fig.* 1, is Station 0. This point should be located in the same relative position that it occupies on the plate, namely, at a distance of $\frac{3}{4}$ inch from the lower and left-hand border lines. Starting from the point *A* as thus located, draw the line *A C* of indefinite length and in the same direction that it has in the plate. In order to determine this direction, draw a light pencil line, through *A* parallel to the vertical border line, on both the plate and the drawing, measure with the protractor the angle between this line and the line *A C* in the plate, and lay off the same angle between the lines in the drawing. From the point *A*, scale off on the line *A C* the length of the first course, 525 feet, thus locating the point *B*, which is Station 5 + 25. At this point, an angle of 25° 15′ is turned to the left. In order to lay off this angle, prolong *A B* to *C*, making *B C* a little longer than the radius of the protractor; then place the center of the protractor on the point *B*, with the zero point on the line *B C*, and on its graduated edge lay off the angle 25° 15′ to the left of *B C*, marking the point *D* of angle measurement with a needle point. Through the points *B* and *D* draw a straight line. The angle *C B D* is 25° 15′, and the line *B D* is the direction of the next course. Prolong the line *B D* to *F*, and on the line *B F* scale off from *B* the length of the second course *B E*, which is found by subtracting 525 from 1,172, giving a difference of 647 feet. The second angle point, Station 11 + 72, is thus located at *E*. Any number of minutes smaller than the number contained in the smallest subdivision of the protractor should be estimated and laid off by the eye.

The remaining angles and courses are platted in a similar manner. At *E*, lay off to the left of the line *E F* the second angle *F E G*, equal to 60° 30′. Prolong *E G* to *K*, and from *E* scale off on the line *E K* the length of the third course *E H*, which is equal to 1,553 − 1,172 = 381 feet. This determines the position of the third angle point, Station 15 + 53. At *H* lay off to the right of *H K* the third angle *K H L* equal to 44° 10′, and through the point *L* thus

determined draw the line HLN; the length of the fourth course is 2,194 − 1,553 = 641 feet, and this distance, scaled off from H along that line, fixes the position of Station 21 + 94 at M, the fourth angle point. Finally, lay off to the left from MN the angle NMO equal to 32° 35′, and from M, along the line MOP, scale off the length of the last course, which is equal to 2,584 − 2,194 = 390 feet. The fifth and last point P, which is Station 25 + 84, is thus established.

When commencing to plat the lines b and c, the notes for which are given in the accompanying table, care should be

NOTES FOR LINE *b*

Station	Angle
23 + 10	End of line
16 + 35	R. 25° 10′
12 + 82	L. 15° 15′
8 + 50	L. 30° 40′
4 + 40	R. 15° 20′
0	

NOTES FOR LINE *c*

Station	Angle
26 + 66	End of line
21 + 46	R. 34° 30′
17 + 09	R. 53° 28′
11 + 96	L. 25° 10′
5 + 33	R. 21° 10′
0	

exercised to locate the starting point of the line b at a distance of ¾ inch from the lower border line, and 4¼ inches from the left-hand border line, and Station 0 of line c at ¾ inch from the lower border line, and 7¼ inches from the left-hand border line. The first part of each line should be drawn in the same direction with reference to the border lines as on the plate, so as not to have the lines run too near together in platting. The rest of the work is similar to that explained for the line a.

9. To Lay Off an Angle by Its Chord.—In platting an angle by means of its chord, the length of the chord is obtained from a table of chords. Such a table gives the lengths of chords for all angles from 0° to 90° in a circle whose radius is 1. A radius of any convenient length may

be assumed, and the chord length corresponding to that radius obtained by multiplying the length of the chord given in terms of radius 1 by the length of the assumed radius. Thus, let it be required to lay off an angle of 40° 10′ to the left from the line AB, Fig. 3. The line AB is prolonged to C, making BC = 400 feet, the length assumed for the radius.

A table of chords gives .6868 as the chord of an angle of 40° 10′ in terms of a radius 1. Multiplying this chord by 400, the length of the assumed radius, gives 274.72 feet as the length of the required chord. From B as a center, and with a radius BC = 400 feet (to scale), describe to the left of BC the indefinite arc CD, making the length of the arc slightly greater than the length of the required chord. Then from C as a center, with a radius of 274.72 feet, describe an arc intersecting the arc CD at C', and connect B and C' by a straight line. The angle CBC' thus formed is the required angle measuring 40° 10′. This method of platting angles is more accurate, though less rapid, than platting with a protractor.

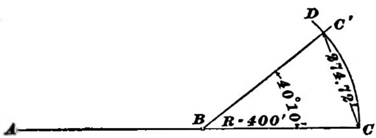


FIG. 3

The table of chords used for these calculations may be found in almost any engineers' pocketbook. If the student does not possess a pocketbook that contains this table, he can easily find the required chord from his table of natural sines; it is equal to twice the sine of half the given angle. Thus, the chord of 40° 10′ = 2 sin $\frac{40° \ 10'}{2}$ = 2 sin 20° 05′ = 2 × .34339 = .68678, which, written to only four decimal places, is .6868, the same as given in a table of chords.

10. Platting a Survey Line by Chords.—The method of platting a survey line by chords is illustrated in Fig.* 2 of this plate, in which the angles for the lines a, b, and c are laid off by this method. The notes for the line a are

given in the accompanying table, and from these notes the line is platted to a scale of 200 feet to the inch in the following manner:

The starting point or initial point *A* of this line, also numbered *0*, should be located on the drawing $\frac{3}{4}$ inch from the lower border line, and 11 inches from the left-hand border line, and the line *A B* should be drawn in the same direction, with respect to the border line, as is shown on the plate. The direction of this line once established, the point *B*, which is Station 3 + 60, is located by scaling off from *A* on the line *A B*, to the adopted scale, the length of the first course, which is 360 feet. At *B*, the line deflects 30° 30′ to the right from the line *A B*. Prolong *A B* 400 feet, which is the length of radius assumed in calculating the chord length for laying off the angles in these examples. Then from *B* as a center, with a radius of 400 feet, describe to the right of *A B* prolonged the indefinite arc *C C′*, containing at least 30° 30′. The chord of an angle of 30° 30′ corresponding to a radius 1 is .5261, which, when multiplied by 400, the length of the assumed radius, gives 210.44 feet as the length of the required chord. From *C* as a center, with a radius of 210.44 feet, describe an arc intersecting the arc *C C′* at the point *E*. The line joining *B* and *E* forms with *B C* an angle *C B E* = 30° 30′, the required angle. Prolong *B E* and scale off along this line the length of the second course, which is equal to 913 − 360 = 553 feet, thus locating the point *F*, which is Station 9 + 13. Prolong *B F* 400 feet to *G*. With *F* as a center, and a radius *F G* of 400 feet, describe to the right of *F G* the indefinite arc *G G′*, containing at least 39° 26′. The chord of 39° 26′ corresponding to a radius 1 is .6747, which, multiplied by 400, gives 269.88 feet as

NOTES FOR LINE *a*

Station	Angle
25 + 80	End of line
20 + 38	L. 37° 20′
15 + 18	L. 31° 08′
9 + 13	R. 39° 26′
3 + 60	R. 30° 30′
0	

the length of the required chord. With G as a center, and this chord as a radius, describe an arc intersecting the arc $G G'$ at H. The line joining F and H forms with the radius FG the required angle $GFH = 39° 26.'$ Prolong FH and lay off along this line the length of the third course, equal to 1,518 − 913 = 605 feet, thus locating the point K, which is Station 15 + 18. Prolong FK 400 feet to L. With K as a center and KL as a radius, describe to the left of KL the indefinite arc LM. The chord corresponding to 31° 08′ is .5367, which multiplied by 400 gives 214.68 feet as the length of the required chord. With this chord as a radius and L as a center, describe an arc intersecting the arc LM at the point N. Join K and N, forming with KL the

NOTES FOR LINE *b*

Station	Angle
22 + 40	End of line
16 + 50	L. 18° 20′
8 + 60	R. 25° 14′
3 + 25	R. 8° 10′
0	

NOTES FOR LINE *c*

Station	Angle
25 + 34	End of line
19 + 94	L. 51° 22′
14 + 81	R. 21° 20′
10 + 38	R. 39° 18′
4 + 13	L. 64° 30′
0	

angle $LKN = 31° 08'$, and along the continuation of KN scale off the fourth course equal to 2,038 − 1,518 = 520 feet. Station 20 + 38 is thus located at O. Produce KO 400 feet to P. Then with O as a center and OP as radius, describe the indefinite arc PQ. The chord of 37° 20′, the last angle, is .6401, which multiplied by 400 gives the length of the required chord as 256.04 feet. With P as center and this chord as radius, describe an arc intersecting PQ at R. The line joining O and R forms with OP the required angle $POR = 37° 20'$. Along the continuation of OR, scale off the length of the last course equal to 2,580 − 2,038 = 542 feet, thus locating the final Station 25 + 80 at the extremity S of this course.

In a similar manner, plat the lines *b* and *c* according to the accompanying notes, locating the starting point of line *b* 1 inch from the lower border line and 13 inches from the left-hand border line, and the starting point of line *c* $\frac{3}{4}$ inch from the lower border line and 14 inches from the left-hand border line.

DRAWING PLATE, TITLE: PLATTING ANGLES—II

11. To Lay Off an Angle by Its Tangent.—In laying off an angle by its tangent, the line from which the angle is turned is prolonged a certain distance, and the tangent of the given angle, as obtained from a table of natural tangents, is multiplied by the distance that the line is prolonged; this result, commonly called the **calculated tangent**, is platted on a line drawn perpendicular to the line prolonged at its extremity. A line joining the angular point with the extremity of the calculated tangent gives the direction of the required line, on which the given distance is then laid off to scale.

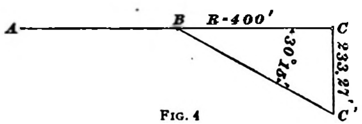


FIG. 4

Let *D B*, Fig. 4, be a given line from which an angle of 30° 15′ is to be laid off to the left at the point *B*. Prolong *D B* to *C*, making *B C* = 400 feet, the distance chosen

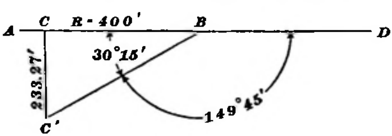


FIG. 5

in this case. Any other distance would do just as well. The tangent of 30° 15′, as given in a table of natural tangents, is .58318, which, when multiplied by 400 feet, gives 233.27 feet as the length of the calculated tangent. At *C* draw the perpendicular *C C′* to the right of *A C*, making its length equal to that of the calculated tangent, and join *B*

and C'. The angle $C B C'$ thus formed is the required angle of 30° 15′, and the line $B C'$ gives the direction of the required line.

To plat an obtuse angle $D B C'$, Fig. 5, turned off to the right of $B D$, produce $D B$, and construct, by the method explained above, the angle $C B C'$ equal to the supplement of $D B C'$, that is, equal to 180° minus the required obtuse angle.

12. Platting a Survey Line by Tangents.—This method is illustrated by the platted traverse lines *a* and *b* in Fig.* 1 of this plate. The notes for the line *a* are given in the accompanying table and are platted as follows: The starting point A, which is Station 0, should be located in the same relative position that it occupies on the plate, namely, ¾ inch from the left-hand border line and 3¾ inches from the upper border line. The direction of the line $A B$ can be determined in a manner similar to that described in Art. 8. After drawing the line $A B$ in the required direction, lay off on it from A, to a scale of 200 feet to the inch, the length of the first course, 500 feet, thus locating the point B, which is Station 5. At this station, the line deflects 43° 30′ to the right. Prolong $A B$ 400 feet. The natural tangent of 43° 30′ is .94896, which, when multiplied by 400, gives 379.58 feet as the length of the calculated tangent. At C, the extremity of the line prolonged, draw the perpendicular $C D$ to the right, and on this perpendicular scale off from C the calculated tangent distance, 379.58 feet, to the point D. The line joining B and D forms an angle of 43° 30′ with the line $A B$ prolonged and gives the required direction of the second course of the traverse. On the line $B D$ prolonged, scale off the distance 1,322 − 500 = 822 feet, thus locating the point E, which is Station 13 + 22. At this point the line

NOTES FOR LINE *a*

Station	Angle
25 + 00	End of line
19 + 97	L. 40° 10′
13 + 22	R. 32° 15′
5 + 00	R. 43° 30′
0	

deflects 32° 15′ to the right. Prolong *BE* 400 feet to the point *F*. The natural tangent of 32° 15′ is .63095, which, when multiplied by 400, gives 252.38 feet as the length of the calculated tangent. At *F*, the extremity of the prolongation of *BE*, draw the perpendicular *FG* to the right, and on this perpendicular scale off the calculated tangent distance, 252.38 feet, to the point *G*, and join *E* and *G*. The angle *FEG* thus formed is 32° 15′, and the line *EG* is the required direction of the third course of the traverse. Scale off on the line *EG* prolonged the distance 1,997 − 1,322 = 675 feet to *H*, thus locating Station 19 + 97, at which point the line deflects 40° 10′ to the left. Prolong *EH* 400 feet to *K* and at this point draw the perpendicular *KL* to the left. The natural tangent of 40° 10′ is .84407, which, when multiplied by 400, gives 337.63 feet as the calculated tangent. Scale off on the line *KL* the calculated tangent distance of 337.63 feet to *L*, and join *H* and *L*. The angle *KHL* thus formed is 40° 10′. Scale off on the line *HL* the distance 2,500 − 1,997 = 503 feet, thus locating the point *M* at the end of the line, which is Station 25 + 00. In a similar manner plat the line *b* according to the notes here given, locating the starting point, Station 0, ½ inch below the point *A* of the preceding line, and ¾ inch from the left border line.

NOTES FOR LINE *b*

Station	Angle
27 + 47	End of line
20 + 97	R. 42° 20′
12 + 73	R. 49° 10′
6 + 63	L. 62° 15′
0	

13. To Lay Off Angles by Bearings.—By this method of laying off angles, the direction of each line is referred to a meridian line. In platting the traverse of a survey, a pencil line giving the direction to the meridian is drawn through each station at which the bearing of a line is taken. Usually, it is convenient to make the meridian parallel to the sides of the drawing sheet. The lines showing the direction of the meridian at each station can then be drawn by

means of the ordinary **T** square and triangles, and the angles can be laid off by means of a protractor, by chords, or by tangents, as may be desired.

Let it be required to plat a line having a bearing of N 60° E followed by a line having a bearing of N 30° E. The point *A*, Fig. 6, is assumed as the position of the station at which the first bearing is taken. Through this point the line *A B* is drawn along the edge of the triangle, perpendicular to the straight edge of the **T** square, to represent the direction of the meridian. Since the bearing is east, the angle *C A B*, equal to 60°, is laid off to the right of *A B*. From

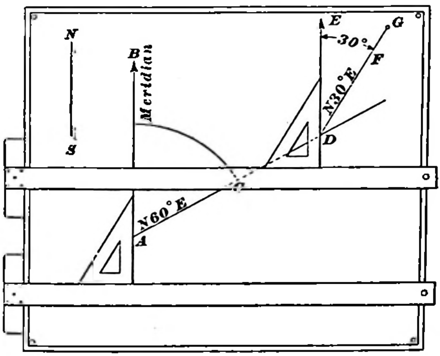


FIG. 6

the notes, find the length of the first course, and lay it off along *A C* to the adopted scale, thus locating at its extremity the point *D*, where the next bearing is taken.

The bearing of the next line, as taken at *D*, is N 30° E. Slide the **T** square upwards, and also slide the triangle across the board until the edge that is perpendicular to the **T** square passes through *D*, and then draw the meridian *D E* through *D*. From *D*, lay off an angle *E D G* equal to 30°, to the right of *D E*.

14. Platting Bearings by Protractor.—This method of platting bearings is illustrated in Fig.* 2. The four angle

lines or traverse lines shown in this figure, designated as *a*, *b*, *c*, and *d*, respectively, are all platted by bearings. The bearings of the lines *a* and *b* are platted by means of the protractor, and those of the lines *c* and *d* are platted by the tangent method; the latter method is the more accurate, though it is much the slower. For the lines *a* and *c*, the distances are given in stations of 100 feet, and should be platted to a scale of 200 feet to the inch; for the lines *b* and *d*, the distances are given in chains, and should be platted to a scale of 2 chains to the inch. The notes for line *a* are here given in tabular form.

NOTES FOR LINE *a*

Station	Bearing
28 + 15	End of line
23 + 55	S 45° 00′ E
18 + 92	S 70° 45′ E
14 + 20	N 80° 30′ E
10 + 40	S 81° 20′ E
6 + 90	N 83° 30′ E
3 + 75	N 60° 00′ E
0	N 10° 15′ E

The point *A*, which is Station 0 of the line *a*, should be located in about the same relative position as it has on the plate, that is, $\frac{3}{4}$ inch from the left-hand border line and $7\frac{1}{2}$ inches from the lower border line. The arrow *N S* represents the direction of the meridian. The bearing of the first course is N 10° 15′ E. Through *A*, draw a line *A B* parallel to the meridian line *N S*. The line so drawn represents the direction of the meridian at the point *A*, and the direction of the first course is then at an angle of 10° 15′ to the right of the line *A B*. At the point *A*, lay off an angle of 10° 15′ to the right of *A B* by means of the protractor, and then draw the line *A C*, which thus has a bearing N 10° 15′ E and represents the direction of the first course. Scale off the length of the first course, 375 feet, from *A* along the line *A C*, thus locating the point *C*, which is Station 3 + 75. Draw the line *C D* through *C* parallel to the meridian line *N S*; the line *C D* then represents the direction of the meridian at the point *C*. Since the bearing of the second course is

N 60° 00′ E, lay off at C an angle of 60° 00′ to the right of CD, and draw the line CE, which thus represents the direction of this course. Scale off from the point C along the line thus drawn the length of the second course, which is equal to 690 − 375 = 315 feet, thus locating the point E, which is Station 6 + 90. Through E draw the line EF parallel to the meridian line NS; and at the point E lay off to the right of the line EF the bearing of the third course, which is N 83° 30′ E, and draw the line EG representing the direction of this course. Scale off on this line the distance 1,040 − 690 = 350 feet, thus locating the point G, which is Station 10 + 40. Draw the meridian line GH, and lay off from the line GH the bearing of the next course, S 81° 20′ E. Since this course is south and east, the angle is laid off to the right of GH from the south end of the protractor, in a direction contrary to the movement of the hands of a watch. After laying off this angle, draw the line GK representing the direction of this course, and scale off on it the distance 1,420 − 1,040 = 380 feet, thus establishing the point K, which is Station 14 + 20. In a similar manner locate the remaining stations on this line.

The notes for line *b* are given in tabular form below, and should be platted according to the method just described. In the notes of this line, the stations at the angles of the line are numbered consecutively, beginning with Station 1 at the starting point, which is located 1¼ inches below the point *A* of the preceding line and ¾ inch from the left-hand border line. This station is written in the first or top line of the notes, which read from the top downwards, as is not unusual in the notes of ordinary land surveys with the compass, instead of from the bottom upwards, as in surveys where more or less sketching

NOTES FOR LINE *b*

Station	Bearing	Distance Chains
1	N 40½° E	4.22
2	N 65¼° E	6.75
3	S 75½° E	8.70
4	S 45¼° E	6.60
5	N 81¼° W	5.18
6	End of line	

is necessary. In these notes, the numbers of the stations do not indicate the lengths of the various courses; these lengths are given in the third column of the notes.

15. Platting Bearings by Tangents.—Lines *c* and *d* of Fig.* 2 are platted by means of the tangents of the bearings of the various courses. The direction of each line is laid off from the meridian by means of the tangent of its bearing in substantially the same manner that an angle is laid off by its tangent, as explained in Art. **11.** This is one of the best methods of platting bearings, and when a T square and triangles are used, the work can be performed expeditiously. The notes given here for line *c* are platted in the following manner:

NOTES FOR LINE *c*

Station	Bearing
29 + 15	End of line
23 + 65	S 30° 45′ W
15 + 85	S 70° 15′ E
11 + 25	N 25° 30′ E
6 + 00	S 65° 30′ E
0	N 45° 00′ E

The point *A*, which is the initial point of this line, should be located in the same relative position that it occupies on the plate, that is, $\frac{3}{4}$ inch from the left-hand border line and $2\frac{1}{2}$ inches below the point *A* of the line immediately preceding. The line *N S* is assumed to have the direction of the meridian. The bearing of the first course is N 45° 00′ E. Through *A* draw the meridian line *A B* parallel to the meridian *N S*, and on this line scale off from *A* the distance by which the tangent of the bearing is multiplied, in this case 400 feet, to the point *B*. At *B*, the extremity of this distance, draw *B C* to the right perpendicular to the meridian line *A B*. The tangent of 45° 00′ is 1.0000, which when multiplied by 400 gives 400 feet as the length of the calculated tangent for this bearing and distance. Scale off the distance of 400 feet on the perpendicular *B C*, thus fixing the point *C*, and draw the line *A C*. The angle *B A C* that the line *A C* forms with the meridian is 45° 00′, and the bearing of the line *A C* is N 45° 00′ E. On the line *A C*

prolonged, scale off the length of the first course, 600 feet, thus locating the point D, which is Station 6. The bearing of the second course is S 65° 30′ E. Through D draw the meridian line DE. Since the bearing is south and east, it should be laid off below the point D and to the right of DE. On the line DE scale off the distance, 400 feet, to the point E. At E draw the perpendicular EF to the right of DE. The tangent of 65° 30′ is 2.19430, which, when multiplied by 400, gives 877.72 feet for the calculated tangent. On the perpendicular EF, scale off the distance of 877.72 feet to the point F, and draw the line DF; the angle EDF thus formed is 65° 30′, and the bearing of the line DF is S 65° 30′ E. On the line DF scale off the length of the second course, which is equal to 1,125 − 600 = 525 feet, thus locating the point G, which is Station 11 + 25. In a similar manner plat the directions of the remaining courses, and locate the stations at their extremities.

NOTES FOR LINE *d*

Station	Bearing	Distance Chains
1	N 30° E	6.18
2	S 33¼° E	5.15
3	N 70¾° E	8.42
4	S 55½° E	6.63
5	N 26¾° E	7.89
6	End of line	

The notes for line *d* are given here in tabular form; they are platted in substantially the same manner as those for line *c*, except that, as the distances are in chains, a distance of 4 chains, equal to 400 links, is laid off on the meridian line through each angular point, and is used as the multiplier of each tangent. The point A of line *d* is located ¾ inch from the left-hand border line and ½ inch above the lower border line, and is marked as Station 1. Through this point, draw the meridian line AB parallel to NS, and on this line scale off from A the distance of 4 chains to the point B. At B erect a perpendicular to the right of AB. The bearing of the first course is N 30° 00′ E, and the tangent of 30° 00′ is

.57735, which, when multiplied by 4, gives 2.3094 chains as the length of the calculated tangent for this bearing and distance. Scale off this calculated tangent distance on the perpendicular and draw a line joining the point thus determined with the point *A*; the line thus drawn forms with the line *A B* an angle of 30° 00′, and has a bearing of N 30° 00′ E. On this line, scale off the length of the first course, 6.18 chains, thus locating Station 2. In a similar manner plat the remaining courses and locate Stations 3, 4, 5, and 6, marking them as shown in the plate.

The bearing of each course should be written above it plainly and distinctly, the letters reading in the same direction in which the course is measured. In all railroad surveys and in the more recent highway surveys, the distances are reckoned in station intervals of 100 feet, and each station is numbered according to its distance in station intervals from the initial point of the survey. When this system is used, the length of each course is given by the difference between the numbers of the stations at its two extremities. But in land surveys, and in the older highway surveys, the lengths of the courses are recorded in surveyors' chains, and the stations at the angular points or extremities of the courses are numbered consecutively, beginning with Station 1 at the initial point of the survey. In this system, the lengths of the courses are not indicated by the station numbers, but must be recorded separately.

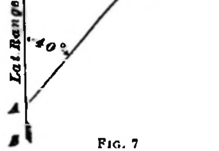


Fig. 7

16. To Lay Off an Angle by Latitude and Longitude Ranges. Suppose that the bearing of a course is N 40° 00′ E, and its length is 300 feet. The ranges of the course, which are calculated from the length and bearing, are: latitude range, +229.8 feet; longitude range, +192.8 feet.

Let *A*, Fig. 7, be the station at which the bearing is taken.

Through A draw the meridian $N S$. From A lay off along $N S$ the distance $A B$ equal to the calculated latitude range, or 229.8 feet, and at the extremity of this distance draw the perpendicular $B C$ to the right, or toward the east. On this perpendicular, scale off the calculated longitude range, 192.8 feet, to the point C, and draw the line $A C$. The angle $B A C$ thus formed is an angle of 40°, and the length of $A C$ is 300 feet.

17. Platting a Survey by Ranges.—In this method, each course is platted by means of its latitude and its longitude range, the operation described in the last article being repeated at every station. As will be observed, it is not necessary to scale off the distances, since the extremities of each course are determined by its ranges.

18. Platting a Survey by Latitudes and Longitudes.—This method of platting is fully explained in another section. It is, by far, more accurate than any of the methods that have been described, because the position of each station is referred directly to the initial point or first station of the survey and is platted independently.

In surveys of such character as preliminary railroad surveys, ordinary land surveys, etc., angles and bearings are often platted by tangents; but on such work as difficult railroad location, where dependence must be placed on accurate platting for the purpose of obtaining a satisfactory paper location, the method by latitudes and longitudes should be used, and the line should be platted to a scale large enough to show complete topographical details.

19. Parallel Rulers.—There are two general kinds of parallel rulers. One kind, known as the **folding parallel**

FIG. 8

ruler, consists of two rulers connected to each other by two light bars of equal length and having jointed ends that

permit the rulers to be opened apart or folded together, but hold them constantly parallel to each other. This form of parallel ruler is shown in Fig. 8; it is not much used at the present time. The other kind, called the **rolling parallel ruler,** is very convenient for platting and is used very extensively. An illustration of it is given in Fig. 9. It consists of an ordinary ruler fitted with milled rollers of equal diameter attached to a common axis; it is usually made of metal, and is therefore of considerable weight. This instrument is used for the purpose of transferring the direction of a line from one part of the plat to another, such as drawing a line through any point on a map parallel to the meridian. For instance, if it is required to draw a meridian line through a certain point on a plat, the straight edge of the parallel ruler is made to coincide with the line representing the given meridian, and the ruler is then rolled across the paper until

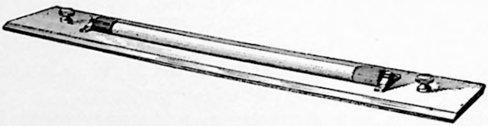


FIG. 9

its straight edge passes through the desired point. A line drawn through the point along the straight edge of the ruler will then be the required meridian line.

20. Protractor Sheets.—In platting surveys by means of the ordinary movable protractor, more or less error is likely to occur in transferring the meridian lines from point to point on the plat and in adjusting the protractor to the meridian line for laying off the bearing of each course. This liability to error is reduced to a minimum by drawing the plat on what is commonly called a **protractor sheet.** This is a sheet of drawing paper, tracing paper, or bristol board having a protractor printed in the center of the sheet, and is sometimes called a **paper protractor.** The protractor is a full circle, from 8 to 14 inches in diameter, graduated to half or quarter degrees, according to size, and is printed from

accurately engraved plates. It is usually printed in red on Whatman's drawing paper. The numbers of the divisions of the protractor are not printed on the sheet, but are written

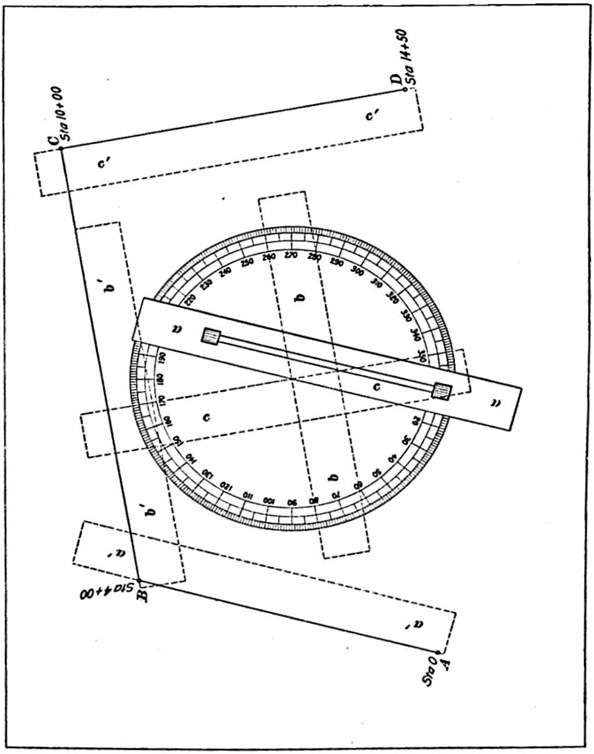


Fig. 10

on afterwards by the draftsman in the manner most convenient for the plat for which the sheet is used. Fig. 10 represents a protractor sheet, and illustrates the manner in

which an azimuth traverse is platted by means of the parallel ruler. The notes of the traverse are given in the accompanying table. In this case the meridian line is assumed to be parallel to the left edge of the drawing paper, and the divisions on the protractor are numbered accordingly. As this is for the platting of an azimuth traverse, the degrees are numbered from 0 to 360 around the complete circle, with the zero mark toward the north or the south edge of the paper, according as azimuths are reckoned from the north or from the south. The point *A* is chosen for the initial point, or Station 0 of the traverse. Since the forward azimuth of the first course from Station 0 is 195°, the parallel ruler is placed on the protractor in the position *a a*, so that the straight edge of the parallel ruler is on the division marked 195° and the division directly opposite, which is the division marked 15°. The number of the division on the protractor directly opposite the division representing the given azimuth will always be equal to the difference between 180° and the given azimuth. When the straight edge of the parallel ruler is placed on two such divisions, it will always pass through the center of the protractor. By using opposite divisions, instead of one division and the center of the protractor, greater accuracy is obtained. When the edge of the parallel ruler is on the 195° and the 15° division marks, it is in a position parallel to the direction of the line that is to be platted through the point *A* to represent the first course of the traverse, whose azimuth is 195°. The parallel ruler is then rolled to the point *A*, where it will have the position *a′ a′*, and, with its straight edge passing through the point, the line *A B* is drawn. On this line the length of the first course, 400 feet, is laid off to the scale of the plat, thus locating the point *B*, which is Station 4. The azimuth reading for the

Station	Azimuth	Magnetic Bearing
14 + 50		
10 + 00	350° 00′	N 10° W
4 + 00	260° 00′	S 80° W
0	195° 00′	S 15° W

course from Station 4 to Station 10 is 260°. The parallel ruler is therefore placed in the position *b b*, so that its edge is on the division of the protractor marked 260° and the opposite division, marked 80°, and is then rolled to the point *B*, in the position *b' b'*, and the line *B C* is drawn. On the line thus drawn, the distance 1,000 − 400 = 600 feet is scaled off, thus locating the point *C*, which is Station 10. In a like manner the line *C D* is platted, locating the point *D*, which is Station 14 + 50. If a parallel ruler is not at hand, two triangles can be used for the same purpose, though not quite so expeditiously. The method of drawing parallel lines by means of two triangles is fully explained in *Geometrical Drawing*.

DRAWING PLATE, TITLE: MAP OF RAILROAD LOCATION

21. The Line of a Railroad Survey.—The line of a preliminary railroad survey is usually a succession of straight lines; such a line is not uncommonly called an *angle line*, but it is correctly designated as a *traverse line*. The **located line,** as the final line of a railroad survey is called, is a succession of straight lines and curves. In platting a located line, the straight lines, commonly called **tangents,** should be laid out and measured from one point of intersection to another, and the angles between the tangents, that is, the intersection angles, should be platted either by tangents or by ranges. The points of tangency, that is, the point of curve and point of tangent for any required curve joining two tangents, should then be located by calculating the tangent distances and scaling them off from the point of intersection backwards and forwards on the two connecting tangents. The center of each curve is best determined by describing intersecting arcs from the points of tangency as centers, with a radius equal to that of the given curve.

22. General Description of the Plate.—On this plate are shown two platted lines, each illustrating the operation of platting the line of a railroad location. This map is

called a map of railroad location, but the following description will apply as well to the platting of any similar alinement consisting of straight lines and curves, such as that for a canal or pipe line. In this plate, all the angles are laid off by the method of tangents described in Art. **11**, though any other satisfactory method may be used. The notes from which these two lines are to be platted are given in detail on pages 28 to 33; they should be gone over carefully and each calculation verified before beginning the platting. The magnetic meridian is assumed to be parallel to the right and left border lines of the plate, and the starting point, or Station 0, of each line should be platted in the same position with reference to the border lines as it occupies on the plate. The direction of the first tangent of each line should be determined from the direction of the magnetic meridian, and should be platted carefully with reference thereto. Without these precautions, the lines are liable to run off the paper, necessitating a repetition of the work, and involving the erasure of lines, which soils and injures the paper, and mars the appearance of the drawing. The notes should be platted to a scale of 300 feet to the inch.

23. Drawing the Plate.—As has been stated, the notes of two lines of railroad location are given in detail in the following pages. All calculations necessary for platting these notes are assumed to have been made, and written on the right-hand page of the notebook, as shown. All these calculations for each line should be verified by the student before beginning to plat the line. The lines are then platted from the notes as follows: First draw a meridian line parallel to the right-hand and left-hand border lines, then locate the starting point *A*, Fig.* 1, $\frac{3}{4}$ inch from the left-hand border line, and $1\frac{3}{4}$ inches from the lower border line. Through this point, which is Station 0 of this line, draw the line *A B* parallel with the meridian line. From the notes, it is found that Station 0 is the P. C. of an 8° curve to the right, and that the direction of the tangent *A′ A*, preceding the curve, is due north and south; this tangent is assumed to be part of

a line already constructed. The intersection angle is 63° 10′, and the radius of an 8° curve is 716.78 feet; from these the tangent distance is found to be 440.7 feet, as recorded in the notes. Using a scale of 300 feet to the inch, this distance is scaled off upwards from the point *A*, on the tangent *A′A* prolonged, thus locating the point of intersection *C* of the back and forward tangents, or tangents preceding and following the curve. In order to determine the direction of the forward tangent *CE*, it is necessary to lay off to the right, at the point of intersection *C*, the intersection angle of 63° 10′. On the prolongation of the back tangent, scale off, from the angle of intersection to the point *D*, the distance *CD*, equal to the multiplier of the tangent, which in this case is taken as 400 feet. The intersection angle of this curve is 63° 10′, and the tangent of this angle is 1.97681. The length of the calculated tangent is therefore $400 \times 1.97681 = 790.7$ feet. Since this curve is to the right, draw from *D* a line to the right perpendicular to *AB*, and on this perpendicular scale off the calculated tangent, 790.7 feet, locating the point *E*. A line drawn from *C* through the point *E* gives the direction of the forward tangent. On the line *CE*, scale off from *C* the tangent distance of the curve, 440.7 feet, thus locating the P.T. of the first curve at the point *F*, which is at Station 7 + 89.6, since the P. C. is at Station 0 and the length of the curve is 789.6 feet. From *A* and *F* as centers, with a radius of 716.78 feet, the radius of an 8° curve, describe arcs intersecting at *G*; then, with *G* as a center, and the same radius, describe a curve joining the points *A* and *F*. The curve *AF* is an 8° curve tangent to the lines *AA′* and *FE* at the points *A* and *F*.

From the notes it is found that the tangent *FE* extends to Station 13 + 16, which is the P. C. of a curve of 6° to the right, whose central angle is 44° 20′. The radius of this curve is 955.37 feet, and its tangent distance is 389.2 feet, as given in the notes. The distance from the point of intersection of the first curve to the point of intersection of the second curve is calculated next. This distance is composed of three parts; namely, the tangent of the preceding curve, which is 440.7 feet; the intermediate tangent, extending

NOTES FOR FIG.* 1

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
40	12° 00′			
39	6° 00′			
38 + 00	18° 00′	36° 00′		
37	12° 00′			
36	6° 00′			
35 + 00	P. C. 12° L.			
34				
33 + 4.9	10° 40.3′ P. T.	35° 50′	S 36° 30′ E	S 36° 40′ E
33	10° 30′			
32	7° 00′			
31	3° 30′			
30	7° 14.7′	14° 29.4′		
29	3° 44.7′			
28	0° 14.7′			
27 + 93	P. C. 7° R.			
24				
21				
20 + 54.9	10° 38.8′ P. T.	44° 20′	S 72° 30′ E	S 72° 30′ E
20	9° 00′			
19	6° 00′			
18	3° 00′			
17	11° 31.2′	23° 2.4′		
16	8° 31.2′			
15	5° 31.2′			
14	2° 31.2′			
13 + 16	P. C. 6° R.			
12				
11				
10				
9				
8				
7 + 89.6	15° 35′ P. T.	63° 10′	N 63° 00′ E	N 63° 10′ E
7	12° 00′			
6	8° 00′			
5	4° 00′			
4	16° 00′	32° 00′		
3	12° 00′			
2	8° 00′			
1	4° 00′			
0	P. C. 8° R.		North	North

NOTES FOR FIG.* 1

Remarks	June 28, 1894
Int. Ang. = 72° 00′	From intersection to intersection.
12° curve, L. R. = 478.34 ft.	Tan preceding curve = 2 6 4.8 ft.
T. = 347.5 ft.	Tan between curves = 1 9 5.1 ft.
P. C. = 35 + 00	Tan 12° curve = 3 4 7.5 ft.
Length of curve = 600 ft.	Total, = 8 0 7.4 ft.
P. C. C. = 41 + 00	tan 72° 00′ = 3.07768
Def. 100 ft. = 6° 00′	400 ft. × 3.07768 = 1,231.1 ft.
Def. 1 ft. = 3.6′	
Int. Ang. = 35° 50′	From intersection to intersection.
7° curve, R. R. = 819.02 ft.	Tan preceding curve = 3 8 9.2 ft.
T. = 264.8 ft.	Tan between curves = 7 3 8.1 ft.
P. C. = 27 + 93	Tan 7° curve = 2 6 4.8 ft.
Length of curve = 511.9 ft.	Total, = 1 3 9 2.1 ft.
P. T. = 33 + 4.9	tan 35° 50′ = .72211
Def. 100 ft. = 3° 30′	400 ft. × .72211 = 288.8 ft.
Def. 1 ft. = 2.1′	
Int. Ang. = 44° 20′	From intersection to intersection.
6° curve, R. R. = 955.37 ft.	Tan preceding curve = 4 4 0.7 ft.
T. = 389.2 ft.	Tan between curves = 5 2 6.4 ft.
P. C. = 13 + 16	Tan 6° curve = 3 8 9.2 ft.
Length of curve = 738.9 ft.	Total, = 1 3 5 6.3 ft.
P. T. = 20 + 54.9	tan 44° 20′ = .977
Def. 100 ft. = 3° 00′	400 ft. × .977 = 390.8 ft.
Def. 1 ft. = 1.8′	
Int. Ang. = 63° 10′	Radius 1 = 400 ft.
8° curve, R. R. = 716.78 ft.	tan 63° 10′ = 1.97681
T. = 440.7 ft.	400 ft. × 1.97681 = 790.7 ft.
P. C = 0	
Length of curve = 789.6 ft.	
P. T. = 7 + 89.6	
Def. 100 ft. = 4° 00′	
Def. 1 ft. = 2.4′	

NOTES FOR FIG.* 1—*Continued*

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
69 + 10.1		End of line		
61 + 65.1	15° 40′ P. T.	31° 20′	N 39° 45′ E	N 39° 40′ E
61	12° 44.1′			
60	8° 14.1′			
59	3° 44.1′			
58 + 17	P. C. 9° R.			
55				
54				
53				
52				
51				
50 + 00	17° 30′ P. T.	63° 00′	N 8° 15′ E	N 8° 20′ E
49	14° 00′			
48	10° 30′			
47	7° 00′			
46	3° 30′			
45	14° 00′	28° 00′		
44	10° 30′			
43	7° 00′			
42	3° 30′			
41 + 00	18° 00′ P. C. C. 7° L.	72° 00′	N 71° 15′ E	N 71° 20′ E

NOTES FOR FIG.* 2

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
13 + 41.7	10° 15′ P. T.	32° 30′	S 79° 00′ E	S 79° 00′ E
13	9° 00′			
12	6° 00′			
11	3° 00′			
10 + 00	6° 00′	12° 00′		
9	3° 00′			
8 + 00	P. C. 6° L.			
5				
3				
0			S 46° 30′ E	

NOTES FOR FIG.* 1—*Continued*

	Remark	June 28, 1894
Int. Ang. = 31° 20′ 9° curve, R. R. = 637.27 ft. T. = 178.7 ft. P. C. = 58 + 17 Length of curve = 348.1 ft. P. T. = 61 + 65.1 Def. 100 ft. = 4° 30′ Def. 1 ft. = 2.7′	From intersection to intersection. Tan preceding curve = 5 0 1.9 ft. Tan between curves = 8 1 7.0 ft. Tan 9° curve = 1 7 8.7 ft. Total, = 1 4 9 7.6 ft. tan 31° 20′ = .60881 400 ft. × .60881 = 243.5 ft.	
Int. Ang. = 63° 00′ 7° curve, L. R. = 819.02 ft. T. = 501.9 ft. P. C. C. = 41 + 00 Length of curve = 900 ft. P. T. = 50 + 00	From intersection to intersection. Tan preceding curve = 3 4 7.5 ft. Tan between curves = 0.0 ft. Tan 7° curve = 5 0 1.9 ft. Total, = 8 4 9.4 ft. tan 63° = 1.96261 400 ft. × 1.96261 = 785 ft.	

NOTES FOR FIG.* 2

	Remarks	June 28, 1894
Int. Ang. = 32° 30′ 6° curve, L. R. = 955.37 ft. T = 278.5 ft. P. C. = 8 + 00 Length of curve = 541.7 ft. P. T. = 13 + 41.7 Def. 100 ft. = 3° 00′ Def. 1 ft. = 1.8′	Radius 1 = 4 0 0.0 ft. From Sta. 0 to P. C. = 8 0 0.0 ft. Tan 6° curve = 2 7 8.5 ft. Total from P. C. to P. I. = 1 0 7 8.5 ft. tan 32° 30′ = .63707 400 ft. × .63707 = 254.8 ft.	

NOTES FOR FIG.* 2—*Continued*

Station	Deflection	Total Angle	Magnetic Bearing	Calculated Bearing
57 + 40		End of line		
47 + 19	8° 46.3′ P. T.	34° 00′	N 11° 15′ E	N 11° 20′ E
47	8° 15′			
46	5° 30′			
45	2° 45′			
44 + 00	8° 13.7′	16° 27.4′		
43	5° 28.7′			
42	2° 43.7′			
41 + 00.8	14° 01.7′ P.C.C. 5° 30′ L.	52° 00′	N 45° 15′ E	N 45° 20′ E
41	14° 00′			
40	10° 30′			
39	7° 00′			
38	3° 30′			
37 + 00	11° 58.2′	23° 56.4′		
36	8° 28.2′			
35	4° 58.2′			
34	1° 28.2′			
33 + 58	P. C. 7°. L.			
32				
30 + 36.6	13° 27.8′ P. T.	48° 10′	S 82° 30′ E	S 82° 40′ E
30	12° 00′			
29	8° 00′			
28	4° 00′			
27 + 00	10° 37.2′	21° 14.4′		
26	6° 37.2′			
25	2° 37.2′			
24 + 34.5	P. C. 8° L.			
24				
23				
22 + 14.4	9° 39′ P. T	44° 30′	S 34° 20′ E	S 34° 30′ E
22	9° 00′			
21	4° 30′			
20 + 00	12° 36′	25° 12′		
19	8° 06′			
18	3° 36′			
17 + 20	P. C. 9° R.			
17				
15				

NOTES FOR FIG.* 2—*Continued*

	Remarks June 28, 1894
Int. Ang. = 34° 00′	From intersection to intersection.
5° 30′, L. R. = 1,042.14 ft.	Tan preceding curve = 399.5 ft.
T. = 318.6 ft.	Tan between curves = 0.0 ft.
P. C. C. = 41 + 00.8	Tan 5° 30′ curve = 318.6 ft.
Length of curve = 618.2 ft.	Total, = 718.1 ft.
P. T. = 47 + 19	tan 34° 00′ = .67451
Def. 100 ft. = 2° 45′	400 ft. × .67451 = 269.8 ft.
Def. 1 ft. = 1.65′	
Int. Ang. = 52° 00′	From intersection to intersection.
7° curve, L. R. = 819.02 ft.	Tan preceding curve = 320.4 ft.
T. = 399.5 ft.	Tan between curves = 321.4 ft.
P. C. = 33 + 58	Tan 7° curve = 399.5 ft.
Length of curve = 742.8 ft.	Total, = 1041.3 ft.
P. C. C. = 41 + 00.8	tan 52° 00′ = 1.27994
Def. 100 ft. = 3° 30′	400 ft. × 1.27994 = 512 ft.
Def. 1 ft. = 2.1′	
Int. Ang. = 48° 10′	From intersection to intersection.
8° curve, L. R. = 716.78 ft.	Tan preceding curve = 260.7 ft.
T. = 320.4 ft.	Tan between curves = 220.1 ft.
P. C. = 24 + 34.5	Tan 8° curve = 320.4 ft.
Length of curve = 602.1 ft.	Total, = 801.2 ft.
P. T. = 30 + 36.6	tan 48° 10′ = 1.11713
Def. 100 ft. = 4° 00′	400 ft. × 1.11713 = 446.9 ft.
Def. 1 ft. = 2.4′	
Int. Ang. = 44° 30′	From intersection to intersection.
9° curve, R. R. = 637.27 ft.	Tan preceding curve = 278.5 ft.
T. = 260.7 ft.	Tan between curves = 378.3 ft.
P. C. = 17 + 20	Tan 9° curve = 260.7 ft.
Length of curve = 494.4 ft.	Total, = 917.5 ft.
P. T. = 22 + 14.4	tan 44° 30′ = .98270
Def. 100 ft. = 4° 30′	400 ft. × .98270 = 393.1 ft.
Def. 1 ft. = 2.7′	

from the P. T. of the preceding curve at Station 7 + 89.6 to the P. C. of the second or 6° curve at Station 13 + 16, a distance of 526.4 feet; and the tangent of the 6° curve, which is 389.2 feet; making a total distance of 1,356.3 feet. On the line *CE* prolonged, scale off from *C* a total distance of 1,356.3 feet, thus locating the point of intersection *H* of the 6° curve, and on the prolongation of the same line lay off the additional distance *HK*, equal to 400 feet, in order to lay off the intersection angle of this curve. The tangent of 44° 20′ is .97700, which, multiplied by 400, gives 390.8 feet as the calculated tangent for laying off this angle. From *K* draw a line to the right perpendicular to *HK*, and on it scale off this calculated tangent, 390.8 feet, thus locating the point *L*. The line joining *H* and *L* determines the direction of the forward tangent of the second curve. Next, from the point of intersection *H*, scale off on both back and forward tangents the tangent distance of 389.2 feet, thus locating the P. C. of this curve at *M*, which is Station 13 + 16, and its P. T. at *N*, which is Station 20 + 54.9. Then, with *M* and *N* as centers, and a radius of 955.37 feet, the radius of a 6° curve, describe arcs intersecting at *O*, and with *O* as a center and the same radius, describe a curve between the points *M* and *N*. The curve *MN* thus drawn is a 6° curve and is tangent to the lines *FH* and *HL* at the points *M* and *N*.

In platting these lines of railroad location, the tangent distances, the radii of the curves, and the tangents for laying off the intersection angles, should be drawn in dotted lines, as they are merely construction lines. The line of survey should be drawn in a full, bold line, as shown in the plate. The points of intersection and the points of curve and tangent are to be marked by small circles, the latter points being also designated by their station numbers. Dotted radial lines should be drawn from the center of each curve to its P. C. and P. T. On one of these radial lines, the length of the radius of the curve should be written, and the amount of the central angle should be marked within the radial lines. No further directions are deemed necessary for

platting the remainder of this line or the notes for example 2, a plat of which is shown in Fig.* 2 of the same plate, except that the starting point, Station 0, of example 2 is to be located ¾ inch from the left-hand border line and 1½ inches from the upper border line.

24. Railroad Curves.—For fitting in and platting the curves of a railroad location, what are known as **railroad curves** are very convenient. These consist of thin curved strips of hard rubber, pearwood, metal, or cardboard, cut to different radii according to a uniform scale, which is usually 100 feet to the inch. The two edges of each curved strip are the arcs of two curves having different radii. The degree of curvature of each arc, and the scale to which it has the given degree of curvature, are stamped distinctly on each strip, as

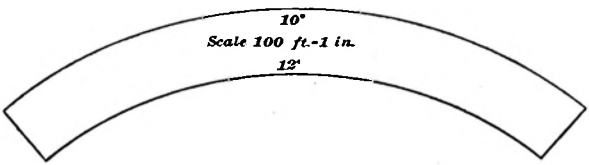


FIG. 11

shown in Fig. 11. A set of such railroad curves contains from 10 to 40 curves of different radii, according to the assortment. The curves can be adapted to any scale. For example, a 10° curve to a scale of 100 feet to the inch will serve for a 5° curve to a scale of 200 feet to the inch, or a 2° 30′ curve to a scale of 400 feet to the inch. In the same way, a 12° curve to a scale of 100 feet to the inch can be used for a 6° curve to a scale of 200 feet to the inch, or a 24° degree curve to a scale of 50 feet to the inch, etc. The principal object of these curves is to enable the engineer to readily select the curve that will best fit the ground lying between two tangents, as platted on the topographical map. The curves are applied directly to the topographical map on which the tangents of the line have been platted, and the curve that is best fitted to the ground and the tangents, and will give the best grades

and most economical construction, is selected. A satisfactory curve having been decided on, the tangent distances are then calculated and the curve is drawn in by means of a railroad curve or a pair of compasses, as may be most convenient.

25. Beam Compass.—When the radius of a curve is of considerable length, it is difficult, and sometimes impossible, to describe the arc of a true circle with ordinary compasses and lengthening bar. In such cases, an accurate and convenient substitute for the ordinary compasses and lengthening bar is found in the instrument known as a **beam compass.** Such an instrument is shown in Fig. 12. It consists of two

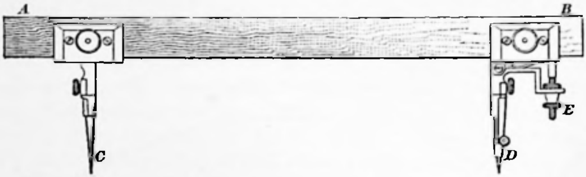


FIG. 12

upright legs *C* and *D* attached to metal pieces that clamp on the wooden beam *A B*. The leg *C* is attached rigidly to the metal clamp and carries a needle point, as shown. The leg *D* is attached to the metal clamp by means of a hinge joint that is adjustable by means of a lever and the milled-headed thumbscrew *E*. It carries either a pencil or a pen, by which the curve is described about the needle point as a center. The two legs *C* and *D* are clamped to the wooden beam *A B* by means of milled-headed screws, the heads of which are shown in the figure. By means of the milled-headed thumbscrew *E*, the pen or pencil can be accurately adjusted to the desired radius.

MAPPING

(PART 2)

TOPOGRAPHICAL DRAWING

REPRESENTATION OF TOPOGRAPHY

1. Topographical maps are graphical representations of the relative elevations, as well as of the dimensions and geographical positions, of the different natural and artificial features included in any given portion of the earth's surface. They show all inequalities of surface, such as hills, hollows, valleys, and plains, and the location of towns, highways, canals, railroads, streams, lakes, etc. Detailed topographical maps also show buildings and other structures, property lines, boundaries of fields, names of property owners, extent and varieties of timber, degree of curvature, character of soil and vegetation, etc. The contour map is valuable in the location of railroads, highways, canals, town sites, reservoirs, dams, parks, etc. In railroad, highway, and canal locations, it is used extensively for determining the best location with regard to alinement and grade.

2. Systems of Representing Topography.—The three most common systems of representing relative elevations on a topographical map are as follows: (1) by contour lines, (2) by hachures, (3) by shade from vertical light. A fourth system of representation is by means of what is called a **relief map**, which is constructed of papier mâché or other suitable material. On such a map is shown a series of

miniature hills and valleys, representing the exact form of the natural surface to a greatly reduced scale.

3. Contour Lines.—The method of representing topography by contour lines is the one generally adopted among engineers. As explained in *Topographic Surveying*, a **contour line** represents the intersection of a horizontal surface with the surface of the earth, and is therefore a line drawn through points of equal elevation. A map containing the outline of a given surface, together with the contour lines representing its form and inequalities, is called a **contour map** of the surface. Contour lines may conveniently be used to represent any form of surface.

Figs. 1 to 5, inclusive, illustrate the most common types of

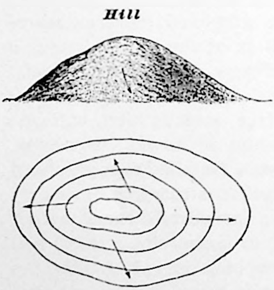


Fig. 1

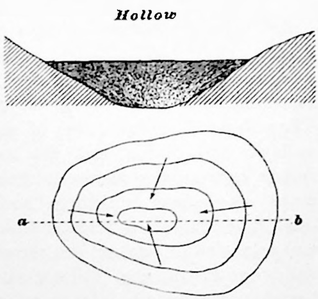


Fig. 2

the natural-surface formations of the ground. In the upper portion of each figure the natural formation is shown in elevation or section, and in the lower part the same formation is represented in plan by means of contour lines, as on a topographical map. In order that these may be more readily understood, arrows are marked on each figure to indicate the direction in which the water would flow. The vertical sections are in each case taken on the line *a b* in the contour plan. Fig. 1 represents a *hill;* Fig. 2, a *hollow;*

Fig. 3, the end of a *ridge*, commonly known as a *shoulder;* Fig. 4, the end of a *valley;* and Fig. 5, the end of two valleys

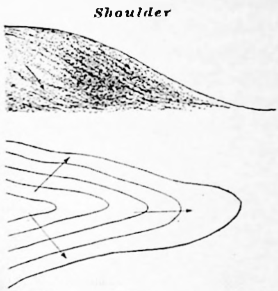


FIG. 3

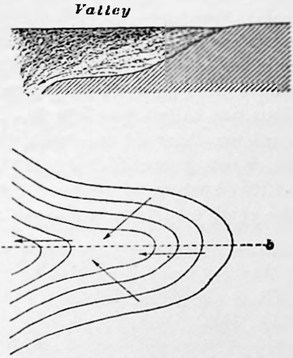


FIG. 4

meeting between two hills, forming what is commonly designated as a *saddle*. From the foregoing figures it is evident that where the contour lines run close together the slope of the natural surface is steeper, and where they are far apart the surface becomes more nearly level. Each contour line must be a distinct and continuous line, until it either runs off the map or closes on itself. In representing a cliff or a steep hillside, the contour lines run close together, and in case the cliff becomes absolutely vertical, two or more contour lines may run

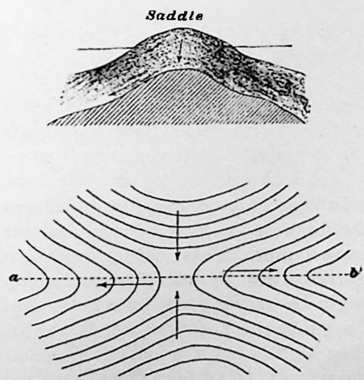


FIG. 5

together in a single line, or may even cross each other, as in the case of an overhanging cliff.

4. Hachures.—Slopes are sometimes represented by a system of short disconnected lines having the directions of lines of greatest slope; that is, the directions that water would take in running off the surface of the slope. Such lines are called **hachures**, or **hatchings**. Since contour lines are lines of constant elevation, hachures must always be drawn perpendicular to the contour lines. An example of this system of representing the form and inequalities of the earth's surface is shown in Fig. 6.

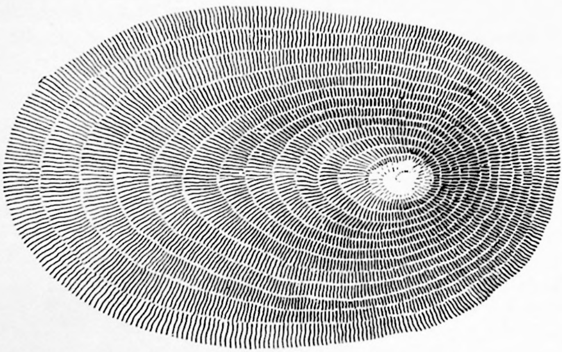


Fig. 6

In sketching topography by this system, the topographer should hold the book directly in front of him so that it will correspond with his position on the ground, and draw the lines toward him. If standing at the top of a slope, he should begin by drawing the lines from the bottom, and vice versa. To guide the hachures, he should first sketch in the contour lines lightly with a pencil, especially when the slopes are steep and irregular. The hachures must be drawn truly perpendicular to the contour lines, which are not drawn in ink, but are indicated by the spaces between the rows of

vertical hachures. Where the contour lines curve sharply, it is often well first to draw in the hachures at considerable intervals, as a guide to the direction of those drawn afterwards. Hachures in adjoining rows should not be continuous, but should be so drawn as to break joints. They must not overlap, and should be drawn in slightly wavy lines. The hachures should have their thickness and distance apart proportional to the steepness of the slope. The lines are made heavier as the slope is steeper, being fine for gentle slopes, while for very steep slopes the blank spaces are but half the breadth of the lines.

5. Shades From Vertical Light.—This system of representing the form and inequalities of the earth's surface depends on the principle that the steeper any slope is the less vertical light it receives, and consequently, the darker it appears. This difference in the degree of light is imitated and much exaggerated by the shading, when slopes are represented by this method. For work of this kind, the United States Coast and Geodetic Survey uses definite amounts of shade for slopes of different degrees. The shading is made dark for steep slopes and light for gentle slopes; it is deep black for slopes of 75° and steeper, and is graded to about midway between black and white for slopes of 30°, and so on, level surfaces being white or unshaded.

The shading is applied in various ways. A rapid method, and one sufficiently accurate for many kinds of work, is to sketch in the contours and then apply the shading in the form of India ink diluted with water. This shading, or India-ink tinting, is applied with a brush, and each varying tint is applied with its particular brush, care being taken not to allow any tint to dry before the succeeding tint is applied. By applying the tints in this way, they can be so blended as to give a smooth and finished effect to the work.

CONVENTIONAL SIGNS

6. Certain conventional signs are commonly used in topographical mapping to represent the natural and artificial features of the surface; the most common are shown in

Figs. 7 to 21. In making these conventional signs, great care should be exercised to draw them neatly and clearly, for if they are made in a slovenly manner they will greatly mar the appearance of the drawing. As a means of economy in making large maps, some offices have stamps for such conventional signs as those for grass, underbrush, woods, swamp, marsh, clearing, orchard, cultivated ground, etc., by means of which the signs can be quickly stamped on the portions of the map where required, thus saving considerable time and expense. It should be noted, however, that the practice regarding conventional signs is not entirely uniform; the same signs are not used in all offices to represent the same topographical features.

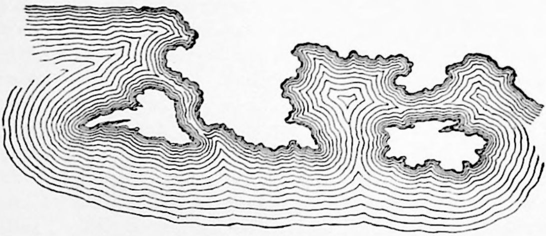


FIG. 7

7. Shore Line.—The shore line of a lake or other body of water is represented by a line that follows all the windings and indentations of the shore. Near the shore line, on the water side, is drawn a parallel line, and near this, but at a slightly greater distance from it, is drawn another parallel line, and so on, the distance between the lines gradually increasing and the lines becoming less irregular, as shown in Fig. 7.

8. Rocky Shore.—An abrupt and rocky shore is represented as shown in Fig. 8. The irregular dotted surfaces surrounded by shore lines represent sand bars, and the dotted outlines beyond the shore line represent shoals or submerged rocks.

9. Sand Shore and Sand Dunes.—A sand shore is represented by fine dots, as shown in Fig. 9. A gravel shore is represented in a similar manner, except that the dots are made larger or heavier. The black line represents the shore line proper, usually the high-

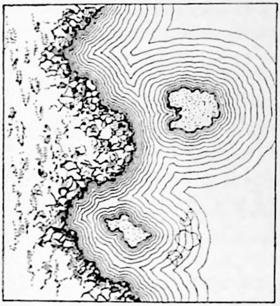


FIG. 8

FIG. 9

water line. The irregular dotted surfaces inland from the shore line represent *sand dunes.*

10. Rivers.—The shore lines of large rivers are usually represented in a manner somewhat similar to the shore lines

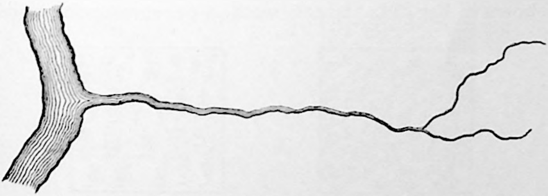


FIG. 10

of other bodies of water, as shown in Fig. 10. Large brooks or creeks are represented by two parallel lines, and small ones by a single line.

11. Grass and Cultivated Ground.—Grass is represented by irregular groups of short diverging lines, as shown in Fig. 11. Cultivated ground is represented by a

series of parallel continuous lines alternating with dotted lines, as shown in Fig. 12.

12. Orchards are represented by irregularly scalloped outlines of approximately circular form, arranged in regu-

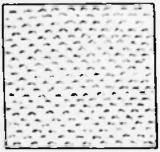


FIG. 11

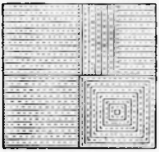


FIG. 12

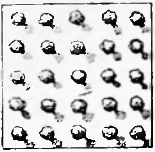


FIG. 13

lar rows to represent the trees, and shaded on the lower right-hand side, as shown in Fig. 13.

13. Woods.—The conventional signs used most commonly for representing woodlands consist of scalloped and shaded outlines similar to those used to represent orchards, except that they are arranged irregularly, being placed close together or far apart according as the forest is dense or open, and intermingled with stars in an irregular manner, as shown in Fig. 14. In this method of representing woods,

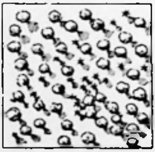


FIG. 14

FIG. 15

the scalloped outlines represent the deciduous trees, and the stars the evergreens. Woods are also sometimes represented by means of trees shown in elevation, as in Fig. 15. By reason of the trees being shown in elevation, this method is not so consistent for a map, and is not used so extensively, as the other, although it is very effective in

distinguishing the different varieties of trees, especially the deciduous from the evergreen trees.

14. Clearings are commonly represented by outlines resembling stumps placed irregularly and interspersed with the signs for grass and bushes, as shown in Fig. 16.

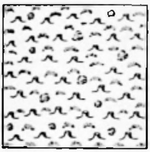


FIG. 16

FIG. 17

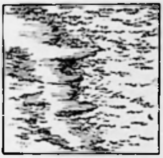


FIG. 18

15. Underbrush is usually represented by very irregular lines that are more or less scalloped and slightly shaded in places, the very irregular outlines somewhat resembling those of the underbrush, as shown in Fig. 17.

16. Swamp.—Swamp land is commonly represented by a combination of the conventional signs for grass, bushes, and water, giving the general effect shown in Fig. 18.

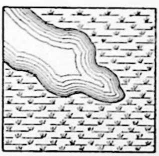


FIG. 19

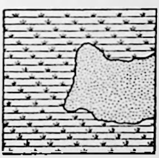


FIG. 20

17. Fresh-Water Pond and Marsh.—A fresh-water pond is represented in the same manner as an ordinary body of water. A fresh-water marsh is represented by irregular groups of short diverging lines and a series of parallel lines broken at irregular intervals, as shown in Fig. 19.

18. Salt-Water Pond and Marsh.—The surface of a salt-water pond is represented by small dots. A salt-water marsh

is represented by irregular groups of short diverging lines and a series of unbroken parallel lines, as shown in Fig. 20.

19. Rice Dikes and Ditches.—A rice dike is represented by two rows of short parallel lines; and a ditch, by two continuous parallel lines. The rice is represented by irregular groups of short diverging lines, having a number of light parallel shade lines drawn below each group, as shown in Fig. 21.

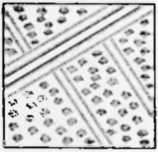


FIG. 21

20. Essentials to Mapping.—In the construction of maps, clearness, accuracy, legibility, and neatness are indispensable to good work. A neat explicit title, including the scale, date of survey, name of surveyor, direction of meridian, and key to any topographical symbols used, is very essential, and a neat border line adds much to the appearance of the map. Where time and cost are not to be considered, the lower sides of the contour lines may be shaded or hatched as though water were draining off them, and the valleys and low places tinted with a light shade of India ink. In sketching the contour lines on a map, gaps or spaces should be left in the lines at suitable intervals for writing the elevations of the contours.

PLATE, TITLE: CONTOUR MAP AND PROFILE

NOTE.—As stated in *Mapping*, Part 1, figures in the plates are referred to by placing a star after the abbreviation Fig. Thus, Fig.* 1 means Fig. 1 in the plate, while Fig. 1 means Fig. 1 in the text.

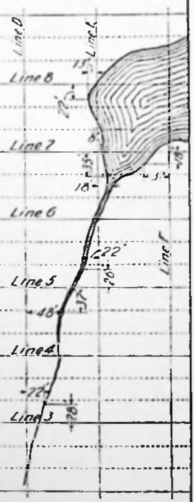
21. Preliminary Remarks and Field Notes.—Fig.* 1 is a contour map of a rectangular area, a portion of which, adjacent to the northeast corner, is occupied by a lake. Fig.* 2 represents profiles along the lines *A A'* and *F F'* of the contour map; the former line extends along the western edge of the field and the latter line extends over the summit of a hill and across an arm of the lake. The vertical scale of these profiles is five times the horizontal scale, thus exaggerating the slope of the surface. Before drawing this plate,

the article in *Topographic Surveying* describing the method of performing the field work and keeping the notes for a survey of this character should be read carefully, so that the work will be thoroughly understood. The area shown in Fig.* 1 is 1,100 feet long by 800 feet wide, and is divided into squares of 100 feet on a side, except, of course, the portion occupied by the lake. The surface of the lake is adopted as the surface of reference, or datum plane. The notes of the levels taken over this area are here given.

Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
Line 1							
	11.2	112		0.0			Surface of Water
0			2.1	9.1			
1			3.8	7.4			
2			4.9	6.3			
3			7.6	3.6			
4			7.6	3.6			
4+80			11.2	0.0			Shore of Lake
Line H							
6+16			11.2	0.0			" " "
6			97	1.5			
5			1.9	9.3			
Line 6							
H+15			11.2	0.0			Shore of Lake
Line 5							
H+72			11.2	0.0			" " "
T.P.	10.8	209	1.1	10.1			
Line H							
4			7.3	13.6			
3			4.9	16.0			
2			6.8	14.1			
1			5.0	15.9			
0			4.4	16.5			
T.P.	11.7	32.0	0.6	20.3			
Line G							
0			4.0	28.0			
1			0.8	31.2			
2			3.0	29.0			
3			0.3	31.7			
4			0.9	31.1			
5			8.2	23.8			
T.P.	1.2	21.8	11.4	20.6			
T.P.	2.7	15.1	9.4	12.4			
6			8.9	6.2			
7			15.1	0.0			Shore of Lake

Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
		15.1					
8+95			15.1	0.0			Shore of Lake
10			11.4	3.7			
11			3.5	11.6			
Line 11							
G+98			15.1	0.0			Shore of Lake
Line 10							
G+53			15.1	0.0			" " "
Line F							
11			0.5	14.6			
T.P.	3.8	17.9	1.0	14.1			
Line 11							
F+60			2.2	15.7			
Line F							
10+45			0.4	17.5			
10			3.2	14.7			
9			13.8	4.1			
8+60			17.9	0.0			Shore of Lake
7+16			17.9	0.0			" " "
7			16.8	1.1			
6			0.9	17.0			
5+80			11.4	6.5			
T.P.	10.2	27.9	0.2	17.7			
5			0.7	27.2			
T.P.	11.6	38.7	0.8	27.1			
T.P.	11.8	50.1	0.4	38.3			
T.P.	10.3	59.2	1.2	48.9			
4			8.4	50.8			
3			2.2	57.0			
2			6.2	53.0			
1			12.0	47.2			
T.P.	1.1	48.9	11.4	47.8			
0			9.8	39.1			
T.P.	2.0	41.3	9.6	39.3			

Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
Line E		41.3					
0			9.7	31.6			
1			12.3	29.0			
2			8.3	33.0			
3			8.1	33.2			
T.P.	0.8	30.9	11.2	30.1			
4			8.0	22.9			
T.P.	1.2	21.2	10.9	20.0			
5			5.1	16.1			
6			12.0	9.2			
7			20.3	0.9			Shore Line 8 ft East of Line E E
7+46			21.2	0.0			Shore Line
8+10			21.2	0.0			
T.P.	3.1	17.9	6.4	14.8			
9			11.5	6.4			
10			3.7	14.2			
10+07			2.9	15.0			
10+44			2.9	15.0			
11			6.7	11.2			
Line D							
11			0.5	17.4			
10			10.9	7.0			
T.P.	11.7	26.8	2.8	15.1			
9			10.6	16.2			
8			9.3	17.5			
7			12.7	14.1			
6			8.6	18.2			
Line 5							
0+64			13.6	13.2			Creek
Line D							
5			4.6	22.2			
Line 4							
0+57			8.4	18.2			Creek
Line D							
4			1.8	25.0			



Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
Line 3		26.8					
D+15			4.1	22.7			Creek
C+70			3.0	23.8			
Line D							
3			3.5	23.3			
2+40			2.6	24.2			Creek
Line 3							
D+50			0.3	26.5			
Line 2							
C+50			3.5	23.3			
Line D							
2			2.2	24.6			
1+72			1.8	25.0			
1			3.5	23.3			
0			2.7	24.1			
T.P.	11.9	37.8	0.9	25.9			
Line C							
0			5.7	32.1			
1			9.5	28.3			
2			10.3	27.5			
3			7.4	30.4			
4			0.7	37.1			
5			6.0	31.8			
6			11.1	26.7			
7			10.2	27.6			
8			10.7	27.1			
T.P.	0.4	26.3	11.9	25.9			
9			7.2	19.1			
10			13.2	13.1			
10+58			17.2	9.1			
11			12.0	14.3			
Line B							
11			12.5	13.8			
10			8.7	17.6			
9			1.3	25.0			

Station	Back Sight	Height of Instrument	Foresight	Elevation of Surface			Remarks
T.P.	11.4	36.9	0.8	25.5			
8			2.6	34.3			
T.P.	11.5	47.2	1.2	35.7			
7			6.7	40.5			
6			8.5	38.7			
5			4.6	42.6			
4			1.2	46.0			
3			2.6	44.6			
2			6.7	40.5			
1			8.8	38.4			
0			5.8	41.4			
Line A							
0			0.6	46.6			
T.P.	10.9	57.7	0.4	46.8			
1			9.5	48.2			
2			3.9	53.8			
3			0.0	57.7			
4			0.7	57.0			
5			4.5	53.2			
6			7.2	50.5			
7			10.6	47.1			
T.P.	0.7	47.2	11.2	46.5			
8			5.6	41.6			
T.P.	0.9	36.4	11.7	35.5			
9			2.6	33.8			
10			11.8	24.6			
T.P.	1.0	26.2	11.2	25.2			
11			8.2	18.0			
T.P.	0.5	15.3	11.4	14.8			
T.P.	1.6	9.3	7.6	7.7			
			9.3	0.0			Check-Surface of Water

LINE *F F′*
Stations 0 to 1
0+11 = contour 40
0+73 = contour 45
Stations 1 to 2
1+48 = contour 50
Stations 2 to 3
2+50 = contour 55
Stations 3 to 4
3+32 = contour 55
Stations 4 to 5
4+03 = contour 50
4+24 = contour 45
4+45 = contour 40
4+66 = contour 35
4+87 = contour 30
Stations 5 to 6
5+09 = contour 25
5+28 = contour 20
5+47 = contour 15
5+66 = contour 10
5+87 = contour 10
5+96 = contour 15
Stations 6 to 7
6+13 = contour 15
6+44 = contour 10
6+75 = contour 5
Stations 7 to 8
7+16 = contour 0 shore line
Stations 8 to 9
8+60 = contour 0 shore line
Stations 9 to 10
9+08 = contour 5
9+55 = contour 10
Stations 10 to 11
10+05 = contour 15
10+92 = contour 15

22. Drawing the Contour Map.—The map shown in Fig.* 1 of this plate should be platted from the preceding level notes. The outlines, or boundary lines, of the area are first drawn in the form of a rectangle 11 by 8 inches in dimensions, representing an area 1,100 feet in length and 800 feet in width, to a scale of 100 feet to the inch. These boundary lines are then each divided into parts 1 inch in length, each part representing 100 feet to scale, as shown in the plate; the lines extending along the sides are numbered, and those extending along the ends are lettered, in consecutive order, as shown. The lines of division are then drawn in pencil, thus dividing the area into squares 100 feet on each side. The elevations of the intersections of the lines, as given in the notes, should be written in their respective positions with pencil, and the contour points then calculated and tabulated. The positions of the contour points on each line are calculated and then tabulated in any convenient form, or are located graphically, as may be desired. A simple form for

tabulating the contour distances is shown on the preceding page. The distances to contours given in this form are those calculated for the line FF' of Fig.* 1. The calculations are made as follows: From the notes, the elevation of Station F-0 is 39.1 feet, and that of Station F-1 is 47.2 feet, giving a rise equal to $47.2 - 39.1 = 8.1$ feet from the former station to the latter. Since the horizontal distance between the stations is 100 feet, the rate of slope is equal to $\frac{100}{8.1}$, or 12.3 feet horizontal for 1 foot rise. The contour interval is taken at 5 feet, and, consequently, the elevation of each contour is some multiple of 5 feet. Hence, the first contour above Station *F-0* is contour 40, and to locate this contour a rise of $40.0 - 39.1 = .9$ foot above this station must be made. Since the rate of slope is 12.3 feet horizontal for 1 foot rise, the horizontal distance from Station 0 on this line to contour 40 is equal to $12.3 \times .9 = 11.1$ feet. The rise from contour 40 to contour 45 is 5 feet. As the rate of slope continues the same, contour 45 will intersect line F at a distance of $12.3 \times 5 = 61.5$ feet from contour 40, or $61.5 + 11.1 = 72.6$ feet from Station 0. In the notes, these distances are given to the nearest whole foot.

From an inspection of the elevations, it is evident that contour 50 must occur between Stations 1 and 2, since the elevation of the former is 47.2 feet and that of the latter is 53 feet. The rise from Station 1 to Station 2 is equal to $53.0 - 47.2 = 5.8$ feet. Since the horizontal distance giving this rise is 100 feet, the rate of slope is equal to $\frac{100}{5.8} = 17.2$ feet horizontal for 1 foot rise. To locate contour 50, a rise of $50.0 - 47.2 = 2.8$ feet must be made, and since the rate of slope is 17.2 feet horizontal for 1 foot rise, contour 50 will intersect line F at a distance of $17.2 \times 2.8 = 48.2$ feet from Station 1.

Contour 55 will evidently occur between Stations 2 and 3 on this line, and is located in the same manner as just described for contours 40, 45, and 50. This operation is continued until all the contour points on the line FF' have been located. The contour points on all the lines parallel to FF' are located in a similar manner. Since the contour

lines intersect the lines *1–1*, *2–2*, etc., which are perpendicular to the line FF', as well as the lines that are parallel to it, their points of intersection must be located on these lines also. The positions of the contour points on these lines are calculated in substantially the same manner as those on the line FF', just described.

As the positions of the contour points are determined along the different lines, they are marked lightly in pencil with the elevation of the contour. The points of equal elevation, that is, all points having the elevation of any contour, are then joined by a continuous line, drawn lightly in pencil freehand, and curved in such manner as to represent the form of the surface as accurately as possible. These form the contour lines, and should afterwards be inked, either with a ruling pen* or with an ordinary writing pen, leaving gaps or spaces at suitable intervals in which to write the contour elevations. Usually, the lines dividing the area into squares and the elevations of the stations are only marked lightly in pencil and are erased after the contour lines have been inked and their respective elevations have been written in the gaps left at suitable intervals for that purpose.

The plate shows the division lines dotted, and also shows the elevations of the stations on the lines E, F, and G. In drawing this plate, however, the student will not show any of the division lines, except the line FF', and will not show the elevations of any stations whatever. These lines and elevations are shown merely as a guide in laying out the map, and should not appear on the finished drawing, except in the case of the line FF', which is shown because the profile of the surface along this line is constructed on the same drawing. The finished drawing will show the elevations of all contours, but will not show the elevations of any stations.

23. Platting Profiles From the Contour Map.—It occasionally becomes necessary to construct the profile of a

*A special ruling pen is made for this kind of work; it is commonly known as a **curve pen** or **contour pen.**

vertical section along some line of a given tract from a contour map of the tract that has previously been made. The profile is constructed in the following manner:

The contour map is placed on a drawing board, and the line along which the profile is required is then platted on the map. This line is, for convenience, here called the *line of profile*. If the profile is to be constructed on a separate sheet of paper, this is placed on the map with its edge parallel to the line of profile, and is fastened to the drawing board by thumbtacks or held in place by heavy paper weights. On this sheet a series of lines is drawn parallel to the line of profile as platted on the contour map, and are spaced at equal distances apart. The common distance between the parallel lines thus drawn will correspond to the contour interval, and, beginning at the lower one, the lines are numbered consecutively in multiples of the contour interval corresponding to the elevations of the required contours; the lowest parallel line usually has the elevation of the lowest contour intersecting the line of profile on the contour map. At each point where a contour line intersects the line of profile on the contour map, a projection line is drawn perpendicular to the line of profile. The intersection of this projection line with the parallel line corresponding in elevation to the contour from which it is drawn, is a point on the profile of the surface along the desired line.

In constructing *Profile F*, Fig.* 2, the lines *0*, *5*, *10*, etc. are drawn parallel to the line FF' on the contour map, and, to a scale of 20 feet to the inch, are spaced 5 feet apart, a distance equal to the contour interval. The lowest point on the line FF' is at the shore of the lake, the surface of which has been assumed as datum. The lowest line of the profile is therefore marked zero, and the lines above it are marked successively upwards in multiples of 5, until the point of highest elevation on the line FF' is reached. The points of intersection of the different contours with the line FF' are then projected on the lines of corresponding elevation on the profile sheet; the points so determined are points in the profile of the surface along the line FF'.

Thus, on the contour map, contour 40 intersects the line FF' at the point a, and from this point the line $a\,a'$ is drawn at right angles to $F\,F'$, intersecting the line on the profile sheet representing an elevation of 40 at the point a'. The point a' thus located is a point in the surface line of the profile of the line FF', having an elevation of 40 feet. In a similar manner the lines $b\,b'$, $c\,c'$, $d\,d'$, $e\,e'$, $f\,f'$, etc. are drawn from the points b, c, d, e, f, etc., thus locating the points b', c', d', e', f', etc., whose elevations are 45, 50, 55, 55, 50, etc. feet, respectively, in the surface line of the profile of the line FF'. Beyond the point f, the slope becomes steeper and the projection lines are necessarily drawn nearer together; on this portion of the map and profile, the letters designating the projection lines are omitted for clearness. The points thus located on the surface line of the profile are joined by a line drawn freehand, as described in *Leveling*. This line will represent the surface line of the profile, or the surface of the ground along the line FF', referred to the lake as a datum. In constructing the profile, the projection lines $a\,a'$, $b\,b'$, $c\,c'$, etc., should be drawn lightly in pencil, and after the surface line of the profile is drawn and inked, they should be erased. They are shown on the plate merely for the purpose of explanation.

The profile along the line $A\,A'$ is constructed in a similar manner, but is shown on the plate without the construction lines. The student will draw both profiles, finishing them in the same general manner as this profile. In each case the finished profile should show the horizontal lines representing the elevations, with the elevations written on them, the surface line, and the horizontal and vertical scales of the profile, but should not show any of the construction lines that are shown dotted in the plate.

The map and profile should be so arranged on the sheet as to leave a margin 1 inch wide inside of the border line, which should enclose a surface 13 by 17 inches.

PLATE, TITLE: STADIA SURVEY AND HACHURES

24. Preliminary Remarks and Field Notes.—Fig.* 1 of this plate shows the plat of a small portion of a village, made from the notes that are described in *Topographic Surveying*. These notes are reprinted for convenient reference on pages 24 and 27.

Before beginning to plat the survey, the student should make all the calculations for reducing the notes, and verify the results given in the text. He should calculate also the ranges of all the courses in the traverse, as well as the latitudes and longitudes of all the courses, computed from Station 1.

The stadia traverse line is then platted; this can be done easily by means of latitudes and longitudes. The side shots from each stadia station are next platted by means of the protractor and scale. A circular protractor, graduated from 0° to 360°, is more accurate and expeditious than the ordinary semicircular protractor for platting azimuth readings, and the protractor sheet is much more expeditious than either. The points where positions are located by the side shots are marked on the plat by a dot and circle, and the elevation of each point is written close to it in pencil. The highways, streams, boundary lines, buildings, etc. are then platted from the points located and the measurements given in the sketches. After all the artificial features are platted, the contour lines are sketched in from the elevations of the different points located on the map, assuming the slope to be uniform between any two known elevations.

25. Platting the Stadia Notes.—For the map shown in Fig.* 1, Station 1 is the starting point of the survey. Since the latitudes and longitudes of all other stations have positive signs, these stations must lie to the north and east of Station 1. A point is chosen on the sheet for the location of Station 1, in the same relative position as in the plate, and Stations 2, 3, 4, 5, and 6 are located by their latitudes and longitudes with respect to axes through

Station 1. The successive points thus located are then connected by dotted lines, representing a closed traverse.

If a movable protractor is used, a meridian line is drawn through each station of the traverse, by which to orient the protractor. The protractor is then oriented at Station 1, by placing it in such a position that the 0° and 180° marks coincide with the meridian line, and the center of the protractor is directly over Station 1. The directions of the side shots from Station 1 are then laid off around the circumference of the protractor by making dots on the paper at points corresponding to the azimuths of the different pointings from this station. A pencil line is then drawn from Station 1 through each dot, and the stadia measurement for that pointing is scaled off from Station 1 on the line so drawn, thus locating the points marked *a*, *b*, *c*, *d*, etc. in the notes. The letter designating each point, and the corresponding elevations are marked lightly in pencil beside the point. The protractor is oriented at Station 2, the side shots are platted in the same manner as just described for Station 1, and each point platted is marked with the corresponding letter and elevation. The side shots from Stations 3, 4, 5, and 6 are platted in a similar manner, thus completing the platting of the side shots. The roads, streams, property lines, buildings, etc. are then platted from the measurements given in the sketches.

The next step is to plat the contour lines. In order to determine the positions of the contours, the slope of the surface is assumed to be uniform between the located points whose elevations are determined, and the positions of points on the contours between any two located points are determined by making the distances to the located points proportional to the differences of elevation. The calculations are the same as those for locating the contours on the contour map when the levels are taken at regular station intervals, as described in Art. 22, except that the distances between the points whose elevations are taken are varying instead of being uniformly 100 feet. The contour points having been located, the contour lines are sketched in pencil through the points of equal elevation. The platting of the survey is

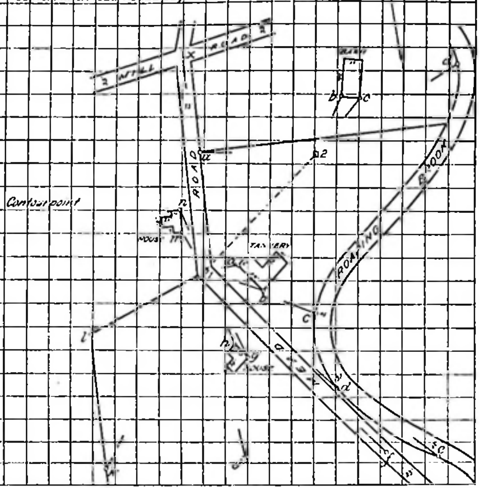
Stadia Survey of Meadville Suburb
May 27th 1901

Observer - J.R. Gavin
Recorder - Gonaver

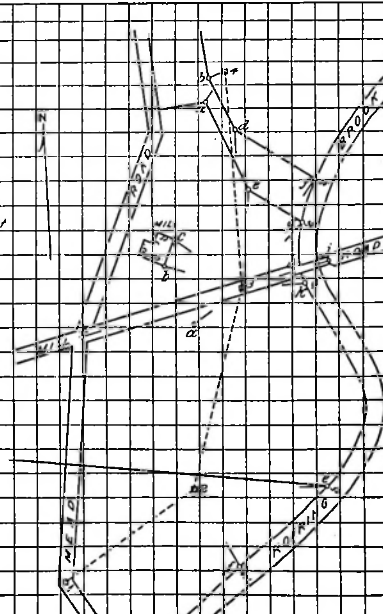
Rodmen – Griffith, Steele

Sta	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation
	Readings	from □1,	Elev. 177.42		
x	7° 30′	6.30	− 0° 23′	631	173.2
a	90° 43′	.91	− 2° 04′	92	174.1
b	120° 18′	1.66	− 2° 12′	167	171.0
c	126° 31′	3.15	− 2° 06′	316	165.8
d	143° 49′	4.60	− 1° 25′	461	166.0
e	141° 32′	7.85	− 0° 38′	786	168.7
f	148° 04′	6.74	− 0° 34′	675	170.7
g	167° 19′	2.47	− 0° 50′	248	173.8
h	172° 22′	1.97	− 1° 20′	198	172.8
j	181° 17′	4.99	+ 0° 12′	500	179.2
k	221° 45′	5.79	+ 1° 02′	580	187.8
l	256° 01′	3.47	+ 1° 50′	348	188.5
m	342° 03′	1.17	+ 1° 16′	118	180.0
n	350° 16′	1.71	+ 0° 52′	172	180.0
□2	60° 00′	4.20	− 1° 33′	M - 422	165.77
	Readings	from □2,	Elev. 165.77		
□1		4.22	+ 1° 37′		
a	286° 01′	3.21	+ 2° 01′	322	177.1
b	32° 02′	2.36	− 0° 48′	237	162.5
c	41° 49′	2.60	− 1° 03′	261	161.0
d	68° 32′	4.61	− 1° 22′	462	154.8

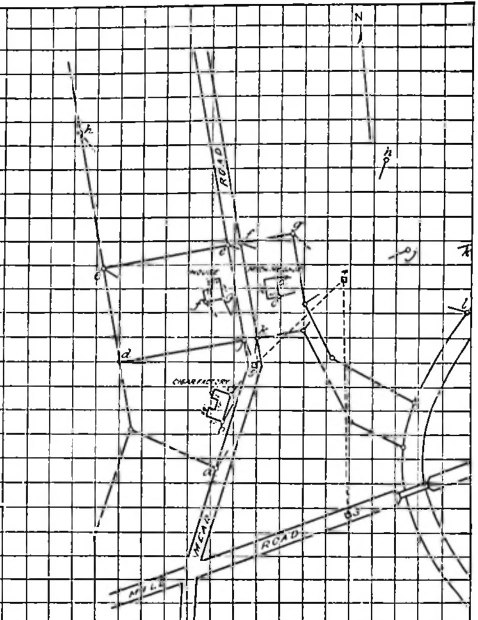
□1 Angle in Center Mead Road, South of Mill Road.
- Center Mill & Mead Roads as per Wilson's Survey



Sta	Azimuth	Stadia	Vertical Angle	Hor Distance	Elevation	
	Readings	from ⊡2	(cont.) Elev. = 165.77.			
e	90° 45′	3.59	− 1° 18′	360	157.6	
f	154° 53′	2.28	− 0° 54′	229	162.2	
⊡3	17° 30′	5.47	+ 0° 17′	M = 545	168.61	
	Readings	from ⊡3	Elev. = 168.61			
⊡2		5.42	− 0° 19′			
a	245° 30′	1.73	− 1° 53′	174	162.9	Contour Point
x	256° 45′	4.58	+ 0° 40′	459		Tie Line
b	287° 43′	2.19	− 1° 16′	220	163.7	
c	307° 24′	2.21	− 1° 41′	222	162.1	
d	359° 04′	4.06	− 1° 26′	407	158.4	
e	7° 33′	2.49	− 2° 50′	249	156.3	
f	38° 21′	3.39	− 3° 07′	339	150.1	
g	47° 20′	2.31	− 4° 18′	231	151.2	
j	76° 45′	2.39	− 0° 20′	240	167.2	
h	76° 46′	1.39	− 0° 34′	140	167.2	
k	82° 52′	1.66	− 5° 41′	165	152.9	
⊡4	357° 35′	5.61	− 0° 46′	M = 559	161.29	
	Readings	from ⊡4,	Elev. = 161.29			
⊡3		5.55	+ 0° 44′			
a	215° 41′	0.96		97	161.3	
b	255° 10′	0.53	+ 1° 13′	54	162.5	



Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation
	Readings	from □4.	(cont) Elev. =	161.29	
c	275° 17′	1.09	+ 1° 19′	110	163.8
d	301° 11′	1.34	+ 0° 54′	135	163.4
e	303° 46′	2.84	+ 0° 47′	285	165.2
f	308° 36′	2.57	+ 0° 43′	258	164.5
g	339° 25′	1.79	+ 0° 08′	180	161.7
h	32° 19′	4.29	− 1° 24′	430	150.8
j	70° 03′	2.67	− 1° 38′	268	153.7
k	77° 32′	6.61	− 1° 21′	662	145.7
l	102° 12′	4.03	− 1° 54′	404	147.9
□5	230° 54′	2.53	+ 0° 44′	M = 252	164.51
	Readings	from □5,	Elev. = 164.51		
□4		2.49	− 0° 45′		
a	208° 02′	2.73	+ 0° 28′	274	166.7
b	218° 53′	1.63	+ 0° 38′	164	166.3
c	228° 02′	0.96	+ 0° 53′	97	166.0
d	277° 15′	3.68	+ 1° 59′	369	177.3
e	307° 47′	4.87	+ 1° 04′	488	173.6
f	326° 20′	2.18	+ 1° 02′	219	168.5
g	338° 43′	1.99	+ 0° 48′	200	167.3
h	327° 47′	2.71	− 0° 15′	272	161.1
j	343° 48′	0.71	+ 0° 19′	72	164.9
k	13° 31′	0.72	− 0° 34′	73	163.8



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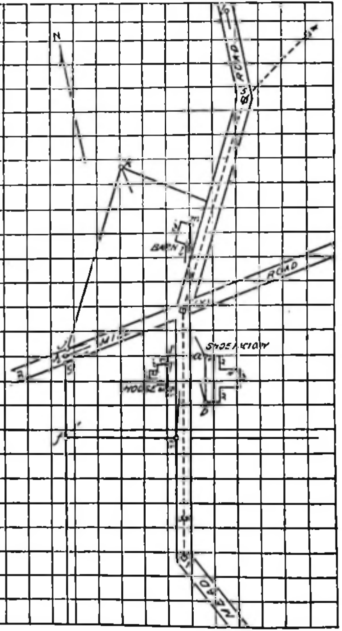
Sta.	Azimuth	Stadia	Vertical Angle	Horz Distance	Elevation	
	Readings from ⊡5, (cont) Elev. = 164.51					
l	355° 45'	6.02	+ 0° 27'	603	169.2	
x=⊡6					173.35	Should be 173.20
						as per Wilson's Survey
	Readings from ⊡6, Elev. = 173.20					= Original B.M.
⊡5		5.55	− 0° 59'			
a	159° 12'	1.27	− 0° 14'	128	172.7	
b	172° 20'	2.34	+ 0° 22'	235	174.7	
c	190° 14'	3.22	+ 0° 53'	323	178.2	
d	201° 46'	2.02	+ 1° 12'	203	177.5	
e	207° 24'	1.51	+ 1° 13'	152	176.4	
f	232° 18'	4.52	+ 1° 25'	453	184.4	
g	254° 03'	3.44	+ 1° 18'	345	181.0	
h	256° 45'	3.39	+ 0° 50'	340	178.1	
j	260° 02'	3.29	+ 1° 14'	330	180.3	
k	342° 47'	3.96	+ 0° 04'	397	173.7	
l	10° 14'	1.65	− 1° 07'	166	170.0	
m	13° 11'	2.18	− 1° 07'	219	168.9	
⊡1	187° 30'	6.27	+ 0° 25'			
	Survey finished May 28th 1901					
	Stadia Readings reduced by Gonaver – June 1st 1901					
	Notes platted by Griffith – June 1st 1901					



now completed in pencil. The lines defining the natural and artificial features of the surface surveyed, including the contour lines, are inked in, and the elevations of the contour lines are written in gaps left in the lines at suitable intervals for that purpose; all pencil lines are then erased. The plate is completed by lettering it in a neat and plain manner.

From this map, as shown on the plate, the title and the names of property owners are omitted for the sake of clearness, and in order that the more important matters relating to the platting of the survey can be shown plainly and be readily understood. After the map is completed, the student may show a title and the names of property owners, if he so desires, using fictitious names and being careful to place them in such positions on the map as not to interfere with any of the features platted from the notes of the survey.

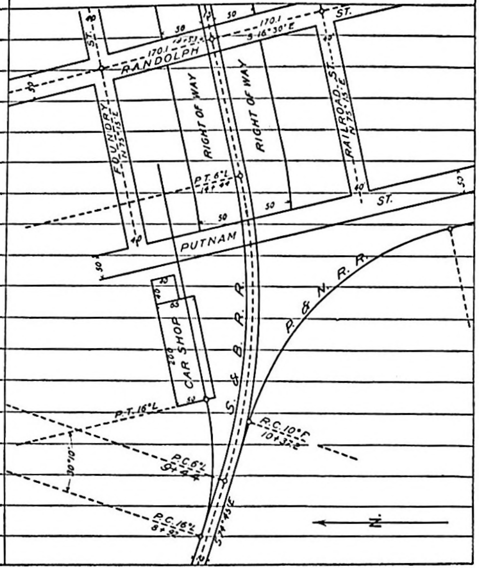
26. Representing Slopes by Hachures.—An example of this system is given in Fig.* 2. The figure represents an abrupt promontory whose base marks the channel of a river. The ground on the opposite side of the river is generally level with occasional undulations. The degree of the slope is indicated by the spacing of the contours and the corresponding lengths and numbers of hatchings. The more abrupt the slope, the closer together are the contours and hachures. The preliminary work necessary for such a topographical map is as follows: A traverse or meander line is run, defining the windings of the stream. The topographer sketches this meander line and stream in his notebook; he then sketches the main features of the surface from the promontory itself. A hand level is of great service in determining relative elevations. From these notes the final map is made up, the work being done in the office. Fine topographical drafting should not be attempted in camp. The facilities of a well-equipped office are necessary for rapid and satisfactory work. The student is not expected to reproduce the exact outline of the figure, but it is expected that his work will show a proper understanding of the subject. Having drawn the outline of the river, he

should draw the contours in light pencil lines, spacing them to conform to the different slopes. It will be evident that within the space represented by Fig.* 2, the surface of the river at *C* and *D* will be practically the same. Hence, if the distance from the summit *A* to the river at *E* is but half the distance from *A* to *F*, the slope *A E* must be twice as abrupt as the slope *A F*, and the contours that mark equal heights will be twice as far apart on the slope *A F* as on the slope *A E*. The student should draw all the contours, outlining the summits at *A* and *B*, before commencing the hachures. The short hachures on each side of the river mark its banks. On the promontory side they are shorter than on the opposite side, as the former has the more abrupt banks.

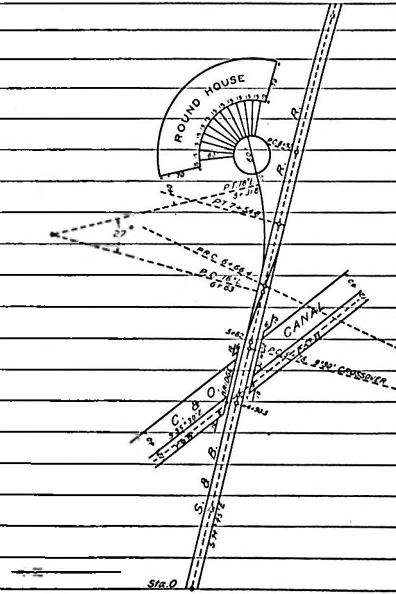
Fig.* 3 represents an irregular and abrupt sea coast. The survey for such an area would embrace a traverse of the entire shore line, including the shore line of the island, as well as that of the mainland. This traverse line should be used as a base line for auxiliary traverse or triangulation lines, by means of which the summits *A*, *B*, *C*, *D*, and *E*, and any other important objects could be located. The heights of these summits could be determined either by triangulation or by the aneroid barometer. With this information as a basis, the shore line is located, the contours are sketched with pencil, and the hachures drawn. As in the case of Fig.* 2, the student is not expected to produce an exact copy, but to show his proficiency by furnishing a clear and finished drawing.

Figs.* 2 and 3 should each be $5\frac{3}{4}$ by $7\frac{3}{4}$ inches in dimensions, and be arranged side by side, as shown on the plate, thus occupying a rectangular space $11\frac{1}{2}$ by $7\frac{3}{4}$ inches on the drawing. The border lines should enclose a rectangular space 13 by 17 inches, as usual.

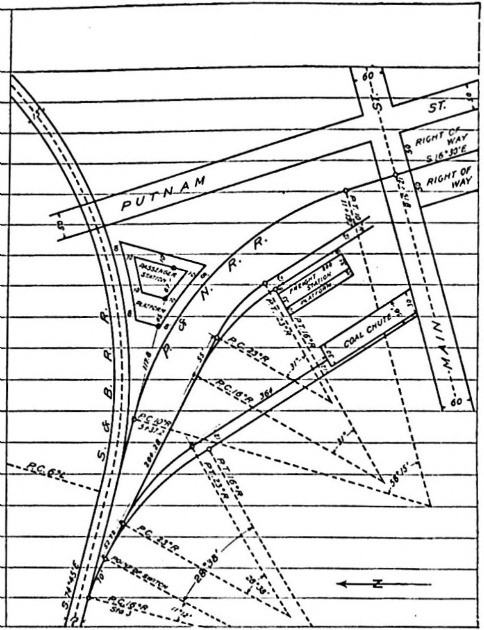
Sta.	Degree of Curve	Intersection Angle	Tangent	Radius	Magnetic Bearing
19+93.4	Center of Randolph Street				
14+44	P.T. 6°L				N 75°15' E
10+37.2	P.C. 10°R	58°15'	319.64	573.7	
9+44	P.C. 6°L	30°	255.9	955.3	
8+92	16° turnout to Left				



8+92	P.C. 16° turnout to Left			
7+54.8	P.T. 9°30' crossover			
6+66.4	P.R.C. crossover			
6+63	P.C. 16° turnout to Left			
5+82	East End of Bridge (30 ft. wide)			
5+78	P.C. 9°30' crossover			
5	Center of Towpath, C. & O. Canal			
4+90.5	West End of Bridge (30 ft. wide)			
0	Center Line S. & B. R.R.			S74°45'E



Sta.	Degree of Curve	Intersection Angle	Tangent	Radius	Magnetic Bearing
12+51.8	Center of Main Street				
11+19.7	P.T. 10°R				S 16° 30' E
5+37.2	P.C. 10°R	58° 15'	319.64	573.7	
3	P.C. 16° turnout to Right				



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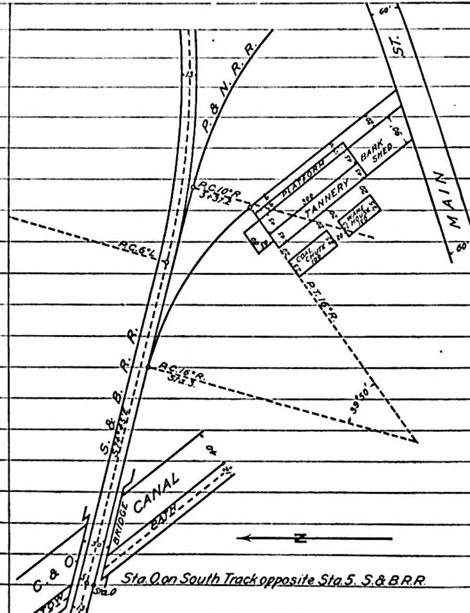
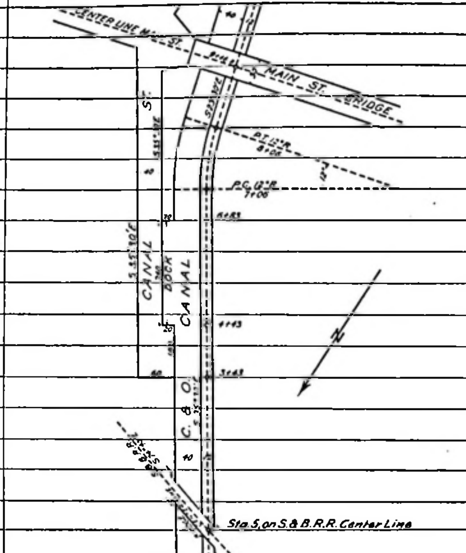
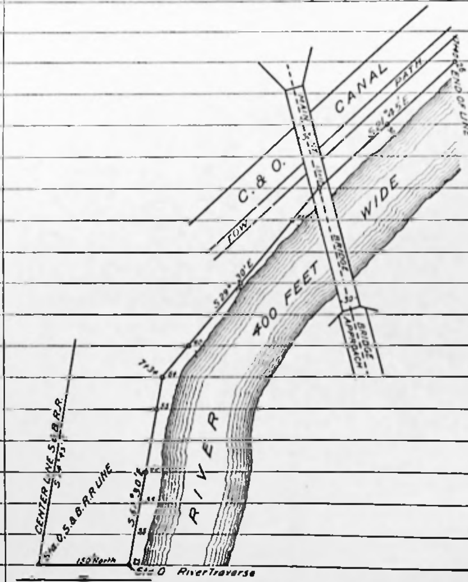
5+37.2	P.C. 10° R	58°15'	319.64	573.7		
3	P.C. 16° turnout to Right					
0					S 74°45'E	Sta. 0 on South Track opposite Sta. 5. S. & B. R.R.



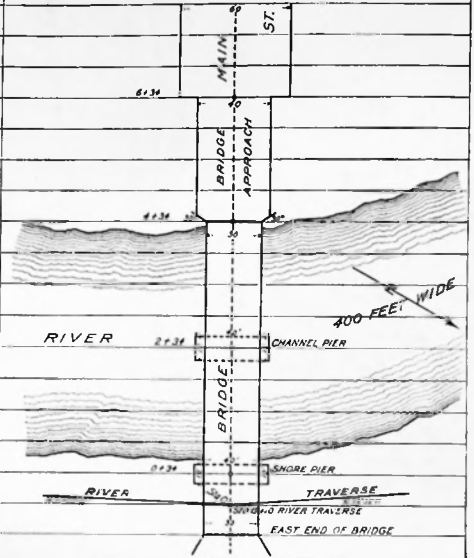
Sta.	Magnetic Bearing	Remarks
9+46.3		Center of Main Street
8+06	S 23° 30′ E	P.T. 12° Curve to Right
7+06		P.C. 12° Curve to Right for 12°
6+83		South End of Dock
4+43		North End of Dock
3+43		End of Canal Street
0	S 35° 30′ E	Sta. 5, on the S.& B. R.R. Center Line



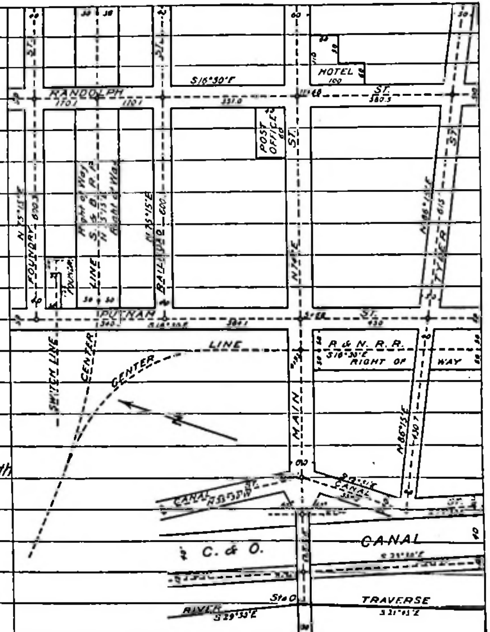
17+40		26' to Right, River Bank, End of Line
15		30' to Right, River Bank
13+10	S 21° 45' E	Center of Main Street
12		25 to Right, River Bank
10		10' to Right, River Bank
8		40' to Right, River Bank
7+34	S 29° 30' E	64' to Right, River Bank
7		55' to Right, River Bank
5		60' to Right, River Bank
3+50		55' to Right, River Bank
1+80		35' to Right, River Bank
0	S 67° 30' E	25' to Right, River Bank



Sta.	Magnetic Bearing	Remarks
6+34		West End of Approach
4+34		West End of Bridge
2+34		Center of Channel Pier
0+34		Center of Shore Pier
0	S 74° W	Center of Main St. and Sta. 13+10 River Traverse



11+68		Center of Randolph Street
5+68		Center of Putnam Street
4+93		Center Line P.& N. R.R.
1+70.7		Center of Canal St. leading North & South
0+90		East End of River & Canal Bridge
0+36.2		Center of Towpath C.& O. Canal
0	N 74° E	Center of Main St. & Sta. 13+10 River Traverse



PLATE, TITLE: MAP OF A PORTION OF A TOWN

27. Preliminary Remarks and Field Notes.—This map represents a portion of a town, together with its facilities for transportation by railroad and canal. The map is to be $11\frac{1}{2}$ by $15\frac{1}{2}$ inches in dimensions, leaving a margin of $\frac{3}{4}$ inch inside the border line, and should be platted entirely from the field notes, to a scale of 150 feet to the inch. The notes given on pages 30 to 37 show all the data necessary for platting this map. The plate should therefore not be referred to except for the purpose of obtaining an intelligent idea of the general form of the map or in case of doubt regarding the interpretation of some part of the notes. The left-hand pages of the notes give all details of the alinement of the different survey lines run to locate the required points, and on the corresponding right-hand pages are shown sketches of the lines as run, with the measurements locating all accessible features adjacent to the lines. The several survey lines are used as base lines for the location of the streets, railroads, canal, river bank, and such other features as are included in the map. The starting point of each survey line is numbered zero, and the measurements to all points on the line are referred to the beginning or zero point of that line and are recorded by the station number and plus. The entire map should first be platted carefully in pencil, and then, if it is found that all parts of the map fit together harmoniously, so as to indicate that the notes have been platted correctly, the drawing should be inked. The foregoing field notes of the various survey lines necessary for drawing the map are supposed to have been taken while those lines were being run.

28. Drawing the Map.—In this map, the meridian is assumed to be parallel to the left-hand border line, with the north point toward the top of the map. The following is a general description of the manner in which the map should be platted and the order to be followed in taking the data from the notes:

First, plat the center line of the S. & B. R. R., assuming Station 0 of the notes to be on the west border line of the plate, 250 feet south of the northwest corner. Beginning at this point, plat the center line of the S. & B. R. R., locating the P. C. at Station 9 + 44 and the P. T. at Station 14 + 44, in the manner described in *Mapping*, Part 1, and continue the forward tangent to the east boundary of the map. The sketch shows the center lines of the north and south tracks of this railroad to be 13 feet apart; plat these center lines at a distance of 6.5 feet on each side of the main center line and parallel to it. Assume a point on the center line of the south track on the S. & B. R. R. opposite Station 5 on the main center line as the starting point of the survey line of the P. & N. R. R. From this point, plat the center line of the P. & N. R. R., locating the P. C. at Station 5 + 37.2 and the P. T. at Station 11 + 19.7, in the manner that has been described, and continue the forward tangent to the south boundary of the map.

Having platted the center lines of both railroads, the points located along these lines for determining the positions of objects, for the initial points of other survey lines, and from which to make measurements for filling in details, are platted from the notes. At Station 4 + 90.5 on the main center line of the S. & B. R. R., plat the west abutment of the railroad bridge over the canal 30 feet wide, and the east abutment at Station 5 + 82, as given in the notes. At Station 6 + 63 on the same line, plat the turnout to the turntable and roundhouse, which is a 16° curve to the left for 27°, locating the roundhouse and turntable from the measurements given on the sketches in the notes. At Station 8 + 92 of the same main center line, plat the turnout to the car shop, which is a 16° curve to the left for 30° 10′, locating the car shop from the measurements given in the sketch. At Station 3 on the center line of the P. & N. R. R. plat the turnout leading to the tannery, which is a 16° curve to the right for 39° 50′, and from the end of the curve continue the tangent to the line of Main Street, locating the tannery, bark shed, and coal chute from the measurements given in the

sketches. From a point on this turnout curve at the distance of 70 feet from the P. C., draw a line tangent to the curve, thus giving the direction of the straight portion of the track leading to the freight station, which makes an angle of 11° 12′ with the preceding tangent of the S. & B. R. R. From a point on this tangent at the distance of 52.72 feet from its initial point, plat as the turnout leading to the coal chute, a 23° curve to the right for 28° 38′, terminating in a tangent 364 feet long; this tangent should be parallel to the tangent of the tannery track and spaced 12 feet from it. The coal chute at the end of this tangent is platted from the measurements given in the sketches. At the distances of 257 feet and 312 feet, respectively, from the initial point of the track leading to the freight station, locate the P. C's. of two curves, the first a 16° curve to the right for 31°, and the second a 23° curve to the right for 31°, both curves terminating in parallel tangents spaced 12 feet apart. The freight station is next platted from the measurement given in the sketch.

From the notes, it is known that Station 5 on the main center line of the S. & B. R. R. is the starting point of the traverse line along the towpath of the C. & O. canal. Beginning at this point, plat the center line of the towpath, locating the P. C. of a 12° curve to the right for 12° at Station 7 + 6 and the P. T. at Station 8 + 6. Continue the forward and backward tangents to the south and north boundaries of the map, respectively. Plat the canal, canal dock, and towpath from the measurements given in the sketch. The west border line, being parallel to the meridian as established, bears directly north and south. On this line at the distance of 150 feet south of Station 0 of the S. & B. R. R. is the starting point of the traverse line along the river bank. Beginning at this point, plat this traverse line and locate both banks of the river from the offset measurements given in the notes.

Station 13 + 10 of this traverse line is in the center of the Main Street and is Station 0 of the Main Street traverse. Beginning at this point, plat the center line of Main Street and continue it to the east and west boundaries of the map.

On this line, to the east of Station 0, locate the center of the canal towpath at Station 0 + 36.2 and the east end of the river and canal bridge at Station 0 + 90, the abutment wing walls of which diverge at angles of 30° with the center line of the street. Also, locate the center of Canal Street at Station 1 + 70.7, the center of the P. & N. R. R. at Station 4 + 93, the center of Putnam Street at Station 5 + 68, and the center of Randolph Street at Station 11 + 68. On this same line to the west of Station 0, locate the center of the shore pier of the bridge at Station 0 + 34, the center of the channel pier at 2 + 34, the west end of the bridge at 4 + 34, the abutment wing walls of which diverge at angles of 60° with the center line of the street or 30° with the face of the abutment, and the west end of the approach at Station 6 + 34. From the points thus fixed, locate the centers of Canal Street, Putnam Street, and Randolph Street, plat the center lines of these streets, and on them locate the center lines of Tyler Street, Railroad Street, and Foundry Street from the measurements and courses given in the sketches. The lines thus drawn will represent the center lines of the streets shown in the sketch.

The boundary lines of the streets, the bridge, and its approach. are next platted parallel to their respective center lines and of the widths indicated in the sketches, and the bridge piers are platted of the sizes and in the positions there shown. The post office and hotel are next platted at the northwest and southeast corners, respectively, of Main and Randolph Streets, and at the southeast corner of Foundry and Putnam Streets the foundry is platted, all dimensions being taken from the sketches. The right of way of the P. & N. R. R. is next platted; this extends 50 feet each side of the center line and from the south side of the Main Street to the south boundary of the map. The right of way of the S. & B. R. R. is then platted; this also extends 50 feet each side of the center line, and from the east side of Putnam Street to the east boundary of the map.

The passenger station is next located; its westerly end is at the distance of 162.8 feet from the P. C. of the 10° curve to

the right on the P. & N. R. R., measured on the tangent to the curve at that point. The east and west walls of the station are at right angles to the prolonged tangent of the curve, and the dimensions of the station and platforms in the direction of the tangent are as shown in the sketch. The positions of the north and south walls of the station are located by placing the corners of the station 10 feet from the corresponding edges of the platform, which edges are parallel to the adjacent tracks and 8 feet from the center line of each, as shown in the sketches. Returning now to the notes for the S. & B. R. R., the crossover from Station 5 + 78 is platted as a 9° 30′ curve to the right, reversing at the main center line to a 9° 30′ curve to the left, and terminating in the south track and tangent thereto.

The platting of the map should now be complete, and the lettering should next be done neatly in pencil, including the names of streets, railroads, buildings, etc., the bearings of lines, numbers of stations at important points, widths of streets, and such other matters as it may be essential to designate by lettering. The student should then compare the drawing, as constructed in pencil, with the plate, and if, after a careful comparison of all details, he is satisfied that his work is platted correctly and the drawing is complete, he should ink the drawing, making the lines smooth, sharp, and well defined, and giving them the same relative weights as the corresponding lines on the plate. The small circles that in the plate designate points of tangency, angle points and points located in the traverse lines, may be omitted in inking the drawing, if desired. Such circles are not usually shown on drawings of this character, but are shown on the plate in order to clearly designate the positions of the points located.

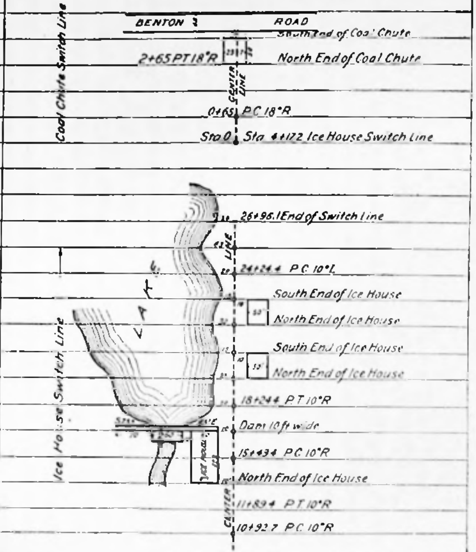
PLATE, TITLE: MAP OF A VILLAGE

29. Preliminary Remarks and Field Notes.—This plate represents a topographical map of a village. In making a survey of this description, some well-defined landmark is selected for a starting point. As there are usually a number of points to choose from, the choice will depend on the judgment of the surveyor. The intersection of the center lines of two highways or of a railroad and highway, or the head-block of a railroad switch, are excellent points from which to commence a survey. The center lines of highways and railroads are the base lines from which the minor details, such as houses and other buildings, are located. The quickest and best method of locating a building is to set a temporary plug on the survey line near the building, then set up the transit at this point and measure the angles between the survey line and two consecutive corners of the building, measuring the distances from the instrument point to the corners of the building. These angles and distances will locate one side of the building. A small freehand sketch is then made, giving the survey line, the station of the plug, or its distance from some known point, and the angles and distances to the side of the building. The remaining sides of the building are added to the sketch and their several lengths measured in consecutive order and marked on the sketch. Such notes are quickly made and as quickly platted, but are omitted here in order to avoid too great an amount of detail.

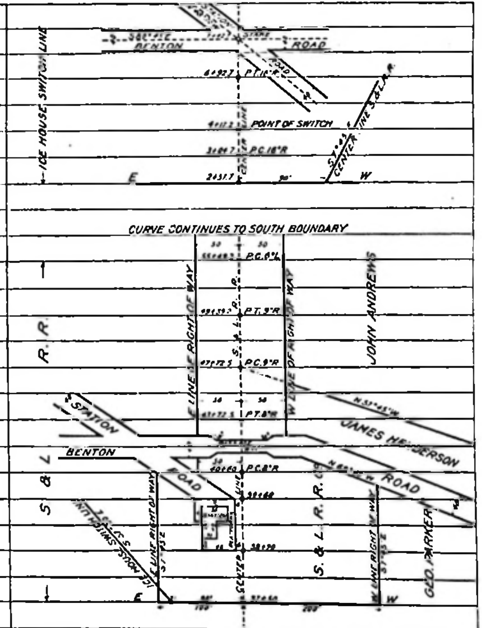
Sketches are of the greatest value in taking topographical notes. They can be made in much less time than is required for writing full descriptions, and are always more intelligible to the draftsman. Each surveyor has his individual methods, both in order of work and form of notes, and often one will consume much less time than another in performing the same work; but expedition is of no value if had at the cost of accuracy.

In drawing this map, the center lines of the railroads and highways are platted from the bearings and distances of the

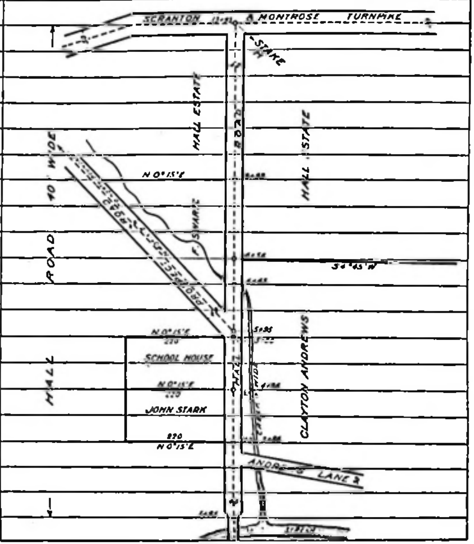
Sta.	Degree of Curve	Intersection Angle	Tangent	Radius	Magnetic Bearing
3+15					
2+65	P.T. 18°R				
0+65	P.C. 18°R	36°	103.8	319.6	
0	P.T.				S 39° 30'E
26+96.1					
25+14.4					
24+24.4	P.C. 10°L			573.7	
23+74					
22+74					
22+24					
21+24					
18+24.4	P.T. 10°R				S 41° 45'W
16+39.4					
15+49.4	P.C. 10°R	27°30'	140.4	573.7	
15+04.4					
11+89.4	P.T. 10°R				S 14° 15'W
10+92.7	P.C. 10°R	9°40'	48.5	573.7	



7+42.7	Center Benton Road				
6+92.7	P.T. 16°R				S 4°35'W
4+17.2	Point of Switch to Coal Chute				
3+04.7	P.C. 16°R	62° 05'	216.2	359.3	
2+57.7					S 57°30'E
55+49.2	P.C. 6°L			955.4	
49+39.2	P.T. 9°R				S 32°15'W
47+72.5	P.C. 9°R	15°	89.9	637.3	
43+72.5	P.T. 8°R				S 17°15'W
40+60	P.C. 8°R	25°	158.9	716.8	
39+60	South End of Platform				
38+70	North End of Platform				
37+50	Center Line S&L.R.R. beginning point				S 7°45'E



Sta.	Magnetic Bearing	Remarks
13+93	S 89° 45′ E	Center of Hall Road & Scranton & Montrose Turnpike
8+93		Division between F. Swartz & Hall Estate
6+56		Division between Clayton Andrews & Hall Estate
6+45		Center of Creek Crossing
5+95		Center of Prospect Road (40′ wide)
5+86		East Boundary School House Lot, Creek 25′ R- of Center Line.
4+86		Division between School House Lot & John Stark, Creek 30′ R- of Center Line.
2+86		West Boundary John Starks Lot, Creek 40′ R- of Center Line.



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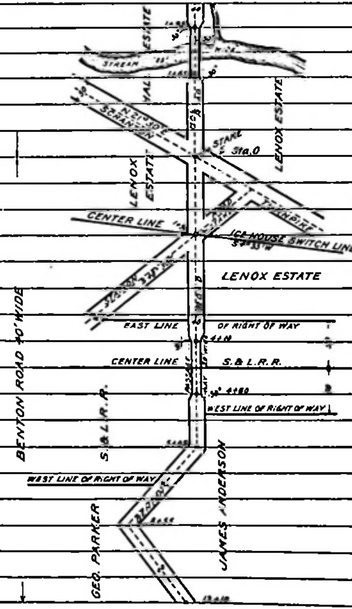
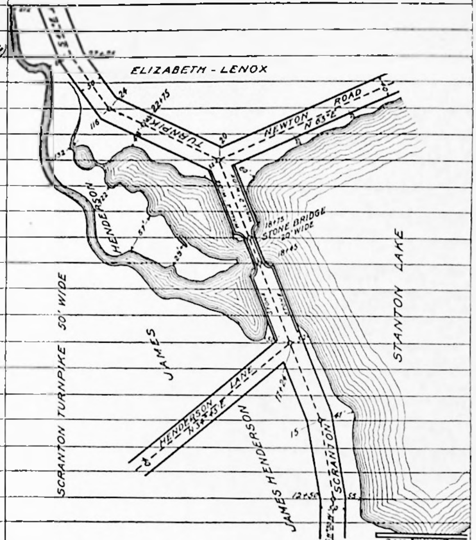
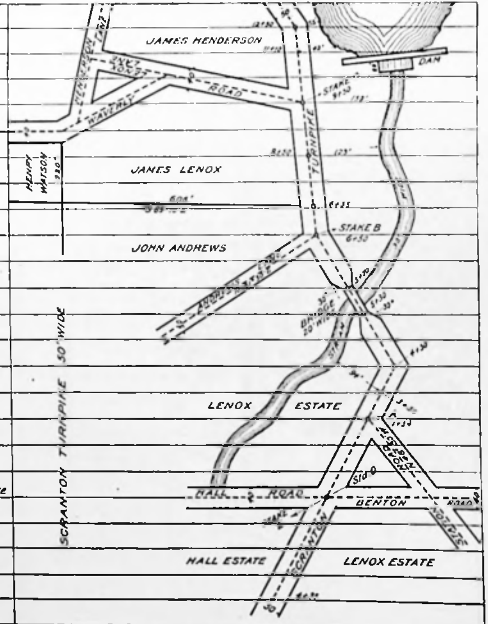
1+95		East End of Bridge
		Mouth of Creek 30' R. of Center Line
1+65		West End of Bridge
	S 89° 45' E	
0		Center of Hall Road, Benton Road & Scranton Turnpike
	N 89° 45' W	
1+90		Station 7+427 Ice House Switch Line (Stake)
4+10		East End of Passage Way
4+60		West End of Passage Way
5+05	N 69° 45' W	Angle in Benton Road
8+55	S 80° 15' W	Angle in Benton Road
12+10		End of Line



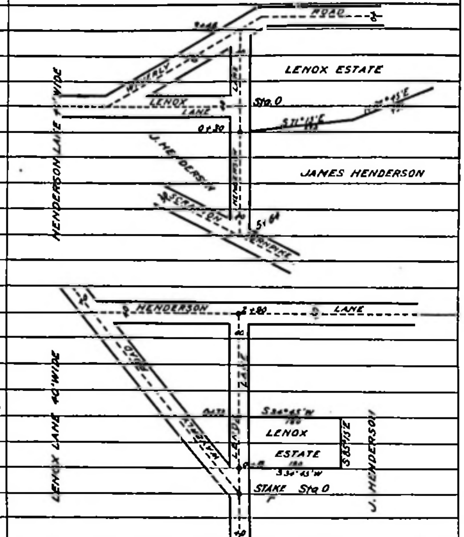
Sta.	Magnetic Bearing	Remarks
28+10	End of Line	Shore of Stream 212' left of Center Line
27+25	S 46° E	Angle in Scranton Turnpike, Shore of Stream (4' wide) 60' left of Center Line
24	S 76° E	Angle in Scranton Turnpike, Shore of Stream (10' wide) 116' left of Center Line
22+75		Shore of Lake 48' left of Center Line
20	N 83° E	Center of Newton Road, Angle in Scranton Turnpike
18+75		South End of Stone Bridge
18+45		North End of Stone Bridge
17+24		Center of Henderson Lane, Shore of Lake 50' left of Center Line and 32' Right of Center Line
15	S 27° E	Angle in Scranton Turnpike, Shore of Lake 41' Right of Center Line
	S 8° 30' E	
	S 11° 30' W	



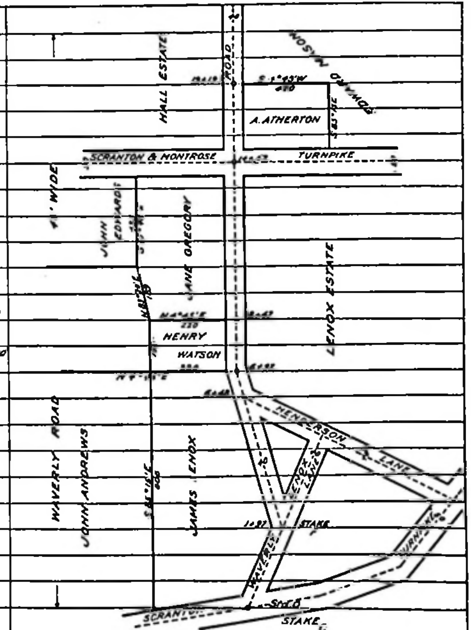
12+50	S 6° 30' E	Angle in Scranton Turnpike. Shore of Lake 55' R. of Center Line
11+10		Shore of Lake 42' R of Center Line
9+50		Center of Waverly Road. Shore of Stream 152' R of Center Line
8+50		Shore of Stream 123' R of Center Line
6+55		Division Line between James Lenox & John Andrews
6+50	S 11° 30' W	Angle in Scranton Turnpike & Center Andrews Lane
5+70		East End of Bridge
5+30		West End of Bridge
4+50	S 58° 30' E	Angle in Scranton Turnpike
3+50		Shore of Stream 94' L. of Center Line
1+34		Center of Station Road (40' wide)
↑	S 21° 30' W	
0		Center of Hall Road, Benton Road & Scranton Turnpike
↓	N 21° 30' E	
4+35		End of Line



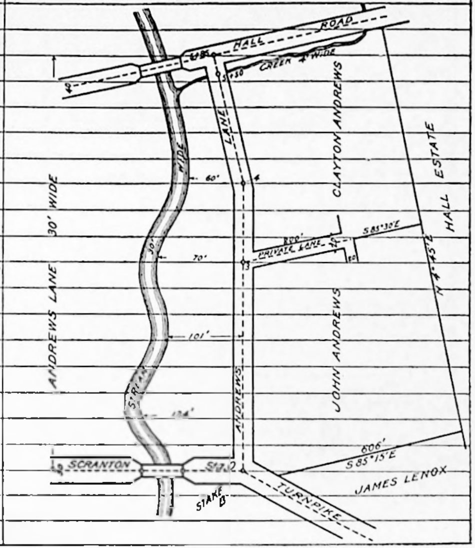
Sta.	Magnetic Bearing	Remarks
3+46		Center of Waverly Road
	N 34° 45' E	
0		Center of Henderson Lane & Lenox Lane
0+30	S 34° 45' W	Division Line between Lenox Estate & J. Henderson
5+64		Center of Scranton Turnpike
2+90		Center of Henderson Lane & Lenox Lane
0+73		Division Line between Lenox Estate & J. Henderson
0+13		Division Line between Lenox Estate & J. Henderson
0	S 55° 15' E	Center of Waverly Road & Lenox Lane



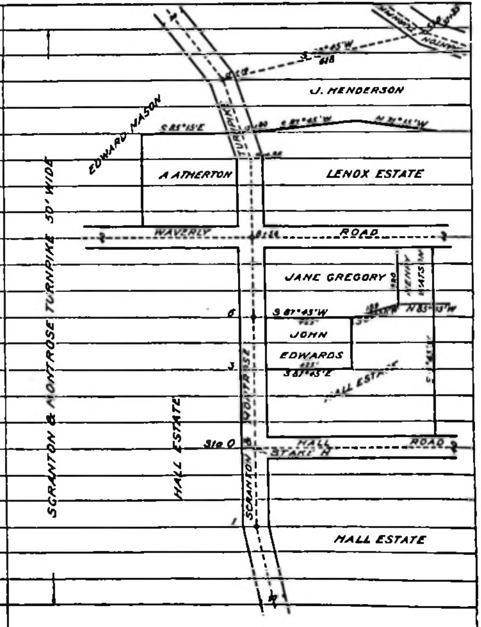
		East Boundary	
19 + 19		Division Line between A Atherton & Edward Mason	
14 + 69		Center of Scranton & Montrose Turnpike	
8 + 47		Division Line between Jane Gregory & Henry Watson	
6 + 97	S 85° 15'E	Angle in Waverly Road Division Line between Henry Watson & James Lenox,	
6 + 48		Center of Henderson Lane	
1 + 97	N 74° 45'E	Angle in Waverly Road, Center of Lenox Lane	
0	S 55° 15'E	Center of Scranton Turnpike & Waverly Road	



Sta.	Magnetic Bearing	Remarks
5+85		Center of Hall Road
5+50		Center of Creek, 4' wide
4	N 11° 30'E	Angle in Andrews Lane Stream 60' Left of Center Line
3		Center of Private Lane, Division Line between John & Clayton Andrews, Stream 70' Left of Center Line
2		Stream 101' left of Center Line
1		Stream 124' left of Center Line
0	N 31° 30'E	Center of Andrews Lane & Scranton Turnpike



19+48		South End of Line
15+10	S 27° 45'E	Angle in Scranton & Montrose Turnpike
		Division Line between A Atherton & Edward Mason
12+80		Division Line between Lenox Estate & J Henderson
11+86	S 7° 45'E	Angle in Scranton & Montrose Turnpike
8+56		Center of Waverly Road
6		Division Line between Jane Gregory & John Edwards
3		Division Line between John Edwards & Hall Estate
	S 2° 15'W	
0		Center of Hall Road & Scranton & Montrose Turnpike
	N 2° 15'E	
1	N 5° 45'W	Angle in Scranton & Montrose Turnpike
4+10		North End of Line



traverse lines, as recorded in the notes. The property lines are located at the points where they intersect the center lines of the highways. The directions of these lines are platted from the bearings given in the notes, and their lengths are platted from the measurements shown in the sketches, the measurements being taken from the center lines of the highways. In platting the various courses of the highway traverse lines, it would be well first to plat each course by the method of tangents, as described in *Mapping*, Part 1, and then check it by the protractor. The right of way of the railroad and the boundary lines of the highways are then platted to their respective widths, as indicated in the sketches, thus completing the framework of the map.

The student will then sketch in the topography, such as the water courses, buildings, etc., from observation, giving them the same relative forms and positions as are shown in the plate. The lettering should then be done in a neat and uniform manner. The conventional signs are the last to be put on the map. The student should bear in mind that the worth of a drawing depends nearly as much on its neatness and uniformity as on its accuracy, and that, therefore, nearly as much care should be exercised in the sketching and lettering as in the platting.

The field notes of the survey made for the purpose of obtaining the data necessary for drawing this map are given on pages 44 to 53.

30. Drawing the Map.—From the preceding notes of the survey, draw the map to a scale of 200 feet to the inch, making the dimensions $11\frac{1}{2}$ by $15\frac{1}{2}$ inches for the map itself and 13 by 17 inches within the border line. The magnetic meridian is assumed to be parallel to the right and left border lines of the plate, with the north point toward the top of the map. The following is a description of the general order in which the map should be drawn:

From the northwest corner of the map, marked K on the plate, scale off 700 feet to the scale of the map along the

north boundary line, locating the point *L* at the extremity of the measurement. Assume the point thus located to be Station 37 + 50 on the center line of the S. & L. R. R., and the starting point of the platting. From this point, draw a line having a magnetic bearing S 7° 45′ E; the line thus drawn represents the direction of the center line of the S. & L. R. R. at this point. The entire center line of this railroad should now be platted from the notes in the manner described in *Mapping*, Part 1. The notes show that the point *M* on the center line of the ice-house switch line is at a distance of 90 feet due east of Station 37 + 50 of the S. & L. R. R. Locate this point and assume it as Station 2 + 57.7 on the center line of the ice-house switch line. From this point draw a line whose magnetic bearing is S 57° 30′ E; the line so drawn represents the direction of the center line of the ice-house switch line at this point. This entire line is now platted from the notes in the manner that has been described.

At Station 4 + 17.2 of the ice-house switch line is the point of switch to the coal chute. This point can be located in the curve by means of the central angle of that portion of the curve preceding the point. From the rule for the length of curve given in *Circular Curves*, the central angle of a curve, which is equal to the angle of intersection, is known to be equal to the length of the curve, expressed in stations of 100 feet, multiplied by the degree of curve. Since the P. C. of the 16° curve is at Station 3 + 04.7, and the point of switch is at Station 4 + 17.2, the length of that portion of the curve preceding the point of the switch to the coal chute is equal to $4.172 - 3.047 = 1.125$ stations, which, when multiplied by the degree of curve, gives a value of $1.125 \times 16 = 18°$ for the corresponding central angle. This central angle is laid off at the center of the curve, to the right of the radial line drawn to the P. C., thus giving the direction of the radial line to the switch point; the intersection of this radial line with the curve is the zero point of the switch line to the coal chute. At this point, draw a line tangent to the curve, that is, at right angles to the radius.

This line has a bearing S 39° 30′ E and is the direction of the short tangent in the first part of the switch line leading to the coal chute. The remainder of this line is platted from the notes in the usual manner.

A sketch in the notebook shows that at Station 7 + 42.7 of the ice-house switch line a stake was driven in the center of Benton Road. By referring to the notes of the Benton Road traverse, this stake is seen to be marked as Station 1 + 90 of that line, and the magnetic bearing of Benton Road at this point is N 89° 45′ W. Therefore, draw a line through this point having a bearing of N 89° 45′ W, for the center line of the Benton Road. On this line, scale off 190 feet toward the east; the extremity of this measurement locates a point that is in the center line of the Hall Road, the Benton Road, and the Scranton Turnpike, and is also Station 0 of the Benton Road traverse. Mark this point E. Beginning at the point E, plat the center line of Benton Road westwards, locating the angle points at Stations 5 + 05 and 8 + 55. The last course is continued to the west boundary of the map.

Beginning at the same point E, plat the center line of the Hall Road traverse eastwards, locating the center of Prospect Road at Station 5 + 95 and the center of the Scranton and Montrose Turnpike at Station 13 + 93, and also locating the property lines at Stations 2 + 86, 4 + 86, 5 + 86, 6 + 56, and 8 + 93. At Station 5 + 95, plat the center line of the Prospect Road by its magnetic bearing, as given in the sketch, and continue this line to the north boundary of the map. Mark Station 13 + 93 with the letter H. Returning to the same point E, which is also taken as Station 0 of the Scranton Turnpike traverse, plat the center line of this turnpike from the notes, locating the center and terminus of the station road at Station 1 + 40, the angle point at Station 4 + 50, the second angle point and center of Andrews Lane at Station 6 + 50, the center of Waverly Road at Station 9 + 50, the angle points at Stations 12 + 50 and 15, the center of Henderson Lane at Station 17 + 24, the angle and center of Newton Road at Station 20, the angle points at Stations 24 and 27 + 25, and continue the last course to the south

boundary of the map. Mark the Stations at 1 + 40, 6 + 50, and 9 + 50 by the letters *A*, *B*, and *C*, respectively. At Station 20 of this traverse, plat the center line of Newton Road from the magnetic bearing given in the sketch, and continue this line to the south boundary of the map.

The point *C*, in the Scranton Turnpike traverse, is Station 0 of the Waverly Road traverse. Beginning at this point, plat the center line of Waverly Road, locating the angle and the center of Lenox Lane at Station 1 + 97, the center of Henderson Lane at Station 6 + 48, the angle at Station 6 + 97, the center of the Scranton and Montrose Turnpike at Station 14 + 94, and continue the center line of Waverly Road to the east boundary of the map. Mark the angle at Station 1 + 97 by the letter *F*; this point is Station 0 of the Lenox Lane traverse. Beginning at the point *F*, plat the center line of Lenox Lane, locating the terminus of Lenox Lane, and the center of Henderson Lane at Station 2 + 90. The last point is Station 0 of the Henderson Lane traverse. From this point, plat the center line of Henderson Lane leading north into Waverly Road and also leading south into the Scranton Turnpike.

The point *H*, located on the Hall Road traverse, is Station 0 of the Scranton and Montrose Turnpike traverse. Beginning at this point, plat, from the notes, the center line of this turnpike northwards, locating the angle at Station 1, and then continue the center line to the north boundary of the map. From the same point *H*, plat the center line of the turnpike southwards, intersecting the center of Waverly Road at Station 8 + 56, and locating the angles at Stations 11 + 86 and 15 + 10; continue the last course to the south boundary of the map. The point *B*, located on the Scranton Turnpike, is Station 0 of the Andrews Lane traverse. Beginning at this point, plat, from the notes, the center line of Andrews Lane, locating the center of a private lane leading to the east, at Station 3, an angle at Station 4, and the terminus of the lane at Station 5 + 85, which is in the center of Hall Road.

Now return to the notes for the S. & L. R. R. and plat the station and platform as located from the center line. Then

plat the right-of-way line on each side of the center line at the distances indicated in the sketch. From the bearing given in the sketch, plat the property line between James Henderson and John Andrews extending west from the railroad right-of-way line. From the notes for the ice-house switch line, plat the ice house and dam on the east side of the line, and plat the two ice houses on the west side of the line from the measurements given in the sketches. Then plat the shore line of the lake on the east side of the center line from the offset measurements given in the sketch. The coal chute is next platted at the terminus of the switch line to the coal chute from the measurements given in the sketch.

Beginning at the point *E* of the Benton Road traverse running westwards, plat the retaining walls and abutments of a passageway 20 feet wide under the railroad, locating the east end at Station 4 + 10 and the west end at Station 4 + 60, as given in the notes, platting the wing walls at an angle of 30° with the direction of the passageway. Beginning at the same point *E*, plat the bridge between Stations 1 + 65 and 1 + 95 of the Hall Road traverse, as shown in the notes, platting the wing walls at an angle of 30° with the direction of the bridge. The property lines should next be platted, giving them the directions as indicated by the magnetic bearings in the sketches, and scaling the depths of lots where these are given. Beginning at the same point *E*, plat the bridge extending from Station 5 + 30 to Station 5 + 70 of the Scranton Turnpike traverse, as located in the notes, platting the wing walls of the bridge at an angle of 30° with the direction of the bridge. Also, plat the property line at Station 6 + 55, giving it the direction and depth as indicated in the sketch. The bridge extending from Station 18 + 45 to Station 18 + 75 should next be platted as shown in the sketch.

Beginning at the point *C*, which is the starting point of the Waverly Road traverse, the property lines at Stations 6 + 97, 8 + 47, and 19 + 19 of this traverse should be platted, giving them their respective depths and directions as indicated in the sketch. From the point *F*, which is the

beginning of the Lenox Lane traverse, the property lines at Station 0 + 13 and 0 + 73 of this traverse should be platted, giving them their respective depths and directions as indicated in the sketch. At Station 0 + 30 of the Henderson Lane traverse running south, the property line between the Lenox estate and James Henderson's land should be platted to the angle in the line, and its continuation then platted of the length and in the direction indicated in the notes. This line should terminate in the center of the Scranton and Montrose Turnpike. The point *B* of the Scranton Turnpike traverse is Station 0 of the Andrews Lane traverse. At Station 3 of the latter traverse, the property line and the private lane 10 feet wide each side of this line should be platted of the depth and in the direction indicated in the sketch. As has been stated, the point *H* is Station 0 of the traverse of the Scranton and Montrose Turnpike running southwards. At Stations 3, 6, and 12 + 80 of this traverse, the property lines should be platted of the depths and in the directions indicated in the sketch. When all the property lines have been platted according to the measurements in the notes and sketches, the boundaries of the various roads should be drawn, making their respective widths as indicated in the sketches and terminating them as there shown.

The framework of the map is now finished, having been platted entirely from field notes, and the map is to be completed by sketching in the details, such as water courses, shore lines, and buildings from the plate, putting in the names of property owners, bearings, etc., and the title, and making the conventional signs. After the traverse lines and property lines have been platted, the buildings should be drawn. They should be drawn in the same relative positions and of substantially the same form and dimensions as on the plate. The positions of important buildings are usually located by measurements made from convenient points in the surveys, and the buildings are platted on the map from the measurements, as recorded in the notes. For this map, however, the measurements locating buildings have in most cases been omitted, in order that the notes will not be too voluminous

and that the more important portions of the platting will not be obscured by too many details. After the buildings are drawn, the names of the various property owners should be written neatly in their proper places. The names and bearings of the highways, the alinement of the railroad, the bearings of property lines, and other necessary or important details should also be written. A neat, plain system of lettering should be used. In a really complete map that is drawn to a sufficiently large scale, the lengths of all courses along highways and property lines, as well as their bearings, should be shown on the map. But in the present case the distances are omitted for brevity.

All this work should be done lightly in pencil, and all details completed. Then, if, after a careful inspection, it is considered to be correctly drawn, the lines should be inked in. The letters should then be carefully gone over in ink, and finally the conventional signs should be made and the entire map finished substantially as shown on the plate.

MATTERS RELATING TO MAPPING

31. Tinting and Coloring.—All conventional signs thus far described are made with a pen. Where surveys cover extensive areas, it sometimes happens that the labor and time for pen work cannot be spared, and colors applied with a brush are used instead. With a skilful hand, work of this character may be rapid and very effective. However, owing to the fact that colors cannot be reproduced by blueprinting or any similar cheap and expeditious process for reproducing drawings, the tinting and coloring of maps and drawings is not now practiced extensively. For this reason, the subject is not treated at length here, and no examples are given. But the process is described sufficiently in detail, it is believed, to give the student a clear understanding of the subject, and enable him to do work of this character if he so desires.

Only a few colors besides India ink are required; the most essential colors are gamboge (yellow), indigo (deep blue),

new blue (light blue), burnt sienna (brown), and scarlet lake. By mixing these colors, almost any color, shade, or tint desired can be obtained. Any shade of green can be produced by mixing blue and yellow colors, such as indigo and gamboge. A pleasing brown can be obtained by mixing scarlet lake, gamboge, and a very small amount of India ink, and a very dark brown by increasing the amount of India ink. Purple can be made by mixing scarlet lake and indigo. Orange tints can be produced by adding a small amount of scarlet lake to gamboge; and so on in almost endless variety.

The colors used in the drafting room are of two kinds, namely, dry colors and moist colors. **Dry colors** are sold in the form of rectangular cakes, wrapped in tin-foil. **Moist colors** are packed in small dishes of porcelain, rectangular in form and open at the top. The surface of the color is covered with wax and the entire dish wrapped in tin-foil.

In using moist colors, the cake of color is rubbed lightly with a moistened brush, which will take up sufficient color, and the color is then diluted in water to the proper tint, which should always be light and delicate. A different dish, preferably shallow and saucer-shaped, should be provided for each color used. For this purpose what is known as a **nest of color saucers** is very convenient. This consists of a number, usually half a dozen, of saucers of the same size that fit on top of each other. A dish for water should also be provided; an ordinary glass tumbler, or a teacup, serves well for this purpose.

To cover a surface with a uniform tint, a camel's hair or sable brush should be used, and a separate brush used for each tint, or the brush should be washed thoroughly in clear water when changing from one tint to another. Confusion in the use of brushes is sure to spoil a tint. For large masses of the same tint, a large brush should be used, but for small details, small brushes are indispensable. Heavy and labored strokes should be avoided. Light and rapid strokes produce smooth and pleasing effects. The map should be pinned to a light drawing board, so that it can readily be inclined at an angle. Keep the brush well filled with color and begin

at the top of the surface, inclining the board toward you. If the outline is very irregular, moisten the edge with water. Apply the tint the full length of the surface and continue it down the surface, never allowing the edge to dry.

In representing topography by colors, woods are commonly colored yellow; grass land, green; cultivated land, brown; brushwood, marbled green and yellow; vineyards, purple; lakes and rivers, light blue with a darker tint at the shore line; seas, dark blue with a little yellow added; marshes, the water blue, with patches of green applied horizontally; and roads, dark brown. Woods may be made very effective by drawing the trees, coloring the angle toward the light (the upper left hand) with a touch of yellow, and the opposite, or lower right-hand, side with indigo.

Skill and judgment in mixing and applying colors can be acquired only by practice. When a combination tint, such as brown, is required, the draftsman must estimate how much coloring is required and provide accordingly. He is likely to use too much color, thus producing a heavy tint that is almost certain to become streaked when applied. If sufficient brushes are available, a separate brush should be used to take up each color, and a separate tinting dish or color saucer should be provided for each color, with sufficient water in it for mixing the required amount. A dish of either glass or porcelain contains the water. The brush is first moistened by dipping it in the water, without, however, letting it take up too much water, and then it is rubbed on the cake of color; a very small amount of color is sufficient. The brush is next dipped into the water in the color saucer, giving off its proportion of color. This water is then stirred until every particle of color is dissolved. If the tint is too light, add more color; if too heavy—a common fault—add more water, until the proper shade is obtained.

The tint should then be applied rapidly, beginning at the top of the paper and not allowing the edge of the tinted surface to become dry before carrying it further down the paper, until the bottom of the surface to be tinted is reached. If the edge of the tinted surface should become dry before

the tint is extended, when the tint is applied to the adjacent surface, a streak of deeper tint will appear where the two tinted surfaces unite. Should this occur unavoidably, however, the streak can be mostly removed and the tint rendered reasonably uniform by a careful application of a sponge rubber after the paper is thoroughly dry. Any tint is deepened by repeating the application of it. The paper must not be allowed to dry between the successive applications of the tint. If from any cause it should become dry, the entire surface must be moistened with clear water before another application of the tint. Careful practice will enable the student to produce a smooth tint.

When a marbled effect is desired, the entire surface is first covered with one tint, and then the other is applied in shorter or longer strokes of the brush, according to the effect that it is desired to produce. In tinting shore lines, the outline of the shore is first traced with a brush moistened with clear water, extending the wash as far as the tint is to be used. A dark-blue color having been prepared, a fine brush is dipped in the color and the outline of the shore is traced. The adjoining paper being moist will cause the color to run. Then a brush is moistened in clear water and the shore line thus traced is washed, the strokes of the brush being drawn from the shore. The effect will be a dark-blue shore line shaded to light blue. For roads, a dark-brown color is used.

32. Scales.—The scale of a topographical map should depend on the character of the work involved, but should always be large enough to clearly admit all necessary details without making the map unwieldy. The work should be so well done that dimensions may be accurately scaled from the map without any calculation. For small plats, such as public squares and small parks, 50 feet to the inch would be a suitable scale, admitting the smallest detail. For larger areas, such as town sites, extensive parks, suburban resorts, etc., scales of from 100 to 400 feet to the inch are commonly used, according to the amount of detail to be shown. The

scale must be reduced in proportion as the area is increased. Published topographical maps are usually made to scales varying from about 1 mile to the inch to 800 feet to the inch, according to the amount of detail represented. The smaller scale permits only the location of cities and towns and the prominent geographical features to be shown, while the larger scale permits the representation of all towns, villages; farms, woods, isolated buildings, roads, foot-paths, streams, and hills of 500 or 600 feet in horizontal extent. On a scale of 2 inches to the mile, the various features of the ground can be clearly represented. In all cases, the character of the surface, the amount of detail included in the survey, and the purpose of the map should determine the scale.

33. Size of Maps.—Maps for use in the field may vary in size from 18 by 24 inches to 24 by 30 inches. Both sizes are

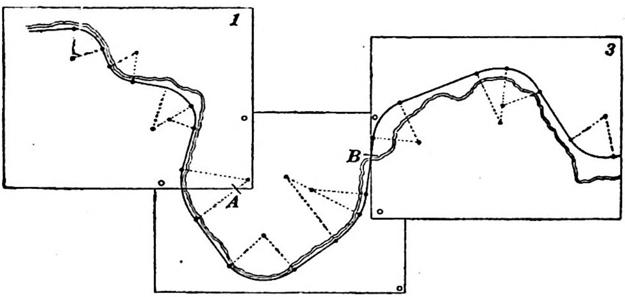


FIG. 22

suitable for railroad work. The line should be so arranged on the different sheets that they can be fitted together, making a continuous map of the line of survey, and the sheets should be numbered in regular succession, appearing somewhat as shown in Fig. 22.

When possible, the sheets should be arranged so that each curve with its center and limiting radii will come on the same sheet. Sometimes this cannot be done. The points where the different sheets join each other should be fixed

by a line drawn at right angles to the center line or radial line at the point of junction, as at *A* or *B*. This simplifies the work of fitting the sheets and greatly increases accuracy.

If the entire map is to be contained on a single sheet, judgment must be exercised in fixing the direction of the first course so as to attain that result, that is, so that the platting will not run off the edge of the sheet. When possible so to arrange the map on the sheet, the points of the compass should be in their natural order, namely, north at the top and south at the bottom of the map. Very fine lines on a map are a blemish rather than a merit, and heavy lines are likewise to be avoided, except when used for shading or boundaries. Boundaries of private property are represented by bold, full lines, and those of states, counties, or municipalities by heavy broken and dotted lines.

34. Lettering.—Legibility and uniformity are the chief requisites for good lettering. Ornamental letters are entirely out of place on a map, except for titles, and they are suitable for the titles of very elaborate maps only. All lettering in the body of the map or on details should preferably be in Italics. Small letters should be two-thirds the height of capitals. Ordinary capitals should usually be $\frac{1}{8}$ inch in height, and the small letters two-thirds of $\frac{1}{8}$ or $\frac{1}{12}$ inch in height. Uniformity in the spacing and slant of letters is as important as uniformity in size. All dimensions should be expressed in figures, and all important lines and objects briefly, but accurately described. So far as possible, without disadvantage to the map otherwise, the tops of the letters should be toward the north and west sides of the map. There is no work where practice is more essential, if skill is to be acquired, and nothing adds more to the finish of a drawing, than good lettering, while poor and slovenly lettering will rob of all merit an otherwise perfect drawing. The outline of a map and its position on the sheet will determine the position of the title, which should usually be in plain, bold lettering, but may be in ornamental lettering when the character of the map is such as to justify it.

DETERMINATION OF TRUE MERIDIAN

INTRODUCTION

PRELIMINARY DEFINITIONS AND EXPLANATIONS

1. In all important surveys the directions of the courses are referred to the true meridian at some point on the earth's surface, and the true meridian is determined by astronomical observations. In this Section two methods of determining the direction of the meridian will be described; namely, by observation of the north star and by observation of the sun. But in order that these methods may be understood, it is necessary first to explain some of the principles of geometry and astronomy on which they depend.

THE SPHERE

2. In geometry, a **sphere** is defined as a solid, bounded by a surface every point of which is equidistant from a point within called the *center*. The surface of such a solid is properly called a *spherical surface*, but for the sake of brevity a spherical surface is often called a *sphere*, just as the word *circle* is used instead of the longer word *circumference*.

A **radius** of a sphere is a straight line drawn from the center to the surface. A straight line passing through the center and terminated at both ends by the surface is called a **diameter** of the sphere.

3. Every plane section through a sphere is a **circle.** A section of a sphere made by a plane passing through the center is called a *great circle.* A section made by a plane not passing through the center is called a *small circle.* Thus, $A\,B\,A'\,B'$ and $B\,P\,B'P'$, Fig. 1, are great circles, because their planes pass through the center C of the sphere; while $a\,b\,a'\,b'$ and $c\,c'$ are small circles, because their planes do not pass through C. A great circle divides the sphere into two equal parts, called *hemispheres.*

4. A straight line through the center of either a great or a small circle, and perpendicular to the plane of the circle, is called the **axis** of the circle. The points where the axis of a circle meets the sphere are called the **poles** of the circle. Thus, $P\,P'$, Fig. 1. is the axis of the great circle $A\,B\,A'\,B'$ and of the small circle $a\,b\,a'\,b'$; the points P and P' are the poles of these circles. The axis of any circle of a sphere passes through the center of the sphere.

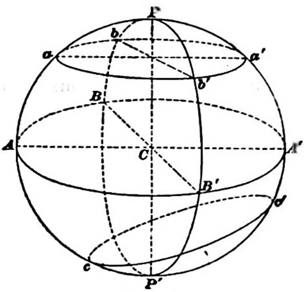


Fig. 1

5. If any great circle of a sphere is taken as a *primary,* or *fundamental,* circle, great circles passing through its poles are called its *secondaries.* Thus, if $A\,B\,A'\,B'$, Fig. 1, is taken as a primary circle $P\,B\,P'\,B'$, a great circle passing through the poles P and P', is one of its secondaries.

It is evident that the plane of a great circle is perpendicular to the plane of each of its secondaries; it follows that, if one circle is a secondary to another, the latter is also a secondary to the former. Thus, the circle $A\,B\,A'\,B'$, Fig. 1, is a common secondary to the two circles $A\,P\,A'\,P'$ and $B\,P\,B'\,P'$.

6. The angle between two great circles, called a *spherical angle,* is the angle between their planes, and is measured by the angle between two lines drawn, one in each plane, perpen-

dicular to the line in which the planes intersect. Each of these lines must meet the line in which the planes intersect at the same point. Thus, the angle between the great circles $A\,P\,A'\,P'$ and $B\,P\,B'\,P'$, Fig. 1, is measured by the angle $A'\,C\,B$; $C\,A'$ lies in the plane of $A\,P\,A'\,P'$ and is perpendicular to the intersection $C\,P$ of the two planes at C, while $C\,B$ lies in the plane of $B\,P\,B'\,P'$ and is also perpendicular to that intersection at the same point C. But the angle $A'\,C\,B$ is measured by its intercepted arc $A'\,B$. Hence, the angle between the circles $A\,P\,A'\,P'$ and $B\,P\,B'\,P'$ is measured by the arc $A'\,B$ intercepted between them on their common secondary $A\,B\,A'\,B'$. This may be stated as a general principle as follows:

The angle between two great circles is measured by the arc intercepted between them on their common secondary.

7. The **angular distance** between two points on a sphere is the angle subtended at the center of the sphere by radii drawn through the points.

8. A **spherical triangle** is a portion of a sphere bounded by three arcs of great circles.

9. **Parallel circles** of a sphere are those whose planes are parallel. Parallel circles have common poles and secondaries.

TERRESTRIAL CIRCLES

10. Earth's Axis and Poles.—The earth revolves, or turns, once in 24 hours about an imaginary line called its *axis*. The points where this line meets the earth's surface are called the earth's *poles*, one being the *north pole* and the other the *south pole*.

11. Equator and Meridians.—The **plane of the equator** is a plane passing through the center of the earth, and perpendicular to the earth's axis. It cuts the surface of the earth in a circle called the *terrestrial equator*.

Any plane containing the earth's axis is called a *meridian plane*, and the circle in which it intersects the earth's surface is called a *meridian line*, or simply a *meridian*.

In Fig. 2, $S N$ is the axis of the earth; S, the south pole; N, the north pole; $E Q R T$, the earth's equator; and the circles $N R S E$, $N G S$, and $N F S$ are meridians. All meridian planes are perpendicular to the plane of the equator. It is evident that a meridian may be passed through any place on the earth's surface; such a meridian is called the meridian of that place. It will be noticed that meridians are circles, not straight lines, and that they all meet at the poles. But, for

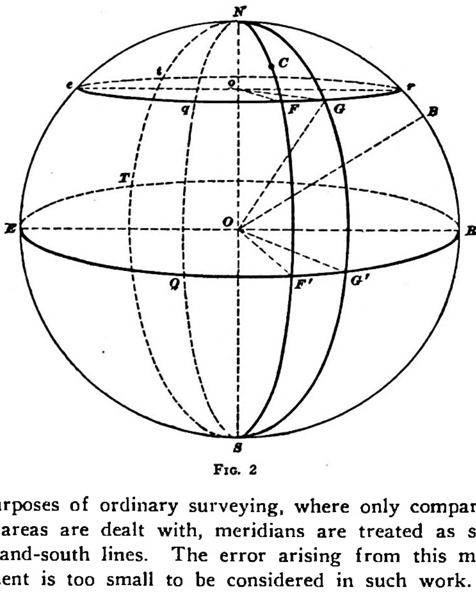


FIG. 2

the purposes of ordinary surveying, where only comparatively small areas are dealt with, meridians are treated as straight north-and-south lines. The error arising from this mode of treatment is too small to be considered in such work.

12. Latitude.—The latitude of a point on the earth's surface is the angle that the radius of the earth passing through that point makes with the plane of the equator. In Fig. 2, the latitude of B is the angle $B O R$. This angle is measured by the arc $R B$ of the meridian passing through the point.

Similarly, the latitude of G is measured by the arc $G' G$. The latitude of a point may, therefore, also be defined as the angular distance of the point from the equator; that distance is measured on the meridian through the point, and is the number of degrees in the arc of the meridian included between the equator and the point.

When it is stated that the latitudes of the points B, G, F, are, respectively, $R B, G'G, F'F$, it should be understood that these arcs are to be expressed in degrees, latitude being an angular quantity, not a linear quantity. The latitude of a point is said to be *north* or *south* according as the point is north or south of the equator. North latitudes are considered positive and are generally indicated by the sign $+$, while south latitudes are negative and are indicated by the sign $-$.

13. Parallels of Latitude.—Any plane parallel to the equator, that is, perpendicular to the earth's axis, cuts the earth's surface in a circle called a *parallel of latitude,* or simply a *parallel.* The circle $e\,q\,r\,t$, Fig. 2, whose plane is parallel to that of $E\,Q\,R\,T$, is a parallel of latitude. All points, as r, G, F, on a parallel have the same latitude, as the arcs $R\,r$, $G'\,G, F'\,F$, are evidently equal.

14. Longitude.—The longitude of a point on the earth's surface is the angle between the meridian plane through that point and another meridian plane assumed as a plane of reference. This angle is also referred to as the angle between the meridian passing through the given point and the meridian determined by the plane of reference. The meridian of Greenwich, England, is generally taken as a reference meridian and is called the *principal meridian.* Suppose that $N\,G\,S$, Fig. 2, is the meridian of Greenwich; then the longitude of the point C, referred to that meridian, is the angle between the planes of the meridians $N\,G\,S$ and $N\,C\,S$. This angle is the same as $G'\,O\,F'$, and therefore may be measured by the arc $G'\,F'$ of the equator. Evidently, the longitude of C is likewise equal to angle $G\,o\,F$ and can be measured by the arc $G\,F$ on parallel $e\,q\,r\,t$. Thus, the longitude of a point is equal to the arc on any parallel included between the meridian

of Greenwich and the meridian through the point considered. The longitude of a point may also be defined as the angular distance of the point west or east of the reference meridian.

Longitude is counted from the reference meridian east or west, from 0° to 180°. When an angle is expressed in degrees, minutes, and seconds, it is said to be *measured in arc*. As will be explained later, longitudes are often expressed in hours, minutes, and seconds, and then are said to be *measured in time*. Degrees, minutes, and seconds of arc are indicated by the symbols °, ′, and ″, respectively, while hours, minutes, and seconds of time are abbreviated *hr.*, *min.*, and *sec.*, or, more simply, *h*, *m*, and *s*.

NOTE.—It will be noticed that some terms, as *latitude* and *longitude*, are here used in a sense somewhat different from that given to them in previous Sections of this Course. The circumstances under which these terms are employed, however, always indicate plainly in what sense they should be taken.

THE CELESTIAL SPHERE

CELESTIAL CIRCLES

15. Celestial Sphere.—Imagine a vast spherical surface enclosing the heavenly bodies and having its center at the earth; this imaginary surface is called the *celestial sphere*, or the *heavens*. The heavenly bodies are so enormously remote that, in comparison to the celestial sphere, the whole earth can be considered as a point; therefore, any point on the earth's surface may be regarded as the center of the celestial sphere.

To one who observes the heavens at night, the celestial bodies appear to be bright points attached to the inner surface of the celestial sphere, but a little reflection is sufficient to establish the fact that the heavenly bodies are not all equidistant from the earth and are not attached to any surface. However, in ordinary astronomical work, it is necessary only to determine the relative directions of the heavenly bodies; and for this purpose, it is convenient to imagine that the heavenly bodies are attached to the celestial sphere, since the daily motion of the stars on the celestial sphere is so small as to be practically

imperceptible even with the most delicate instruments. It is the apparent rotation of the celestial sphere as a whole that causes the heavenly bodies to appear to move.

It will also be seen that the stars are aggregated into more or less definite groups called *constellations*. A very little watching will convince the observer that the forms of the constellations and the relative positions of the stars on the celestial sphere do not change noticeably. Maps of the constellations made centuries ago do not differ materially from those made at present.

16. Celestial Poles.—If the earth's axis is produced in both directions, it will meet the celestial sphere in two points called the celestial poles. The one nearer the north pole of the earth is the north pole and the other is the south pole.

17. Celestial Equator and Hour Circles.—The **celestial equator** is the circle in which the plane of the terrestrial equator intersects the celestial sphere.

An **hour circle** is a circle which passes through the celestial poles. If the equator is taken as a primary circle, the hour circles are its secondaries and, therefore, the planes of all hour circles are perpendicular to the plane of the celestial equator.

18. Zenith and the Celestial Horizon.—The **zenith** of a point on the earth's surface is the point at which a plumb line extended upwards pierces the celestial sphere. The opposite point of the celestial sphere is called the *nadir*.

The **celestial horizon** is the great circle in which a plane through the observer's eye and perpendicular to a plumb-line intersects the celestial sphere. However, the radius of the earth is inappreciable compared with the distance to the celestial sphere; therefore, the plane of the horizon is considered to pass through the center of the earth. In other words, the horizon is considered to be everywhere equidistant from the zenith and the nadir. It is evident that the zenith and the nadir are the poles of the horizon.

Only that part of the celestial sphere above the horizon is visible to an observer; hence, at any position of the observer

on the earth's surface, half of the sphere will be visible. The position of the zenith and of the horizon depends on the direction of the plumb-line and, consequently, on the location of the observer on the earth's surface. Therefore, different parts of the celestial sphere will be visible to an observer as he changes his position on the earth's surface.

19. Vertical Circles.—Circles passing through the zenith and the nadir are called vertical circles. If the horizon is considered as a primary circle, the vertical circles are its secondaries; consequently, the planes of all vertical circles are perpendicular to the plane of the horizon.

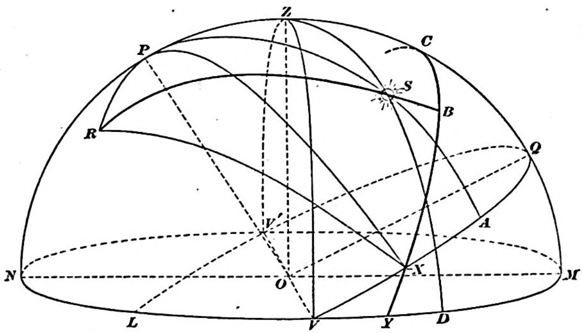


FIG. 3

20. The circle which passes through the poles and the zenith is the *observer's meridian.* The meridian cuts the horizon at two points. The one near the north pole is called the north point and the other is the south point. The meridian is both an hour circle and a vertical circle. The vertical circle whose plane is perpendicular to that of the meridian is called the *prime vertical.*

The prime vertical also cuts the horizon in two points. When the observer looks toward the north, the point on the prime vertical to the right is the east point and that to the left is the west point.

21. In Fig. 3 is shown the celestial hemisphere visible to an observer, with all the imaginary reference lines on it. Point *O* at the center represents the earth; *N V M V'* is the horizon with the north point at *N* and the south point at *M*; *M Z N* is the observer's meridian; *P* is the north pole; *Z* is the zenith; *O P* is half of the axis of the sphere; *V Q V' L* is part of the celestial equator; *P X* and *P A* are parts of hour circles; and *Z D* and *V Z V'* are parts of vertical circles. A knowledge of the various circles is essential in the study of practical astronomy, and, therefore, the lines in the figure should be carefully distinguished.

MOTION OF THE CELESTIAL SPHERE

22. If the line between the celestial poles were luminous, the observer would perceive that it remains stationary and that the entire spherical surface bearing the stars appears to turn slowly about this line as an axis; the direction of this rotation is from east to west. As a matter of fact, however, the celestial sphere is not turning, but the earth itself is rotating on its axis from west to east so that different points of the celestial sphere come successively overhead. The various stars appear above the horizon, or *rise*, toward the east, pass across the sky, and finally disappear below the horizon, or *set*, toward the west.

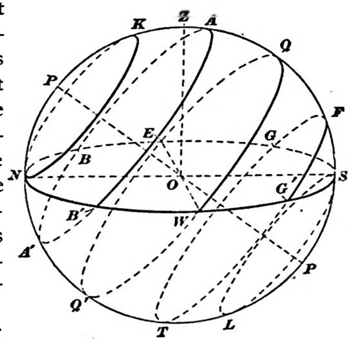


Fig. 4

The apparent motion of the stars is illustrated in Fig. 4, which represents the celestial sphere. The earth is at the center *O*; *S E N W* is the horizon; *P P'* is the axis of rotation; and *E Q W Q'* is the celestial equator. Evidently, a point on the celestial sphere will appear to move in a circle whose plane is perpendicular to the axis and whose center lies on

the axis; thus, the stars will appear to move along circles such as $K N$, $A A'$, $F T$ and $S L$, parallel to the equator. It will be noticed that stars whose paths lie between the circles $K N$ and $S L$, as stars on $A A'$ and $F T$, pass the horizon twice during a revolution, once coming up and once going down; for instance, the star rises at B, crosses the meridian at A, and sets at B'. Stars between the circle $K N$ and P never rise or set, but are always visible; such stars are called *circumpolars.* Stars between circle $S L$ and P' are never visible. A star at P or P' would have no motion at all. There is no star exactly at either pole. The star closest to

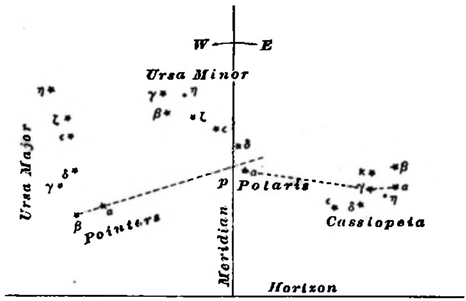


FIG. 5

the north pole is Polaris, which is, therefore, often called the *north star,* or the *pole star.* Its angular distance from the pole is a little more than 1°.

23. The apparent motion of the stars can best be understood by observing the heavens for several evenings and studying the change in the positions of a few prominent stars or groups of stars after a few hours have elapsed. Fig. 5 shows the prominent stars near the north pole p, as viewed from inside the celestial sphere. In order to identify and distinguish the stars of the various constellations, Greek letters are used as shown in the figure (the order of the letters in the alphabet and their pronunciation are not essential). The group

of stars commonly called the *Great Dipper,* which is part of the constellation known to astromomers as *Ursa Major,* is shown toward the west. It consists of seven bright stars so placed that a line through them forms the outline of a dipper. The position of this constellation in the sky varies with the date and time of night, but it can easily be found on any clear night. The two stars in the bowl that are farther from the handle are called the *pointers*; they are marked α (alpha) and β (beta). If a line is imagined drawn through the pointers in the direction indicated in the figure, Polaris will lie nearly on this line at a distance from α approximately equal to five times the distance between the pointers; thus, the north pole may be located closely. The distance between the pointers is about 5 degrees. This gives a rough scale from which other distances may be estimated.

At about the same distance from the pole as the Great Dipper and almost directly opposite the Dipper, is the constellation *Cassiopeia,* five stars of which form a slightly distorted letter W.

On a very clear night, the *Small Dipper, Ursa Minor,* can be seen with Polaris at the end of its handle. This constellation is somewhat similar in form to the Great Dipper, but is much smaller and fainter.

To show the motion of the stars, make a rough diagram of the brighter stars in the northern part of the sky, indicating their positions with respect to the horizon. Then, after an interval of 3 or 4 hours, compare this diagram with the sky. It will be perceived that the stars have moved about the pole in the manner described in Art. 22; to a person facing north, the direction of rotation will appear to be opposite to that in which the hands of a clock move.

MOTION OF THE SUN ON THE CELESTIAL SPHERE

24. Nature of the Motion.—It has been previously explained that the stars may be considered fixed in position on the celestial sphere. However, the earth revolves about the sun and, therefore, the sun appears to move among the stars

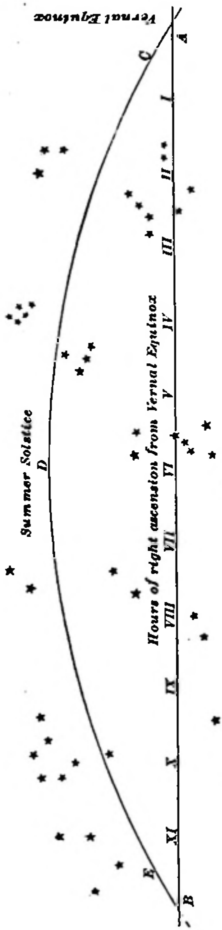


Fig. 6

on the celestial sphere from west to east. The sun makes a complete revolution and returns to its original position in a year, or in a little over 365 days. The sun does not travel along the celestial equator, but the plane of its orbit is inclined to the plane of the equator at an angle of 23° 27′. The path of the sun is called the *ecliptic* and the angle between the planes of the equator and ecliptic is known as the *obliquity of the ecliptic.* Part of the ecliptic is shown at *Y X B C* in Fig. 3.

25. It is evident that the sun crosses the equator twice in each year. On March 21, the sun passes from the southern to the northern hemisphere, and on September 22 it passes from the northern to the southern hemisphere. The point at which the sun appears to cross the equator as it passes from the southern hemisphere to the northern is called the *vernal equinox;* at that time, the season called spring begins, and the day and night are each 12 hours in length. The point at which the sun crosses the equator in passing from the northern to the southern hemisphere is called the *autumnal equinox;* at that time, the season of autumn begins and the day and night are again of equal length. In Fig. 3, *X* is the vernal equinox; the autumnal equinox is not shown. The hour circle

through the equinoxes is called the *equinoctial colure; PX*, Fig. 3, is part of the equinoctial colure.

In Fig. 6 is represented a small band of the celestial sphere lying along the equator *A B*. This view is from inside the sphere, and the sun appears to move among the stars along the line *C D E*. The vernal equinox is at *A* and the autumnal equinox is at *B*; the figure therefore represents the sun's path from March 21 to September 22. The sun is farthest north of the equator on June 21, and the most northerly point of the ecliptic at *D* is called the *summer solstice*. The sun is farthest south of the equator on December 22, and the most southerly point of the ecliptic is called the *Winter solstice;* the winter solstice is not shown in Fig. 6.

POSITION OF A BODY ON THE CELESTIAL SPHERE

26. Systems of Reference Circles.—The exact position of any point on the earth's surface is determined by its latitude and longitude. For the purpose of measuring latitude and longitude, certain imaginary circles on the earth's surface, known as the *equator* and the *principal meridian*, are used as reference circles. In the same way, certain reference circles are conceived to be drawn on the celestial sphere for determining the positions of the heavenly bodies.

In one system, the reference circles are the equator and the equinoctial colure; in another system, the equator and the meridian are used; and in a third, the horizon and the meridian are employed.

27. Right Ascension and Declination.—When the equator and the equinoctial colure are taken as reference circles, a body is located on the celestial sphere by its *right ascension* and *declination*.

The **right ascension** of a celestial body is the angular distance measured along the celestial equator from the vernal equinox eastwards to the hour circle passing through the body. Thus, in Fig. 3, *X A* is the right ascension of the body *S*. Right ascension is usually expressed in hours, minutes, and seconds, continuously from 0^h to 24^h

The **declination** of a heavenly body is the angular distance north or south of the celestial equator measured along the hour circle through the body. Thus, in Fig. 3, the declination of the body *S* is *A S*. The declination of a body is usually expressed in degrees, minutes, and seconds of arc; the declination is considered positive (+) or negative (−) according as the body is north or south of the celestial equator. Sometimes, instead of the distance from the equator, it is more convenient to consider the angular distance from the nearer pole to the body. This is called the *polar distance* and is obviously equal to the complement of the declination. In Fig. 3, *P S* is the polar distance of the body *S*. As seen from Fig. 6, the right ascension and declination of the sun are constantly changing; the method of determining their values will be explained later.

28. Hour Angle.—When the equator and the meridian are taken as the reference circles, a body is located by its *hour angle* and declination. The **hour angle** of a heavenly body is the angular distance measured along the equator westwards from the meridian to the hour circle through the body. In Fig. 3, *Q A* is the hour angle of the body *S*. Hour angles are usually expressed in hours, minutes, and seconds. The hour angles of all bodies are constantly increasing, as the rotation of the earth causes the celestial sphere to appear to move with respect to the meridian.

29. Azimuth and Altitude.—In locating a body from the horizon and the meridian, the *azimuth* and *altitude* of the body must be known. The **azimuth** of a heavenly body is the angular distance measured along the horizon from the meridian to the vertical circle through the body. Azimuths are measured in degrees, minutes, and seconds of arc continuously from 0° to 360°. They are reckoned either from the south point of the meridian toward the west or from the north point toward the east. In surveying, azimuths are usually reckoned from the north. In Fig. 3, *N V′ M D* is the azimuth of the body *S* from the north and *M D* is its azimuth from the south.

The **altitude** of a heavenly body is the angular distance above the horizon measured along the vertical circle through

the body. In Fig. 3 the altitude of the body S is $D\ S$. Altitudes are expressed in degrees, minutes, and seconds of arc. Sometimes it is more convenient to consider the angular distance of the body from the zenith instead of from the horizon. This distance is called the *zenith distance* and is evidently equal to the complement of the altitude; $Z\ S$, Fig. 3, is the zenith distance of the body S. It is evident that the zenith, meridian, and horizon are fixed with reference to the observer, but are not fixed on the celestial sphere. As explained in Art. 22 and as shown in Fig. 4, the celestial sphere appears to rotate, and therefore the altitudes and azimuths of the heavenly bodies are constantly changing. However, the altitude and the azimuth of a body at any instant can be conveniently measured and are frequently used in astronomy.

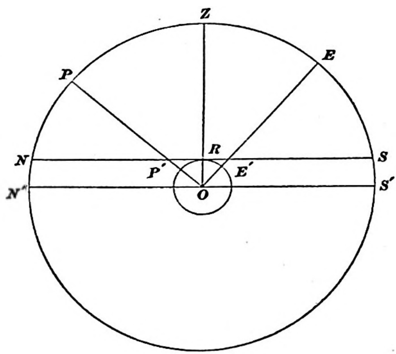


Fig. 7

30. Altitude of Pole.—In Fig. 7 is represented a section of the earth and the celestial sphere cut by a meridian plane. $P'\ R\ E'$ is the earth with its center at O and the observer's position at R; $N\ Z\ S$ is an arc of the celestial sphere; P is the north pole; $O\ E$ is half of the equator; Z is the zenith; $N'\ S'$ is the theoretical horizon; and $N\ S$ is the actual horizon. The size of the earth is greatly exaggerated for clearness in the diagram, but actually the point R may be taken as coinciding with O, and the horizon plane $N\ R\ S$, as coinciding with the plane $N'\ O\ S'$.

The angle $E'\ O\ R$ is the latitude of the observer and since it is the same angle as $Z\ O\ E$, which is measured by arc $E\ Z$, it follows that the arc of the meridian intercepted between the

celestial equator and the zenith measures the observer's latitude. The angle $P\ O\ N'$, measured by arc $N'\ P$, is the altitude of the pole. But arc $E\ P$ and arc $N'\ Z$ are each equal to 90°, and therefore $E\ Z = 90° - Z\ P$ and $N'P = 90° - Z\ P$. Hence, $E\ Z = N'P$; that is, the altitude of the pole is equal to the latitude of the observer. Therefore, in Figs. 3 and 4, arc $N\ P$ is equal to the latitude of the observer.

EXAMPLE.—The latitude of an observer is +41° 48′ 51″, and the declination of Polaris is +88° 55′. Find the zenith distance of Polaris when it is on the observer's meridian (*a*) above the pole, and (*b*) below the pole.

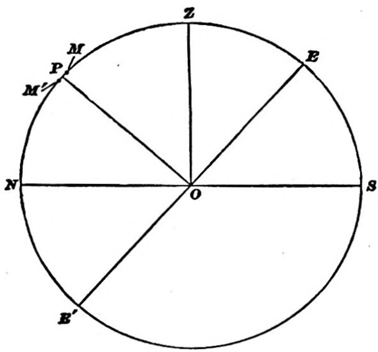


FIG. 8

SOLUTION.—The position of Polaris is shown in Fig. 8 where $N\ Z\ S$ is the meridian; O, the earth; $N\ S$, the horizon; Z, the zenith; $E'\ O\ E$, the equator; P, the pole; M, Polaris above the pole; and M', Polaris below the pole. The altitude of the pole is equal to the latitude of the observer, or $N\ P = E\ Z$ = 41° 48′ 51″. The declination of Polaris is $E\ M$ or $E'\ M'$ and therefore $E\ M = E'\ M'$ = 88° 55′.

(*a*) The zenith distance of Polaris is $Z\ M = E\ M - E\ Z$ = 88° 55′ − 41° 48′ 51″ = 47° 6′ 9″. Ans.

(*b*) $Z\ M' = N\ Z - N\ M' = 90° - N\ M'$

$N\ M' = N\ P - P\ M'$

$P\ M'$ is the polar distance of Polaris or the complement of the declination and is equal to 90° − 88° 55′ = 1° 5′. Hence, $N\ M'$ = 41° 48′ 51″ − 1° 5′ = 40° 43′ 51″; and $Z\ M'$ = 90° − 40° 43′ 51″ = 49° 16′ 9″. Ans.

EXAMPLES FOR PRACTICE

1. What is the zenith distance of the north pole in latitude +30°?
Ans. 60°

2. The latitude of Philadelphia, Pa., is +39° 58′ 2″. (*a*) What is the declination of a star that passes directly overhead at Philadelphia? (*b*) If the declination of Polaris is +88° 55′, find the zenith distance

of Polaris when it is on the meridian of Philadelphia above the pole; (*c*) find the altitude of Polaris when it is on the meridian below the pol

Ans. (*a*) $+39° 58' 2''$; (*b*) $48° 56' 58''$; (*c*) $38° 53' 2''$

3. What is the hour angle: (*a*) of the zenith? (*b*) of the sun at noon? (*c*) of the sun at 8 o'clock in the morning? (*d*) of the sun at midnight?

Ans. (*a*) 0°; (*b*) 0°; (*c*) 20^h; (*d*) 12^h

TIME

SIDEREAL TIME

31. Explanation.—Clocks throughout the world are regulated by comparing them, directly or indirectly, with the clocks of the great national observatories, and these observatory clocks are regulated by means of astronomical observations. To the astronomer, therefore, belongs the duty of the regulation and measurement of time, and this is one of the most important problems of practical astronomy.

32. The earth turns on its axis with an absolutely uniform motion; therefore, the celestial sphere appears to rotate uniformly. The passage of a celestial body across the meridian is called its *transit*, or *culmination*. During one complete revolution of the celestial sphere every celestial body crosses the meridian twice. If the meridian is considered divided into two parts by the axis of the sphere, the passage of a body across that branch of the meridian that contains the observer's zenith is called the *upper transit*, or *upper culmination;* the passage across the other branch of the meridian is the *lower transit*, or *lower culmination*. Thus, in Fig. 4, the upper transits of the stars are at *K*, *A*, and *F*, and the lower transits are at *N*, *A'*, and *T*.

If there could be selected stars or other well-defined points on the celestial equator situated at intervals of 1 hour, and if

they were numbered 1 hour, 2 hours, etc., the time might be read from the celestial sphere exactly as from a clock on which the meridian is the hour hand; obviously, this is impossible. For the purpose of estimating time, it is convenient to consider the angular distance from the meridian to a single point on the celestial sphere. In the system called *sidereal time,* the point selected is the vernal equinox. At the upper transit of the vernal equinox, the sidereal time is $0^h\ 0^m\ 0^s$. The interval between two successive upper transits of the vernal equinox is a sidereal day. A sidereal day is divided into 24 hours and sidereal time is reckoned from 0^h up to 24^h.

Evidently sidereal time is the hour angle of the vernal equinox. In Fig. 3 the sidereal time is the arc of the equator from Q to X.

33. Disadvantage of Sidereal Time.—Sidereal time is equal to the right ascension of the meridian, since it is the angular distance between the vernal equinox and the meridian. Hence, when the sun is on the meridian, that is, at noon, the sidereal time is equal to the right ascension of the sun. Thus, at noon on March 21, the sidereal time is 0^h; at noon on June 21, it is 6^h; at noon on September 22, it is 12^h; and at noon on December 22, it is 18^h. It is seen that sidereal time bears no simple relation to the phenomena of day and night, and therefore, it is not suitable for common use. It is necessary in some astronomical calculations, but is not used for ordinary purposes. However, the length of a sidereal day is used in connection with observations on Polaris, and it is important to know that it is equivalent to $23^h\ 56.1^m$ of mean solar time which will be described presently and which is used for ordinary purposes.

SOLAR TIME

34. Apparent Solar Time.—Instead of determining time by the hour angle of the vernal equinox, it is more convenient for ordinary purposes to consider the hour angle of the center of the sun. Time so estimated is called *apparent solar time.* The instant of the upper transit of the sun's center is *apparent*

noon, and the interval between two successive upper transits is an *apparent solar day.*

The sun is continually moving with respect to the vernal equinox and, therefore, an apparent solar day is longer than a sidereal day by the amount of the daily increase in the sun's right ascension, which is about 4 minutes. Moreover, all apparent solar days are not of equal duration, because the sun in its journey around the sky moves faster at some times than it does at others. Therefore, apparent solar time cannot be measured by a clock whose rate is uniform, and for this reason is unsatisfactory for scientific and practical purposes.

35. Mean Solar Time.—For practical purposes, it is necessary to determine time by the hour angle of what is called the mean sun. The mean sun is not an actual body, but is merely a point that is imagined to start with the true sun and to move with a uniform speed around the celestial equator, completing the entire circuit in the same time that the true sun does. Sometimes the mean sun is ahead of the true sun, and sometimes behind it.

Time that is measured by the hour angle of the mean sun is called *mean solar time,* or *mean time.* Mean time is now carried by all clocks except those used in some kinds of astronomical work, which record sidereal time. The time of the upper transit of the mean sun is *mean noon* and the interval between two successive mean noons is a *mean solar day.* As stated before, a sidereal day is equivalent to $23^h\ 56.1^m$ of mean time.

36. Equation of Time.—When observations are made on the true sun, the apparent time is determined. The difference between apparent time and mean time at any instant is the equation of time. The value of the equation of time at any instant can be found from the United States Government publication called the American Ephemeris, which will be described later in this Section.

37. Civil Time.—In the ordinary way of reckoning time, the day begins at 12 o'clock at night; the hours are counted up

to 12 at noon, and, beginning again at noon, up to 12 at midnight. The day is thus divided into two intervals of 12 hours each, the first interval being known as A. M. time and the second as P. M. time.* When time is reckoned in this manner, it is called *civil time*. Thus, at 6 o'clock in the morning, the civil time is 6^h A. M., and at 6 o'clock in the evening, the civil time is 6^h P. M.

38. Astronomical Time.—In the system called *astronomical time*, the day is considered to begin at noon, and hours are counted from 0 to 24. The astronomical day begins 12 hours later than the civil day of the same date, and this fact should be borne in mind when converting astronomical time into civil, or vice versa. For instance, the astronomical time of 7^h 14^m 3^s on October 17 means 7^h 14^m 3^s after noon on October 17, and the corresponding civil time is October 17, 7^h 14^m 3^s P. M. The astronomical time of February 20, 18^h 6^m 12^s means 18^h 6^m 12^s after noon on February 20, or 6^h 6^m 12^s after midnight on February 20; thus, the corresponding civil time is February 21, 6^h 6^m 12^s A. M. The civil time of 7^h 14^m 3^s P. M. on May 1 means 7^h 14^m 3^s after noon on May 1, and the corresponding astronomical time is May 1, 7^h 14^m 3^s. The civil time of 7^h 14^m 3^s A. M. on May 1 means 7^h 14^m 3^s after midnight of April 30 or $12^h + 7^h$ 14^m $3^s = 19^h$ 14^m 3^s after noon on April 30; hence, the corresponding astronomical time is April 30, 19^h 14^m 3^s.

Astronomical time was formerly used in the American Ephemeris (described later) and similar publications of other countries, but, beginning with the 1925 edition, civil time was introduced instead. The hours, however, are counted from 0 to 24, 0^h being at midnight and 12^h at noon; thus 1 P. M. is called 13^h, 2 P. M. is 14^h, etc.

39. Relation Between Longitude and Time.—The earth rotates on its axis from west to east, and this causes the sun to appear to move from east to west, crossing the various meridians in succession; the mean sun is supposed to have a

*These abbreviations mean, respectively, *ante meridiem* (before meridian transit, before noon), and *post meridiem* (after meridian transit, after noon).

similar apparent motion. Obviously, the hour angle of the sun at any instant, and consequently the solar time, is different for different points on the earth's surface. In Fig. 9, O is the center of the earth, B and C are any two points on the earth's surface, P and P' are the celestial poles, and $E\,E'$ is the celestial equator. If $P\,N\,P'$ and $P\,M\,P'$ are the meridians for points B and C, S is the position of the sun on the celestial sphere at any instant, and $P\,A$ is the hour circle through S, the solar time at B is measured by the arc $N\,A$ and the solar time at C is measured by the arc $M\,A$. Then $N\,A = M\,A + M\,N$, and $M\,A = N\,A - M\,N$. The arc $M\,N$ measures the angle $M\,O\,N$ between the planes of the meridians at B and C and, therefore, measures the difference in longitude between B and C. Hence, the difference in solar time between two points on the earth's surface is equal to the difference in their longitudes.

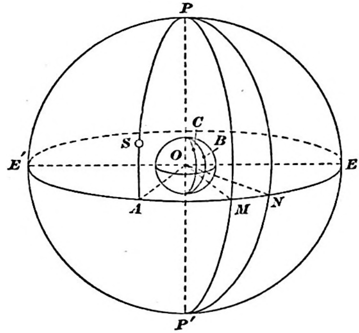


FIG. 9

In order to apply readily the correction for longitude, longitudes are often expressed in time measure instead of arc measure; the method of converting one system to the other will be explained in the following article. Let T and T' be the times at two given points B and C on the earth's surface, and let L be the difference in longitude expressed in time measure. Then, if T at B is known, T' at C can be found by one of the following formulas:

$$T' = T + L \qquad (1)$$

$$T' = T - L \qquad (2)$$

Formula **1** is used when C is east of B and formula **2** is used when C is west of B. Longitudes west of Greenwich are positive and east longitudes are negative.

EXAMPLE 1.—The longitude of Washington, west of Greenwich, is $5^h\ 8^m\ 16^s$ and that of San Francisco is $8^h\ 9^m\ 43^s$. (*a*) What is the time at Washington when it is $9^h\ 3^m$ A. M. at San Francisco? (*b*) What is the time at San Francisco, when it is $7^h\ 54^m\ 30^s$ P. M. at Washington?

SOLUTION.—(*a*) The difference in longitude L is $8^h\ 9^m\ 43^s - 5^h\ 8^m\ 16^s = 3^h\ 1^m\ 27^s$.

Since Washington is east of San Francisco, the time at Washington is later than that at San Francisco, and formula **1** applies. Then

$T' = 9^h\ 3^m + 3^h\ 1^m\ 27^s = 12^h\ 4^m\ 27^s$, or $4^m\ 27^s$ P. M. Ans.

(*b*) San Francisco is west of Washington and the time at San Francisco is earlier than that at Washington. Hence, by formula **2**,

$T' = 7^h\ 54^m\ 30^s - 3^h\ 1^m\ 27^s = 4^h\ 53^m\ 3^s$ P. M. Ans.

EXAMPLE 2.—The longitude of Chicago is $+5^h\ 50^m\ 27^s$ and that of Rome, Italy, is $-49^m\ 50^s$. Find the time at Rome when the Chicago time is $11^h\ 42^m\ 45^s$ P. M. on August 3.

SOLUTION.—Chicago is $5^h\ 50^m\ 27^s$ west of Greenwich, and Rome is $49^m\ 50^s$ east of Greenwich; hence, the difference in longitude between Chicago and Rome is $L = 5^h\ 50^m\ 27^s + 49^m\ 50^s = 6^h\ 40^m\ 17^s$. The time at Rome is later than at Chicago and formula **1** applies. Then,

$T' = 11^h\ 42^m\ 45^s + 6^h\ 40^m\ 17^s = 18^h\ 23^m\ 2^s$

As the day ends at 12 P. M., the true value of T' is $18^h\ 23^m\ 2^s - 12^h = 6^h\ 23^m\ 2^s$; but this is evidently A. M. time of the following day. Therefore, the time at Rome is Aug. 4, $6^h\ 23^m\ 2^s$ A. M. Ans.

40. Conversion of Arc Measure Into Time Measure and Vice Versa.—The sun describes a complete circle around the earth in 24 mean solar hours and, as there are 360° in a circle, the sun moves $\frac{360}{24} = 15°$ in one hour; that is, $15° = 1^h$. Therefore, if the difference in longitude between A and B is 15° and A is east of B, the sun will reach the meridian of A one hour earlier than the meridian of B; thus, if it is noon at B, it is 1^h at A. Also, 1 minute of time is equivalent to $\frac{1}{60}$ of 15°, which is 15′, and 1 second of time represents $\frac{1}{60}$ of 15′, or 15″. Hence,

$1^h = 15°$	$1° = 4^m$
$1^m = 15'$	$1' = 4^s$
$1^s = 15''$	$1'' = \frac{1}{15}^s$

From these relations, time can be converted into arc and arc into time by a simple process of multiplication or division. However, instead of performing these arithmetical operations, which are somewhat laborious, Table I may be used. The relation between degrees of arc and minutes of time is the

TABLE I

CONVERSION OF ARC AND TIME

°	h. m.	°	h. m.	°	h. m.	°	h. m.	°	h. m.	°	h. m.
′	m. s.	′	m. s.	′	m. s.	′	m. s.	′	m. s.	′	m. s.
1	0 4	61	4 4	121	8 4	181	12 4	241	16 4	301	20 4
2	0 8	62	4 8	122	8 8	182	12 8	242	16 8	302	20 8
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16
5	0 20	65	4 20	125	8 20	185	12 20	245	16 20	305	20 20
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36
10	0 40	70	4 40	130	8 40	190	12 40	250	16 40	310	20 40
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56
15	1 0	75	5 0	135	9 0	195	13 0	255	17 0	315	21 0
16	1 4	76	5 4	136	9 4	196	13 4	256	17 4	316	21 4
17	1 8	77	5 8	137	9 8	197	13 8	257	17 8	317	21 8
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16
20	1 20	80	5 20	140	9 20	200	13 20	260	17 20	320	21 20
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36
25	1 40	85	5 40	145	9 40	205	13 40	265	17 40	325	21 40
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56
30	2 0	90	6 0	150	10 0	210	14 0	270	18 0	330	22 0
31	2 4	91	6 4	151	10 4	211	14 4	271	18 4	331	22 4
32	2 8	92	6 8	152	10 8	212	14 8	272	18 8	332	22 8
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16
35	2 20	95	6 20	155	10 20	215	14 20	275	18 20	335	22 20
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36
40	2 40	100	6 40	160	10 40	220	14 40	280	18 40	340	22 40
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56
45	3 0	105	7 0	165	11 0	225	15 0	285	19 0	345	23 0
46	3 4	106	7 4	166	11 4	226	15 4	286	19 4	346	23 4
47	3 8	107	7 8	167	11 8	227	15 8	287	19 8	347	23 8
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16
50	3 20	110	7 20	170	11 20	230	15 20	290	19 20	350	23 20
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36
55	3 40	115	7 40	175	11 40	235	15 40	295	19 40	355	23 40
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56
60	4 0	120	8 0	180	12 0	240	16 0	300	20 0	360	24 0

same as that between minutes of arc and seconds of time. Therefore, each column of the table has two headings. If the value in arc measure is in degrees, the equivalent in time measure is in hours and minutes, and the upper headings of the columns are used; and if the value in arc is in minutes, the equivalent in time is in minutes and seconds, as indicated by the lower headings. Each pair of columns is separated from the others by a double line and is independent of the others; the equivalent values are in the two columns separated by a single line. Thus, the first and second columns are together, the seventh and eighth are together, etc.

The hours and minutes corresponding to degrees, and the minutes and seconds of time corresponding to minutes of arc, can be taken directly from the table. Seconds of arc are reduced to seconds of time by dividing by 15.

It will be noticed that the values in the columns representing time vary by 4. Hence, the method of changing time to arc is slightly different and is best explained by the following examples.

EXAMPLE 1.—Change 278° 18′ 42″ to time measure.

SOLUTION.—The equivalents taken from Table I follow:

From columns 9 and 10,	$278° = 18^h\,32^m\ \ 0^s$
From columns 1 and 2,	$18' = 1^m\,12.0^s$
Dividing by 15,	$42'' = 2.8^s$
Adding,	$278°\ 18'\ 42'' = 18^h\,33^m\,14.8^s$. Ans.

EXAMPLE 2.—Change $7^h\,40^m\,55^s$ to arc measure.

SOLUTION.—From columns 3 and 4 of Table I, $7^h\,40^m = 115°$; but $0^m\,55^s$ does not appear in the table. The next smaller value is $0^m\,52^s$ which is equivalent to 13′. The remaining 3^s is reduced to arc measure by multiplying by 15, or $15 \times 3 = 45''$. Hence, $7^h\,40^m\,55^s = 115°\ 13'\ 45''$. Ans.

A better arrangement of the work follows:

From columns 3 and 4,	$7^h\,40^m = 115°\ 0'\ 0''$
From columns 1 and 2,	$52^s = 13'\ 0''$
Multiplying by 15,	$3^s = 45''$
Adding,	$7^h\,40^m\,55^s = 115°\ 13'\ 45''$. Ans.

EXAMPLE 3.—Change $5^h\,39^m\,26^s$ to arc measure.

SOLUTION.—$5^h\,39^m$ does not appear in Table I, but the next smaller value, $5^h\,36^m$, is equivalent to 84°; the remainder is $3^m\,26^s$. Of

this, $3^m\ 24^s = 51'$, and the remainder, 2^s, is equivalent to $2 \times 15 = 30''$. Therefore, $5^h\ 39^m\ 26^s = 84°\ 51'\ 30''$. Ans.

A more convenient arrangement follows:

From columns 3 and 4,	$5^h\ 36^m = 84°\ 0'\ 0''$
From columns 1 and 2,	$3^m\ 24^s = \ \ 51'\ 0''$
Multiplying by 15,	$2^s = \ \ 30''$
Adding,	$5^h\ 39^m\ 26^s = 84°\ 51'\ 30''$

EXAMPLES FOR PRACTICE

1. The longitudes of New York and Sacramento are, respectively, $73°\ 57'\ 30''$ and $121°\ 27'\ 44''$ west of Greenwich. (*a*) Express these longitudes in time. (*b*) What is the time at New York, when the time at Sacramento is $3^h\ 6^m\ 20^s$ P. M.? (*c*) What is the time at Sacramento when it is $11^h\ 28^m\ 15^s$ A. M. at New York?

Ans. (*a*) $4^h\ 55^m\ 50^s$; $8^h\ 5^m\ 51^s$
(*b*) $6^h\ 16^m\ 21^s$ P. M.
(*c*) $8^h\ 18^m\ 14^s$ A. M.

2. Express in arc measure a difference of longitude of $2^h\ 39^m\ 37^s$.
Ans. $39°\ 54'\ 15''$

3. Express in time a difference of longitude of $49°\ 59'\ 43''$.
Ans. $3^h\ 19^m\ 58.87^s$

4. The longitude of New York is $+4^h\ 55^m\ 50^s$ and that of Paris is $-9^m\ 21^s$. Find the time at Paris when the time at New York is May 8, 9^h P. M. Ans. May 9, $2^h\ 5^m\ 11^s$ A. M.

STANDARD TIME

41. Local Time.—The civil time dealt with in a foregoing article refers to the meridian of the place considered. This time is called local time. Thus, when it is said that Chicago's local mean time is $3^h\ 45^m$ P. M., it is meant that it is $3^h\ 45^m$ since the mean sun crossed the meridian at Chicago. All places on the same meridian have the same local time.

42. Standard Time.—If watches and clocks showed local time at every place, it would be a complicated and cumbersome operation to compare those times for the regulation of the ordinary affairs of life. For this reason, watches and clocks are set, within certain longitudes, to keep time referred to a single meridian between those longitudes; that is, the time shown by a clock or watch between those longitudes is the local time only of the places on the meridian of reference. Time thus reckoned is called standard time.

The United States is divided into four zones, or sections, of standard time, whose reference meridians are, respectively, 75° or 5^h, 90° or 6^h, 105° or 7^h, and 120° or 8^h west of Greenwich. Each of these meridians passes through the center of a zone of standard time, and, therefore, controls the watch time of all places within $7\frac{1}{2}°$ on either side of it; this is shown in Fig. 10. Time referred to the 75° meridian is called eastern time; to the 90° meridian, central time; to the 105° meridian, mountain time; and to the 120° meridian, Pacific time.

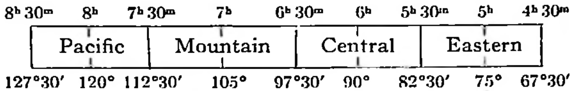


Fig. 10

43. To Change Standard Into Local Time.—In the applications of astronomy to surveying, standard time must frequently be changed into local time. When the standard time at any point within one of the zones is given, the local time is determined by one of the formulas in Art. **39.** But in order to apply the formula it is necessary to know the longitude of the place. When the longitude is given in arc, it must first be reduced to time.

Example.—The standard time at a place whose longitude is 81° 37′, is $9^h\ 37^m\ 45^s$ A. M.; what is the local time at that instant?

Solution.—Since the longitude is 81° 37′, the place lies within the zone of 75° meridian, or eastern, time. The difference in longitude between the given place and the standard-time meridian is $81°\ 37' - 75° = 6°\ 37'$ which is equivalent to $26^m\ 28^s$. Since the place is west of the reference meridian, formula **1** of Art. **39** applies. Then

$$T' = T - L = 9^h\ 37^m\ 45^s - 26^m\ 28^s = 9^h\ 11^m\ 17^s \text{ A. M. Ans.}$$

EXAMPLES FOR PRACTICE

1. The longitude of Cincinnati is 84° 25′ 21″; what is the standard (central) time when the local time is $4^h\ 24^m\ 17^s$ P. M.?

Ans. $4^h\ 1^m\ 58^s$ P. M.

2. The longitude of a place being 113° 49′, what is the local time at the place when the standard time is June 4, $11^h\ 58^m\ 30^s$ P. M.?

Ans. June 5, $23^m\ 14^s$ A. M.

OBSERVATIONS FOR TRUE MERIDIAN

OBSERVATION OF POLARIS

INTRODUCTION

44. It has already been explained that the celestial meridian passes through the north and the south celestial poles, and that the points in which the meridian intersects the horizon are the north and south points. For the purposes of surveying, the term *meridian* refers to a line on the earth's surface which lies in the plane of a celestial meridian. As explained in Art. **11**, such a line is actually part of a circle, but for a relatively small portion of the earth's surface, it may be treated as straight. The direction of the true meridian is determined by a line from the observer to the north pole. There is nothing to mark the north pole, but, as previously mentioned, Polaris is always near it and can be conveniently used for determining the meridian.

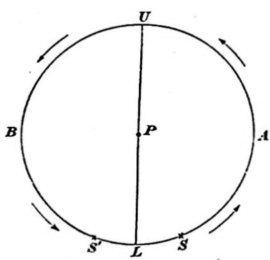


Fig. 11

45. The motion of Polaris about the pole is shown in Fig. 11 where P is the pole, $U\,L$ is the meridian, and the circle $A\,B$ is the path of Polaris. Polaris is on the meridian twice each day, once at upper culmination at U, and once at lower culmination at L. Hence, the meridian can be determined by observing Polaris in either of these positions. However, a star moves most rapidly with respect to the meridian when it is near culmination and, therefore, for an accurate determination of the meridian, it is necessary to observe Polaris

TABLE II
LOCAL MEAN TIME OF UPPER CULMINATION OF POLARIS

	1923		1924		1925		1926		1927		1928		1929		Difference for 1 Day Minutes
	h	m	h	m	h	m	h	m	h	m	h	m	h	m	
January 1	18	51.3	18	52.8	18	50.1	18	51.5	18	52.8	18	54.0	18	51.5	3.95
January 15	17	56.1	17	57.5	17	54.8	17	56.2	17	57.5	17	58.7	17	56.2	3.95
February 1	16	48.9	16	50.3	16	47.6	16	49.0	16	50.3	16	51.5	16	49.0	3.95
February 15	15	53.6	15	55.0	15	52.3	15	53.7	15	55.0	15	56.2	15	53.7	3.94
March 1	14	58.4	14	55.8	14	57.1	14	58.5	14	59.8	14	57.0	14	58.5	3.94
March 15	14	3.2	14	.6	14	1.9	14	3.3	14	4.6	14	1.8	14	3.3	3.94
April 1	12	56.2	12	53.6	12	54.9	12	56.3	12	57.6	12	54.8	12	56.3	3.93
April 15	12	1.2	11	58.6	11	59.9	12	1.3	12	2.6	11	59.8	12	1.3	3.93
May 1	10	58.3	10	55.8	10	57.1	10	58.5	10	59.8	10	57.0	10	58.5	3.92
May 15	10	3.4	10	.9	10	2.2	10	3.6	10	4.9	10	2.1	10	3.6	3.92
June 1	8	56.8	8	54.2	8	55.5	8	56.9	8	58.2	8	55.4	8	56.9	3.91
June 15	8	2.0	7	59.4	8	.7	8	2.1	8	3.4	8	.6	8	2.1	3.91
July 1	6	59.4	6	56.8	6	58.1	6	59.5	7	.8	6	58.0	6	59.5	3.91
July 15	6	4.6	6	2.0	6	3.3	6	4.7	6	6.0	6	3.2	6	4.7	3.92
August 1	4	58.0	4	55.5	4	56.7	4	58.1	4	59.4	4	56.6	4	58.1	3.92
August 15	4	3.3	4	.7	4	1.9	4	3.3	4	4.6	4	1.8	4	3.3	3.92
September 1	2	56.7	2	54.1	2	55.3	2	56.7	2	58.0	2	55.2	2	56.7	3.92
September 15	2	1.8	1	59.2	2	.4	2	1.8	2	3.1	2	.3	2	1.8	3.92
October 1	0	59.0	0	56.4	0	57.6	0	59.0	1	.3	0	57.5	0	59.0	3.93
October 15	0	4.0	0	1.4	0	2.6	0	4.0	0	5.3	0	2.5	0	4.0	3.93
November 1	22	53.3	22	50.7	22	51.9	22	53.3	22	54.6	22	51.8	22	53.3	3.94
November 15	21	58.2	21	55.6	21	56.8	21	58.2	21	59.5	21	56.7	21	58.2	3.94
December 1	20	55.1	20	52.6	20	53.7	20	55.1	20	56.4	20	53.6	20	55.1	3.94
December 15	19	59.9	19	57.3	19	58.5	19	59.9	20	1.2	19	58.4	19	59.9	3.94

TABLE II (*Continued*)
LOCAL MEAN TIME OF UPPER CULMINATION OF POLARIS

	1930		1931		1932		1933		1934		1935		1936		Difference for 1 Day Minutes
	h	m	h	m	h	m	h	m	h	m	h	m	h	m	
January 1	18	52.9	18	51.3	18	55.9	18	53.5	18	55.2	18	56.8	18	58.5	3.95
January 15	17	57.6	17	59.0	18	.6	17	58.2	17	59.9	18	1.5	18	3.2	3.95
February 1	16	50.4	16	51.8	16	53.4	16	51.0	16	52.7	16	54.3	16	56.0	3.95
February 15	15	55.1	15	56.5	15	58.1	15	55.7	15	57.4	15	59.0	16	.7	3.94
March 1	14	59.9	15	1.3	14	58.9	15	.5	15	2.2	15	3.8	15	1.5	3.94
March 15	14	4.7	14	6.1	14	3.7	14	5.3	14	7.0	14	8.6	14	6.3	3.94
April 1	12	57.7	12	59.1	12	56.7	12	58.3	13	.0	13	1.6	12	59.3	3.93
April 15	12	2.7	12	4.1	12	1.7	12	3.3	12	5.0	12	6.6	12	4.3	3.93
May 1	10	59.9	11	1.3	10	58.9	11	.5	11	2.2	11	3.8	11	1.5	3.92
May 15	10	5.0	10	6.4	10	4.0	10	5.6	10	7.3	10	8.9	10	6.6	3.92
June 1	8	58.3	8	59.7	8	57.3	8	58.9	9	.6	9	2.2	8	59.9	3.91
June 15	8	3.5	8	4.9	8	2.5	8	4.1	8	5.8	8	7.4	8	5.1	3.91
July 1	7	.9	7	2.3	6	59.9	7	1.5	7	3.2	7	4.8	7	2.5	3.91
July 15	6	6.1	6	7.4	6	5.1	6	6.7	6	8.4	6	10.0	6	7.7	3.92
August 1	4	59.5	5	.8	4	58.5	5	.1	5	1.8	5	3.4	5	1.1	3.92
August 15	4	4.7	4	6.0	4	3.7	4	5.3	4	7.0	4	8.6	4	6.3	3.92
September 1	2	58.1	2	59.4	2	57.1	2	58.7	3	.4	3	2.0	2	59.7	3.92
September 15	2	3.2	2	4.5	2	2.2	2	3.8	2	5.5	2	7.1	2	4.8	3.92
October 1	1	.4	1	1.7	0	59.4	1	1.0	1	2.7	1	4.3	1	2.0	3.93
October 15	0	5.4	0	6.7	0	4.4	0	6.0	0	7.7	0	9.3	0	7.0	3.93
November 1	22	54.7	22	56.0	22	53.7	22	55.3	22	57.0	22	58.6	22	56.3	3.93
November 15	21	59.6	22	.9	21	58.6	22	.2	22	1.9	22	3.5	22	1.2	3.94
December 1	20	56.5	20	57.8	20	55.5	20	57.1	20	58.8	21	.4	20	58.1	3.94
December 15	20	1.3	20	2.6	20	.3	20	1.9	20	3.6	20	5.2	20	2.9	3.94

very nearly at the exact instant of culmination. This requires a very accurate knowledge of the time of culmination; and since only one sight at the star is allowed, errors of adjustment of the instrument cannot be corrected.

The most accurate method of determining the meridian is by observing Polaris at its extreme position either east or west of the meridian. When, in its motion about the pole, a star reaches its extreme westerly position, it is said to be at *western elongation*. Likewise, the star is at *eastern elongation* when in its extreme eastern position. When a star is near elongation its motion with respect to the meridian is practically imperceptible. Hence, the observer need not know the exact time of elongation; and since there is time for him to take two observations on the star, one with the telescope normal and the other with the telescope inverted, the errors of adjustment of the transit may be eliminated.

It is often inconvenient to observe Polaris at the hours of elongation or culmination. Moreover, in southern latitudes, Polaris is not visible, and there is no star near the south pole. However, the true meridian can also be determined by an observation of the sun at any instant, as will be explained later.

46. Directions for finding Polaris have already been given. Before attempting to locate the star through the telescope, it is important to focus the telescope carefully, as a slight error in focusing may make the star invisible. The objective may be focused by sighting at the moon, a planet, or a very bright star; if none of these is visible, a very distant light will serve.

DETERMINATION OF MERIDIAN BY OBSERVING POLARIS AT CULMINATION

47. Time of Upper Culmination of Polaris.—In Table II are given the local mean times of upper culmination of Polaris for different dates between the beginning of the year 1923 and the end of the year 1936. The times given are local mean times counted from midnight as 0^h and increasing to 24^h. Thus, times less than 12^h are A. M. and those greater than 12^h

are P. M.; for instance, 13^h is 1 P. M., 14^h is 2 P. M., etc. The extreme right-hand column of the table contains the difference between the times of culmination for any two successive days between the date horizontally opposite that difference in the left-hand column, and the following date. Thus, the difference 3.95, which is horizontally opposite January 1, indicates that, between January 1 and January 15, the time of culmination decreases by 3.95 minutes per day. For instance, the time

TABLE III

CORRECTIONS IN MINUTES FOR INTERMEDIATE DATES

(To be subtracted)

Day of the Month	Difference for 1 Day in Minutes				
	3.91	3.92	3.93	3.94	3.95
2 or 16	3.9	3.9	3.9	3.9	3.9
3 or 17	7.8	7.8	7.9	7.9	7.9
4 or 18	11.7	11.8	11.8	11.8	11.8
5 or 19	15.6	15.7	15.7	15.8	15.8
6 or 20	19.5	19.6	19.6	19.7	19.7
7 or 21	23.5	23.5	23.6	23.6	23.7
8 or 22	27.4	27.4	27.5	27.6	27.6
9 or 23	31.3	31.4	31.4	31.5	31.6
10 or 24	35.2	35.3	35.4	35.5	35.5
11 or 25	39.1	39.2	39.3	39.4	39.5
12 or 26	43.0	43.1	43.2	43.3	43.4
13 or 27	46.9	47.0	47.2	47.3	47.4
14 or 28	50.8	51.0	51.1	51.2	51.3
29	54.7	54.9	55.0	55.2	55.3
30	58.6	58.8	58.9	59.1	59.2
31	62.6	62.7	62.9	63.0	63.2

of culmination on January 8, which is 7 days after January 1, is obtained by subtracting from the time of culmination for January 1 the product 3.95×7, or 27.65^m.

In order to facilitate calculation, Table III gives the corrections to be subtracted for all intermediate dates. Suppose, for instance, that it is desired to find the time of upper culmination on October 9, 1932. The difference for 1 day horizontally opposite October 1 in Table II is 3.93. In the vertical column

of Table III with 3.93 as a heading, the value 31.4 is found horizontally opposite the given date 9 (9 or 23). Then, 31.4^m is subtracted from $0^h\ 59.4^m$, which is the time of culmination for October 1, 1932. The difference is $0^h\ 28.0^m$, which is the required time.

In applying the table, either the local mean time is reduced to the corresponding standard time shown by the observer's watch or the watch is set to record local time. The latter method is usually safer.

48. In October the correction given in Table III is sometimes greater than the time in Table II. In such a case, $23^h\ 56.1^m$, which is the mean solar time between two consecutive culminations (Art. **33**), should be added to the time in Table II before the subtraction is performed.

Example 1.—Find the time of upper culmination of Polaris on September 12, 1924.

Solution.—From Table II,
Upper culmination, Sept. 1, 1924.............. $2^h\ 54.1^m$
Difference for 1 day, 3.92.
From Table III,
Correction under 3.92, opposite 12 (12 or 26).. 43.1
Required time .... $2^h\ 11.0^m$ A. M. Ans.

Example 2.—Find the time of upper culmination of Polaris on October 27, 1930.

Solution.—From Table II,
Time of upper culmination, Oct. 15, 1930...... $0^h\ 5.4^m$
Add 1 sidereal day.........................$23^h\ 56.1^m$
$24^h\ 1.5^m$
Difference for 1 day, 3.93.
Correction from Table III, under 3.93
and opposite 27........................... 47.2^m
Time of upper culmination..................$23^h\ 14.3^m$
or $11^h\ 14.3^m$ P. M. Ans.

49. Time of Lower Culmination.—Since Polaris makes a complete revolution around the pole in $23^h\ 56.1^m$, the interval of time between an upper and the next lower culmination is

$\frac{23^h\ 56.1^m}{2}$, or $11^h\ 58^m$, nearly. If the time ot upper culmination for any given date, as found from the table, is less than $11^h\ 58^m$, the time of lower culmination is obtained by adding $11^h\ 58^m$ to the time of upper culmination. If the time of upper culmination is greater than $11^h\ 58^m$, the latter quantity should be subtracted from it, in order to obtain the time of lower culmination for the same date. This will be readily understood by reference to Fig. 11. The star describes either of the semicircles $L\,A\,U$ or $U\,B\,L$ in about $11^h\ 58^m$. If, when Polaris is at U (upper culmination), the time is less than $11^h\ 58^m$, it indicates that, at the beginning of the day, the star was at some point S to the right of L. Before the end of that day, the star describes the path $U\,B\,L\,S$, crossing the meridian at L (lower culmination) $11^h\ 58^m$ after its passage through U. If, when Polaris is at U, the time is greater than $11^h\ 58^m$, it shows that, at the beginning of the day, the star was at some point S' to the left of L; in moving from S' to U, the star evidentiy reached L, $11^h\ 58^m$ before it reached U; hence, in this case, the time of lower culmination is obtained by subtracting $11^h\ 58^m$ from the time of upper culmination.

EXAMPLE.—An observer is located in longitude 110° 12′, and his watch keeps standard mountain time. Find the time on his watch when Polaris is at lower culmination on December 21, 1928.

SOLUTION.—From Table II,

Time of upper culmination, December 15, 1928...	$19^h\ 58.4^m$
Difference for 1 day, 3.94	
Correction from Table III.....................	23.6
Time of upper culmination, December 21, 1928..	$19^h\ 34.8^m$
Subtract	11 58
Time of lower culmination......................	$7^h\ 36.8^m$

The local civil time of culmination is, then, $7^h\ 36.8^m$ A. M. As the observer is in longitude 110° 12′, he is 5° 12′ west of the standard 7-hour meridian. This difference in longitude converted into time is 20.8^m. Then, by formula **1** of Art. **39**, the required standard time of lower culmination is

$$T' = 7^h\ 36.8^m + 20.8^m = 7^h\ 57.6^m \text{ A. M. Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the local time of upper culmination of Polaris: (*a*) on September 5, 1930; (*b*) on October 24, 1931. Ans. $\begin{cases}(a)\ 2^h 42.4^m \text{ A. M.}\\(b)\ 11^h 27.4^m \text{ P. M.}\end{cases}$

2. Find the local time of lower culmination of Polaris: (*a*) on February 24, 1928; (*b*) on April 23, 1927. Ans. $\begin{cases}(a)\ 3^h 22.7^m \text{ A. M.}\\(b)\ 11^h 29.2^m \text{ P. M.}\end{cases}$

3. An observer is located in longitude 118° 4′, his watch keeping standard Pacific time; find the standard time at which Polaris will be at lower culmination: (*a*) on June 4, 1929; (*b*) on April 8, 1927. Ans. $\begin{cases}(a)\ 8^h 35.5^m \text{ P. M.}\\(b)\ 0^h 24.3^m \text{ A. M.}\end{cases}$

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50. Observation of Stars With a Transit.—When a transit is used for observing the stars, it is necessary to illuminate the cross-wires. The simplest method of doing this is by holding a bull's-eye lantern in such a way as to throw the light down the telescope tube through the objective, care being taken not to obstruct the line of sight. A little practice will enable one to do this very easily; the lantern is held in front and a little to one side of the object end of the telescope with the left hand, and the instrument is manipulated with the right. It is more convenient, however, to have some kind of reflector fitted to the object end of the telescope, so that the lantern may be turned from the eyes of the observer rather than toward them.

Fig. 12

A very good reflector may be made from a piece of new tin, cut and bent as shown in Fig. 12. The straight strip is bent about the object end of the telescope tube, so that the annular elliptic piece projects over in front. This piece is then bent to any desired angle, preferably about 45°, and turned so that the light can be thrown down the tube by illuminating the disk from a convenient position. If the reflecting side of the disk is whitened, the effect is very good. The opening should be about $\frac{3}{4}$ inch in its shorter diameter, the longer diameter being such as to make its normal projection equal to the shorter one. There is, of course, no necessity for limiting the outer edges of

the disk. A piece of white paper placed on the end of the telescope in the same manner as a sunshade will serve if nothing better is available.

51. Observing Polaris and Marking the Meridian. Select a date on which Polaris is at either lower or upper culmination during the night (preferably during the early part of the evening). Determine, by means of Table II, the exact time of culmination, being careful either to reduce the tabular values to standard time or to set the watch to show local time. About 15 minutes before the time of culmination, set the transit in such a position that an unobstructed view toward the north may be obtained for a distance of between 300 and 500 feet. Drive a stake, and mark by a tack the exact point occupied by the instrument. About 5 minutes before the time of culmination, direct the telescope to the star. Set both clamps, and with either tangent screw set the vertical cross-wire exactly on the star. The star will appear to be moving toward the left or toward the right according as it is approaching upper or lower culmination. Follow it in its motion by turning the tangent screw until the exact time of culmination, which, preferably, should be called out by an assistant. This completes the observation of the star. Then depress the telescope, direct it to a point on the ground about 400 or 500 feet from the instrument, and have an assistant drive a tack on top of a stake in the line of sight. The line between the two stakes is a true north-and-south line, or true meridian.

DETERMINATION OF MERIDIAN BY OBSERVING POLARIS AT ELONGATION

52. Azimuth of Polaris.—When Polaris is observed at elongation, the line of sight is not in the meridian, but makes an angle with the meridian which is equal to the azimuth of Polaris at that instant. Table IV gives the azimuths of Polaris at elongation for different years and latitudes. If a transit is directed to the star when at elongation, and the corresponding azimuth taken from the table, all that is necessary to determine the meridian is to turn the telescope

TABLE IV

AZIMUTHS OF POLARIS AT ELONGATION

Latitude Degrees	Year 1923 Degrees	1923 Minutes	1924 Degrees	1924 Minutes	1925 Degrees	1925 Minutes	1926 Degrees	1926 Minutes	1927 Degrees	1927 Minutes	1928 Degrees	1928 Minutes
20	1	10.5	1	10.2	1	9.9	1	9.6	1	9.2	1	8.9
21		11.0		10.7		10.3		10.0		9.7		9.4
22		11.5		11.2		10.8		10.5		10.2		9.8
23		12.0		11.7		11.3		11.0		10.7		10.3
24		12.6		12.2		11.9		11.5		11.2		10.9
25	1	13.1	1	12.8	1	12.5	1	12.1	1	11.8	1	11.4
26		13.7		13.4		13.1		12.7		12.4		12.0
27		14.4		14.0		13.7		13.3		13.0		12.6
28		15.1		14.7		14.4		14.0		13.7		13.3
29		15.8		15.4		15.1		14.7		14.4		14.0
30	1	16.5	1	16.2	1	15.8	1	15.5	1	15.1	1	14.7
31		17.3		17.0		16.6		16.2		15.9		15.5
32		18.2		17.8		17.4		17.1		16.7		16.3
33		19.0		18.7		18.3		17.9		17.6		17.2
34		19.9		19.6		19.2		18.8		18.5		18.1
35	1	20.9	1	20.5	1	20.2	1	19.8	1	19.4	1	19.0
36		21.9		21.5		21.2		20.8		20.4		20.0
37		23.0		22.6		22.2		21.8		21.5		21.1
38		24.1		23.7		23.3		22.9		22.6		22.2
39		25.3		24.9		24.5		24.1		23.7		23.3
40	1	26.5	1	26.1	1	25.7	1	25.3	1	24.9	1	24.5
41		27.8		27.4		27.0		26.6		26.2		25.8
42		29.2		28.8		28.4		27.9		27.5		27.1
43		30.6		30.2		29.8		29.4		29.0		28.5
44		32.1		31.7		31.3		30.9		30.4		30.0
45	1	33.7	1	33.3	1	32.9	1	32.4	1	32.0	1	31.6
46		35.4		35.0		34.5		34.1		33.7		33.2
47		37.2		36.7		36.3		35.8		35.4		34.9
48		39.1		38.6		38.1		37.7		37.2		36.8
49		41.0		40.6		40.1		39.6		39.2		38.7
50	1	43.1	1	42.6	1	42.2	1	41.7	1	41.2	1	40.7
51		45.3		44.8		44.4		43.9		43.4		42.9
52		47.6		47.2		46.7		46.2		45.7		45.2
53		50.1		49.6		49.1		48.6		48.1		47.6
54		52.8		52.3		51.7		51.2		50.7		50.2
55	1	55.6	1	55.0	1	54.5	1	54.0	1	53.4	1	52.9
56		58.5		58.0		57.4		56.9		56.4		55.8
57	2	1.7	2	1.1	2	.6	2	.0		59.5		58.9
58		5.1		4.5		3.9		3.3	2	2.8	2	2.2
59		8.7		8.1		7.5		6.9		6.3		5.7
60	2	12.6	2	12.0	2	11.4	2	10.7	2	10.1	2	9.5
61		16.7		16.1		15.5		14.8		14.2		13.6
62		21.2		20.6		19.9		19.2		18.6		18.0
63		26.0		25.3		24.7		24.0		23.3		22.7
64		31.2		30.5		29.8		29.1		28.4		27.7
65		36.9		36.1		35.4		34.7		34.0		33.3

TABLE IV (*Continued*)

AZIMUTHS OF POLARIS AT ELONGATION

Latitude Degrees	1929 Degrees	1929 Minutes	1930 Degrees	1930 Minutes	1931 Degrees	1931 Minutes	1932 Degrees	1932 Minutes	1933 Degrees	1933 Minutes	1934 Degrees	1934 Minutes
20	1	8.6	1	8.2	1	7.9	1	7.6	1	7.3	1	6.9
21		9.0		8.7		8.4		8.1		7.7		7.4
22		9.5		9.2		8.8		8.5		8.2		7.9
23		10.0		9.7		9.3		9.0		8.7		8.3
24		10.5		10.2		9.9		9.5		9.2		8.9
25	1	11.1	1	10.8	1	10.4	1	10.1	1	9.8	1	9.4
26		11.7		11.4		11.0		10.7		10.4		10.0
27		12.3		12.0		11.6		11.3		11.0		10.6
28		13.0		12.7		12.3		11.9		11.6		11.2
29		13.9		13.4		13.0		12.6		12.3		11.9
30	1	14.4	1	14.1	1	13.7	1	13.3	1	13.0	1	12.6
31		15.2		14.9		14.5		14.1		13.8		13.4
32		16.0		15.7		15.3		14.9		14.6		14.2
33		16.8		16.5		16.1		15.8		15.4		15.0
34		17.7		17.4		17.0		16.6		16.3		15.9
35	1	18.7	1	18.3	1	17.9	1	17.6	1	17.2	1	16.8
36		19.6		19.3		18.9		18.5		18.1		17.8
37		20.7		20.3		19.9		19.6		19.2		18.8
38		21.8		21.4		21.0		20.6		20.2		19.8
39		22.9		22.5		22.1		21.7		21.4		21.0
40	1	24.1	1	23.7	1	23.3	1	22.9	1	22.5	1	22.1
41		25.4		25.0		24.6		24.2		23.8		23.4
42		26.7		26.3		25.9		25.5		25.1		24.7
43		28.1		27.7		27.3		26.9		26.4		26.0
44		29.6		29.2		28.7		28.3		27.9		27.5
45	1	31.2	1	30.7	1	30.3	1	29.9	1	29.4	1	29.0
46		32.8		32.3		31.9		31.5		31.0		30.6
47		34.5		34.1		33.6		33.2		32.7		32.3
48		36.3		35.9		35.4		34.9		34.5		34.0
49		38.2		37.8		37.3		36.9		36.4		35.9
50	1	40.3	1	39.8	1	39.3	1	38.8	1	38.4	1	37.9
51		42.4		41.9		41.4		41.0		40.5		40.0
52		44.7		44.2		43.7		43.2		42.7		42.2
53		47.1		46.6		46.1		45.6		45.1		44.6
54		49.7		49.1		48.6		48.1		47.6		47.1
55	1	52.4	1	51.8	1	51.3	1	50.8	1	50.3	1	49.7
56		55.3		54.7		54.2		53.6		53.1		52.5
57		58.3		57.8		57.2		56.7		56.1		55.5
58	2	1.6	2	1.1	2	.5		59.9		59.3		58.8
59		5.1		4.6		4.0	2	3.4	2	2.8	2	2.2
60	2	8.9	2	8.3	2	7.7	2	7.1	2	6.5	2	5.9
61		13.0		12.3		11.7		11.1		10.5		9.8
62		17.3		16.6		16.0		15.4		14.7		14.1
63		22.0		21.3		20.6		20.0		19.3		18.6
64		27.0		26.4		25.7		25.0		24.3		22.6
65		32.5		31.8		31.1		30.4		29.7		28.9

through an angle equal to the azimuth, either to the left (west) if the star is at eastern elongation, or to the right (east) if the star is at western elongation.

The use of Table IV requires a knowledge of the latitude. This can be taken with sufficient accuracy from a good map. If a suitable map is not available, the approximate latitude can be determined in the following manner: Since the latitude is equal to the altitude of the pole, measure the altitude of Polaris at upper culmination, and subtract 1° 5′, which is the distance of Polaris from the pole, or measure the altitude at lower culmination and add 1° 5′.

53. Time of Elongation of Polaris.—Polaris is at eastern elongation about $5^h\ 55^m$ before it reaches its upper culmination, and at western elongation about $5^h\ 55^m$ after upper culmination.

TABLE V

INTERVAL FROM UPPER CULMINATION TO ELONGATION

Latitude	Interval	Latitude	Interval
7° to 19°	$5^h\ 59^m$	52° to 57°	$5^h\ 54^m$
19° to 30°	$5^h\ 58^m$	57° to 60°	$5^h\ 53^m$
30° to 39°	$5^h\ 57^m$	60° to 63°	$5^h\ 52^m$
39° to 46°	$5^h\ 56^m$	63° to 66°	$5^h\ 51^m$
46° to 52°	$5^h\ 55^m$		

More exact values of the interval between culmination and elongation are given in Table V. The times of elongation can, therefore, be readily determined from those of culmination taken from Table II. If the time of eastern elongation comes on the preceding date, add $23^h\ 56.1^m$; if the time of western elongation comes on the following date, subtract $23^h\ 56.1^m$.

Since the star changes its azimuth very slowly at elongation, it is not necessary to know the time of elongation exactly. The time should be known approximately, so that the observer may know about when to make his observation.

EXAMPLE.—Find the times of elongation of Polaris on November 10, 1924, in latitude 48°.

SOLUTION.—From Table II.

Upper culmination, Nov. 1, 1924	$22^h\ 50.7^m$
Difference for 1 day, 3.94	
Correction from Table III	35.5
Upper culmination, Nov. 10, 1924	$22^h\ 15.2^m$
For western elongation, add	5 55.0
	$28^h\ 10.2^m$
As this brings the time on the following day, it is necessary to subtract a sidereal day	23 56.1
Western elongation (to nearest minute), Nov. 10, 1924	$4^h\ 14^m$ A. M. Ans.
Also,	
Upper culmination, Nov. 10, 1924 (to nearest minute)	$22^h\ 15^m$
Subtract	$5^h\ 55^m$
Eastern elongation	$16^h\ 20^m$
	$=4^h\ 20^m$ P. M. Ans.

EXAMPLES FOR PRACTICE

1. Find the time of eastern elongation of Polaris on August 12, 1928, in latitude 58°. Ans. $10^h\ 17^m$ P. M.

2. Find the time of western elongation of Polaris on October 27, 1932, in latitude 22°. Ans. $5^h\ 15^m$ A. M.

54. Making the Observation and Marking the Meridian. Determine the approximate time of elongation as just explained. About 20 minutes before that time, set the transit over a point properly marked, and level it carefully. Set the vernier at zero. With the telescope normal, sight to the star, and, with both clamps set, follow the star by means of the lower tangent screw. If the star is approaching eastern elongation, it will be moving to the right; if western, to the left. About the time of elongation, it will be noticed that the star ceases to move horizontally, and that its image appears to follow the vertical cross-wire of the instrument; the star has then reached its elongation. Depress the telescope and mark another point 400 or 500 feet from that occupied by the instrument. Then plunge the telescope, turn the instrument 180° in azimuth, and, if necessary, relevel the plate by means of the leveling

screws. Set the vertical wire on the star again, depress the telescope, and mark a second point near the one established with the telescope normal.

If the two observations are made within 3 or 4 minutes of elongation, the correct position is midway between the two points marked. The errors of adjustment of the instrument are eliminated and the line thus determined makes an angle with the meridian equal to the azimuth of Polaris which can be taken from Table IV. Then the meridian can be located at any time from the established line by turning off an angle equal to the azimuth, either to the east or to the west, according as the star is at western or eastern elongation.

This is the most accurate method of determining the meridian, and, where possible, should be used in preference to others.

DETERMINATION OF MERIDIAN BY OBSERVING THE SUN

55. Outline of Method.—It has already been explained that it is sometimes impracticable to determine the meridian by observing Polaris and that, under such conditions, the meridian can be located by observation of the sun. In determining the direction of the meridian by observing the sun in any position, it is first necessary to find the azimuth of some convenient reference line by measuring the angle between this reference line and a line from the observer to the sun. Then the direction of the meridian or the azimuth of any other line can be readily found from the azimuth of the reference line.

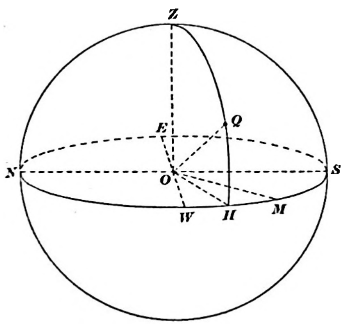


Fig. 13

Let $N\ E\ S\ W$, Fig. 13, represent the horizon plane, and $N\ Z\ S$ the plane of the meridian. Let the observer be at O, and let the straight reference line in the horizon plane have the direction $O\ M$. It is this line whose azimuth $N\ E\ S\ M$ is to be determined. Let Q be the position of the sun on the celestial sphere and $Z\ Q\ H$ the vertical circle through Q. Then $N\ E\ S\ H$ is the azimuth of the sun, and $H\ Q$ its altitude.

When the altitude has been measured, the azimuth of the sun can be computed by the formula given in the following article. The horizontal angle $M\ O\ H$, between the reference mark and the sun, is measured, the angle always being taken toward the right, or in a clockwise direction. The azimuth of the line is determined by subtracting the horizontal angle $M\ O\ H$ from the azimuth $N\ O\ H$ of the sun. If the angle is greater than the azimuth, 360° is added to the azimuth before subtracting.

56. Formula for Finding the Sun's Azimuth.

Let z = zenith distance of the sun;

d = declination;

l = latitude of observer;

a = azimuth, counted from north toward east.

The value of a is computed by the following formula, which is derived by the principles of spherical trigonometry:

$$\sin \tfrac{1}{2}\, a = \sqrt{\frac{\cos \frac{1}{2}\,(z+l+d)\ \sin \frac{1}{2}\,(z+l-d)}{\sin z\ \cos l}}$$

In applying this formula, it should be remembered that there are always two angles corresponding to a given sine. Thus, if $\sin A = .5$, the angle A may be either 30° or 150° $(=180° - 30°)$, since any two supplementary angles have the same sine. Of the two values of $\frac{1}{2}\,a$ given by the formula, one is acute and the other obtuse. For morning observations, the acute angle should be used; for afternoon observations, the obtuse. The sign of the declination must also be considered; if it is negative, the numerical value is subtracted in $\frac{1}{2}\,(z+l+d)$ and added in $\frac{1}{2}\,(z+l-d)$. The methods of determining the values of z and d will be explained in subsequent articles.

THE AMERICAN EPHEMERIS

57. Explanation.—The American Ephemeris and Nautical Almanac is a book published each year by the United States Government which gives accurate information concerning the positions of the sun, moon, stars, and planets during the year, as well as other facts. To any one making observations of the heavenly bodies for any purpose, this book is indispensable. It can be secured from the Superintendent of Documents, Government Printing Office, Washington, D. C., by sending one dollar (not in check or stamps). For convenience in the following explanations, the book will be referred to simply as the *Ephemeris*. It is published about three years in advance, and the year for which it is desired should be specified. Most of the information is given for Greenwich mean time.

58. Ephemeris of the Sun.—The first pages of the Ephemeris give, for Greenwich mean noon of every day in the year, the right ascension and declination of the true sun with the changes in one hour, the equation of time to be added to or subtracted from mean time, the sidereal time, and other important information which will be explained later. A sample from page 2 of the Ephemeris for 1924 is shown in Table VI.

The sidereal time of mean noon is needed in calculations in which the stars are involved, but will not be required in this Section. The other quantities are necessary in the calculations to be explained later.

59. To Find the Right Ascension and Declination of the Sun at Any Instant.

Rule.—*Change the local mean time to Greenwich time. Take from the table the right ascension and declination at the preceding Greenwich mean noon, and determine the corrections to these quantities by multiplying the respective hourly motions by the number of hours elapsed since Greenwich mean noon.*

TABLE VI

EPHEMERIS OF THE SUN, 1924, FOR GREENWICH MEAN NOON

Date	Apparent Right Ascension	Variation Per Hour	Apparent Declination	Variation Per Hour	Semi-Diameter	Hor. Par.	Equation of Time App.-Mean	Sidereal Time
	h m s	s	° ′ ″	″	′ ″	″	m s	h m s
Jan. 1	18 42 42.15	11.053	−23 5 0.7	+11.28	16 17.89	8.95	−3 12.45	18 39 29.70
2	18 47 7.28	11.041	23 0 16.3	12.43	16 17.89	8.95	3 41.02	18 43 26.26
3	18 51 32.10	11.028	22 55 4.3	13.57	16 17.89	8.95	4 9.29	18 47 22.82
4	18 55 56.59	11.013	22 49 24.0	14.71	16 17.88	8.95	4 37.22	18 51 19.37
5	19 0 20.71	10.996	22 43 18.3	15.84	16 17.87	8.95	5 4.78	18 55 15.93
6	19 4 44.41	10.979	−22 36 44.7	+16.96	16 17.85	8.95	−5 31.93	18 59 12.49

Example 1.—Find the right ascension and declination of the sun at 9 A. M., January 2, 1924, Ann Arbor mean time, the longitude of Ann Arbor being $+5^h\ 34^m\ 55^s$.

Solution.—

Local mean time, Jan. 2	$9^h\ 0^m\ 0^s$ A. M.
Longitude of Ann Arbor	$+5\ 34\ 55$
Greenwich mean time, Jan. 2	$2^h\ 34^m\ 55^s$ P. M.

The Greenwich time is, therefore, 2.582^h after noon of Jan. 2.

Right ascension of sun at Greenwich mean noon, Jan. 2	$18^h\ 47^m\ 7.28^s$
Increase of right ascension during 2.582 hr. $=11.041^s \times 2.582$	$+0\ \ 28.51$
Desired right ascension	$18^h\ 47^m\ 35.79^s$ Ans.
Declination of sun at Greenwich mean noon, Jan. 2	$-23°\ 0'\ 16.3''$
Increase of declination during 2.582 hr. $=12.43'' \times 2.582$	$+0\ 32.1$
Desired declination	$-22°\ 59'\ 44.2''$ Ans.

60. To Find the Equation of Time at Any Instant.—To find the equation of time, it is first necessary to determine the Greenwich time corresponding to the given local time. Then the required value of the equation of time can be found by interpolating in Table VI in the following manner: Find the difference between the values of the equation of time corresponding to the preceding and the following Greenwich noons and multiply it by the decimal part of a day elapsed since the preceding noon. Add the result to, or subtract it from, the equation of time at the preceding noon according as the equation of time is increasing or decreasing.

Example.—Find the value of the equation of time at 9 A. M., January 3, 1924, Ann Arbor local mean time, the longitude of Ann Arbor being $+5^h\ 34^m\ 55^s$.

Solution.—

Local time, Jan. 2	$9^h\ 0^m\ 0^s$ A. M.
Longitude of Ann Arbor	$5^h\ 34^m\ 55^s$
Greenwich mean time, Jan. 3	$2^h\ 34^m\ 55^s$ P. M.

The Greenwich time is therefore .107 da. after noon of Jan. 3.

Equation of time for noon, Jan. 4............	$-4^m\ 37.22^s$
Equation of time for noon, Jan. 3............	$-4^m\ 9.29^s$
Change in equation of time for 1 da.	-27.93^s
Total change since Greenwich noon $= -27.93 \times .107$	-2.99^s
Equation of time for preceding Greenwich noon	$-4^m\ 9.29^s$
Desired equation of time......................	$-4^m\ 12.28^s$ Ans.

EXAMPLES FOR PRACTICE

1. The longitude of Chicago is $5^h\ 50^m\ 27^s$. Find the right ascension and declination of the sun (*a*) when the Chicago local mean time is Jan. 4, 1924, $2^h\ 39^m\ 33^s$ P. M.; (*b*) when the standard time there is Jan. 4, 1924, $2^h\ 30^m$ P. M.

Ans. (*a*) and (*b*) $18^h\ 57^m\ 30.20^s$; $-22°\ 47'\ 19.9''$

2. The longitude of Athens, Greece, is $-1^h\ 34^m\ 53^s$. What are the right ascension and declination of the sun when the local mean time there is $9^h\ 30^m$ A. M. on Jan. 6, 1924?

Ans. $19^h\ 3^m\ 59.74^s$; $-22°\ 38'\ 2.8''$

3. The longitude of Cincinnati is $+5^h\ 37^m\ 41^s$. Find the equation of time on January 2, 1924, (*a*) when the local mean time at Cincinnati is 9 A. M.; (*b*) when the standard time there is 3 P. M.

Ans. $\begin{cases} (a)\ -3^m\ 44.10^s \\ (b)\ -3^m\ 51.62^s \end{cases}$

DETERMINATION OF ALTITUDE

61. Use of the Transit.—In determining the azimuth of the sun its altitude must be measured. For this purpose the instrument commonly used is the engineers' transit. Evidently a vertical limb on the transit is essential. For observing the sun, a colored-glass cap is placed on the telescope to protect the eye.

The bubbles on the plate must be very carefully adjusted; the plate bubble parallel to the telescope is especially important because the accuracy of the result depends wholly on that bubble. The telescope bubble must also be in adjustment to indicate when the line of sight is horizontal, and good adjustment of the horizontal cross-wire is necessary. The vertical limb should read zero when the line of sight is horizontal.

The reading of the vertical circle when the plate is leveled and the telescope bubble is brought to the middle of its tube is the *index error*. If the vernier on the vertical circle is adjustable, the reading on the limb may be made zero when the line of sight is horizontal, and then no correction for index error will be required: otherwise the index error should be eliminated or corrected, as explained in the following article.

The adjustment of the telescope bubble, of the plate bubbles, and of the vernier of the vertical circle should always be tested just before and just after any series of observations with the transit.

62. Correction of Index Error.—In case the vertical limb is a complete circle, the error of adjustment of the plate levels and the vernier can be eliminated by first reading the vertical circle with the telescope direct, then revolving the instrument in azimuth through 180°, releveling the plate by means of the leveling screws, plunging the telescope, and reading the vertical angle with the telescope in its reversed position; the average of the two readings of the vertical circle is taken as the measured altitude. Every altitude should be measured in this way when possible; no correction for index error is then required.

In correcting for the index error it is necessary to distinguish carefully whether the value is to be added to or subtracted from the observed vertical angle. If the zero of the vernier lies between the zero of the limb and the objective of the telescope, the index error is − and the correction is added to the observed vertical angle; if the zero of the vernier lies between the zero of the limb and the eyepiece of the telescope, the index error is + and the correction is subtracted from the observed angle. The correction is equal to the index error, but has the opposite sign.

63. Other Corrections to the Measured Altitude.—The index error described in the preceding article is purely instrumental. However, the apparent altitude of a heavenly body is always affected by the refraction of the rays of light from the body in passing through the atmosphere. In case the sun

is observed, corrections for semi-diameter and parallax must also be applied.

64. Refraction.—A ray of light travels in a straight line so long as its path is in a medium of uniform density; but when it passes obliquely from one medium into another of different density, it undergoes a change of direction at the surface of separation. This change of direction or bending of a ray of light is called refraction. Let $E\,E'$, Fig. 14, represent the surface of the earth on which an observer stands at O, and suppose the surface $E\,E'$ is surmounted by an atmosphere $E\,M\,N\,E'$ of uniform density and of a definite height. Then a ray of light from any celestial body S will be bent downwards on reaching the upper surface $M\,N$ of the atmosphere; that is, the ray will move along the broken line $S\,A\,O$ instead of traveling in a straight path.

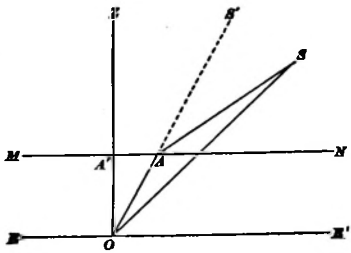


Fig. 14

The apparent position of a body depends on the direction in which light from it enters the observer's eye; hence, the celestial body at S will appear to be at S' instead of in its true position. The difference between the direction $O\,S$ in which the body would be were there no refraction, and the direction $O\,S'$ in which it appears to be, is the refraction. It is thus seen that the effect of refraction is to increase the altitude of all heavenly bodies. The altitude $S\,O\,E'$ is the *true altitude;* the altitude $S'\,O\,E'$ is the *apparent altitude*. To find the true altitude when the apparent altitude has been measured, the amount of the refraction $S\,O\,S'$ must be subtracted from the apparent altitude $S'\,O\,E'$.

This explanation assumes the space above $M\,N$ in the figure to be entirely empty, and the earth's atmosphere $M\,N\,E'\,E$ to be equally dense throughout. In fact, however, the earth's

atmosphere is most dense at the surface of the earth, and gradually diminishes in density to its exterior boundary. The direction of a ray of light traveling in such a medium is constantly changing, and so the path of the ray is a curved line. This curve is represented by line *e d c b a A*, Fig. 15. The ray of light from the star *S* first meets the upper surface of the atmosphere at *e* and is then successively refracted as it passes into layers of greater and greater density, until it finally enters the eye of the observer at *A* in the direction *S′ a A*. As a consequence, *S* is the apparent position of the star.

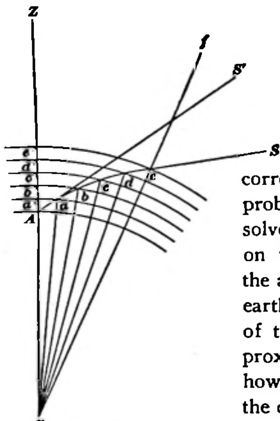


Fig. 15

65. Correction for Refraction.—The exact determination of the amount of refraction corresponding to any altitude is a problem that cannot be completely solved, since the refraction depends on the temperature and density of the air, not only at the surface of the earth, but also along the whole path of the ray *A a b c d e*, Fig. 15. Approximate tables may be constructed, however, that will give the amount of the correction within 1 second or less, provided that the body is not too near the horizon. When the altitude of a body is small, the rays of light from it pass nearly along the earth's surface through many hundred miles of comparatively dense air and the refraction cannot be accurately determined by any method. Hence, when the measured altitude of any body is less than 8° or 10°, the refraction becomes so uncertain that the measurement is of no value for any kind of accurate work.

Table VII gives the amount of refraction corresponding to different altitudes. The apparent altitude of a heavenly body is corrected for refraction by subtracting the corresponding tabular value from the measured angle.

TABLE VII

MEAN REFRACTION TO BE APPLIED TO ALL MEASURED ALTITUDES

(To be subtracted from apparent altitude)

App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
0 0	33 0			6 40	7 40	10 0	5 15	16 40	3 8
						10 10	5 10	16 50	3 6
		3 30	13 6			10 20	5 5	17 0	3 4
						10 30	5 0	17 10	3 3
				7 0	7 20	10 40	4 56	17 20	3 1
						10 50	4 51	17 30	2 59
						11 0	4 47	17 40	2 57
						11 10	4 43	17 50	2 55
		4 0	11 51	7 20	7 2	11 20	4 39	18 0	2 54
						11 30	4 34	18 10	2 52
						11 40	4 31	18 20	2 51
						11 50	4 27	18 30	2 49
1 0	24 29			7 40	6 45	12 0	4 23	18 40	2 47
						12 10	4 20	18 50	2 46
		4 30	10 48			12 20	4 16	19 0	2 44
						12 30	4 13	19 10	2 43
				8 0	6 29	12 40	4 9	19 20	2 41
						12 50	4 6	19 30	2 40
				8 10	6 22	13 0	4 3	19 40	2 38
						13 10	4 0	19 50	2 37
		5 0	9 54	8 20	6 15	13 20	3 57	20 0	2 35
						13 30	3 54	20 10	2 34
				8 30	6 8	13 40	3 51	20 20	2 32
						13 50	3 48	20 30	2 31
2 0	18 35	5 20	9 23	8 40	6 1	14 0	3 45	20 40	2 29
						14 10	3 43	20 50	2 28
				8 50	5 55	14 20	3 40	21 0	2 27
						14 30	3 38	21 10	2 26
		5 40	8 54	9 0	5 48	14 40	3 35	21 20	2 25
						14 50	3 33	21 30	2 24
				9 10	5 42	15 0	3 30	21 40	2 23
						15 10	3 28	21 50	2 21
		6 0	8 28	9 20	5 36	15 20	3 26	22 0	2 20
						15 30	3 24	22 10	2 19
				9 30	5 31	15 40	3 21	22 20	2 18
						15 50	3 19	22 30	2 17
3 0	14 36	6 20	8 3	9 40	5 25	16 0	3 17	22 40	2 16
						16 10	3 15	22 50	2 15
				9 50	5 20	16 20	3 12	23 0	2 14
						16 30	3 10	23 10	2 13

TABLE VII—(*Continued*)

App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction	App. Altitude	Refraction
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
23 20	2 12	26 40	1 53	34 0	1 24	48 0	0 51	68 0	0 23
23 30	2 11	26 50	1 52	34 30	1 23	49 0	0 49	69 0	0 22
23 40	2 10	27 0	1 51	35 0	1 21	50 0	0 48	70 0	0 21
23 50	2 9	27 15	1 50	35 30	1 20	51 0	0 46	71 0	0 19
24 0	2 8	27 30	1 49	36 0	1 18	52 0	0 44	72 0	0 18
24 10	2 7	27 45	1 48	36 30	1 17	53 0	0 43	73 0	0 17
24 20	2 6	28 0	1 47	37 0	1 16	54 0	0 41	74 0	0 16
24 30	2 5	28 15	1 46	37 30	1 14	55 0	0 40	75 0	0 15
24 40	2 4	28 30	1 45	38 0	1 13	56 0	0 38	76 0	0 14
24 50	2 3	28 45	1 44	38 30	1 11	57 0	0 37	77 0	0 13
25 0	2 2	29 0	1 42	39 0	1 10	58 0	0 35	78 0	0 12
25 10	2 1	29 30	1 40	39 30	1 9	59 0	0 34	79 0	0 11
25 20	2 0	30 0	1 38	40 0	1 8	60 0	0 33	80 0	0 10
25 30	1 59	30 30	1 37	41 0	1 5	61 0	0 32	81 0	0 9
25 40	1 58	31 0	1 35	42 0	1 3	62 0	0 30	82 0	0 8
25 50	1 57	31 30	1 33	43 0	1 1	63 0	0 29	83 0	0 7
26 0	1 56	32 0	1 31	44 0	0 59	64 0	0 28	84 0	0 6
26 10	1 55	32 30	1 30	45 0	0 57	65 0	0 26	86 0	0 4
26 20	1 55	33 0	1 29	46 0	0 55	66 0	0 25	88 0	0 2
26 30	1 54	33 30	1 26	47 0	0 53	67 0	0 24	90 0	0 0

EXAMPLE.—The altitude of a star was observed to be 18° 04′ 10″. It is required to correct this altitude for refraction.

SOLUTION.—

Observed altitude	18° 04′ 10″
Refraction	—02 53
Corrected or true altitude.........................	18° 01′ 17″
	Ans.

EXAMPLES FOR PRACTICE

Correct the following measured altitudes for refraction:

(*a*) 39° 48′ 10″.
(*b*) 15° 10′ 20″.
(*c*) 22° 11′ 05″.

Ans. (*a*) 39° 47′ 02″; (*b*) 15° 06′ 52″; (*c*) 22° 8′ 46″

66. Parallax.—The positions of all heavenly bodies are referred to the center of the earth. The position on the celestial sphere that a heavenly body appears to occupy when

viewed from the center of the earth is called its *geocentric place.* The right ascensions and declinations of the celestial bodies published in the Ephemeris are for geocentric places.

The **parallax** of a heavenly body is the difference between its direction as actually observed and the direction that it would have if seen from the earth's center. Thus, in Fig. 16, where the observer is supposed to be at O, the position of P in the sky (as seen from O) would be marked by the point where OP produced would pierce the celestial sphere. Its position as seen from C would be determined by producing CP. The angle POX formed by OP and a line parallel to CP is the parallax of P for an observer at O.

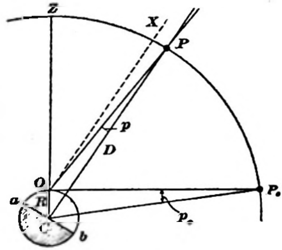


Fig. 16

Since the angle POX equals the angle $OPC = p$, the parallax may also be defined as the angular distance between the observer's station and the center of the earth, as seen from the body observed.

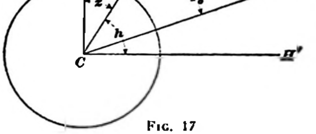


Fig. 17

67. Correction of Altitude and Zenith Distance for Parallax.—Let A, Fig. 17, be the position of the observer; C, the center of the earth; AH, the horizon; CH', a parallel to AH; Z, the zenith; and M, the position of a heavenly body. Then the angle z_a is the apparent or observed zenith distance of M, while z is its geocentric zenith distance. Likewise, h_a and h are, respectively, the apparent and the geocentric altitude of the body, and p is its parallax. From triangle CAM,

$$z = z_a - p \qquad (1)$$

that is, the geocentric zenith distance is equal to the observed zenith distance minus the parallax.

The angle $M N H$, external to the triangle $M N A$, is equal to h, since $N H$ is parallel to $C H'$. Therefore,

$$h = h_a + p \qquad (2)$$

That is, the geocentric altitude is equal to the observed altitude plus the parallax.

Evidently the parallax of a body depends on its distance from the earth and its altitude. Thus, when a body is on the horizon the parallax p_o is greater than the parallax p in any other position. In the case of the stars, no correction of any kind is applied for parallax, as the geocentric parallax of the nearest star is less than one-millionth of a second. In the case of the sun, a small correction is necessary. This correction varies with the altitude and can be taken from Table VIII.

EXAMPLES FOR PRACTICE

Correct the following measured altitudes of the sun for parallax:

(*a*) 25° 10′ 0″.		(*a*)	25° 10′ 08″
(*b*) 60° 09′ 10″.	Ans.	(*b*)	60° 09′ 14″
(*c*) 10° 10′ 5″.		(*c*)	10° 10′ 14″
(*d*) 5° 09′ 10″.		(*d*)	5° 09′ 19″

---

68. Correction for Semi-diameter.—The angular diameter of the sun is about 0° 32′. In the high-power telescopes now generally used on transits, such an image will largely fill up the field of view. It is impossible to bisect such a large image with sufficient accuracy, and, consequently, when the telescope is pointed to the sun, the intersection of the cross-wires cannot be placed accurately at the center of the sun. Whenever the altitude of the sun is observed with the transit, the horizontal wire is placed tangent to the upper or lower edge of the apparent disk. Thus, as a result of the observation, the altitude of either the upper or the lower edge is obtained. The observed altitude must be corrected by adding to or subtracting from it the angular semi-diameter of the sun, according as the lower or the upper edge has been observed. The amount of the semi-diameter for each day is given in the

TABLE VIII

SUN'S PARALLAX IN ALTITUDE TO BE APPLIED TO ALL MEASURED ALTITUDES OF THE SUN

(To be added to observed altitude)

Altitude Degrees	Parallax Seconds	Altitude Degrees	Parallax Seconds
0	9	54	5
6	9	57	5
12	9	60	4
16	8	63	4
20	8	66	3
25	8	69	3
30	8	72	3
34	7	75	2
36	7	78	2
40	7	81	1
45	6	84	1
48	6	87	0
51	5	90	0

Ephemeris, as shown in Table VI; or it may be taken with sufficient accuracy from the following tabulation:

Time of Year (Approximately)	Semi-Diameter of Sun	Time of Year (Approximately)	Semi-Diameter of Sun
January 1.......	16′ 18″	July 1..........	15′ 45″
April 1..........	16 2	October 1.......	16 2

EXAMPLE.—The altitude of the upper edge of the sun was observed with a transit on January 4, 1924, and the circle reading was 23° 9′ 20″; the index error was found to be −1′ 0″. What was the true altitude of the sun's center?

SOLUTION.—

Observed altitude of upper edge................	23° 9′ 20″
Index correction	+1 0
Apparent altitude of upper edge.................	23° 10′ 20″
Correction for refraction (Table VII)..........	−2 13
Correction for parallax (Table VIII)..........	+ 8
True altitude of upper edge...................	23° 8′ 15″
Correction for semi-diameter (Table VI)......	−16 18
Final corrected altitude of center.............	22° 51′ 57″ Ans.

EXAMPLES FOR PRACTICE

1. The altitude of the upper edge of the sun was observed on April 1 and the circle reading was 41° 15′; the index error was found to be +2′. Find the true altitude of the sun's center.

Ans. 40° 56′ 0″.

2. The lower edge of the sun was observed on January 3, 1924, and the vertical circle reading was 36° 42′; the index error was eliminated. What was the true altitude of the sun's center?

Ans. 36° 57′ 8″.

DETERMINING THE AZIMUTH

69. Making the Observations.—It is assumed that the transit is provided with a complete vertical circle, and also with a colored-glass cap to protect the eye. All parts of the instrument should be in good adjustment, but the errors of adjustment can be eliminated by taking several observations, as will be described.

The instrument is set up over station P at one end of the reference line and, with the vernier of the horizontal circle set

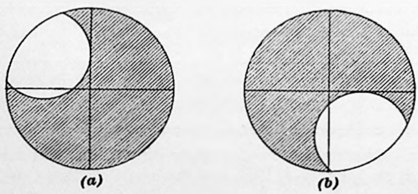
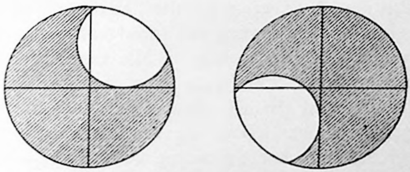


FIG. 18

to read zero, the telescope is directed to the other point M on the line, care being taken to make the final bisection by turning the lower tangent screw, so that the setting of the vernier will not be altered. The upper plate is then unclamped and the telescope is directed to the sun. Since the image of the sun is too large to bisect accurately, the cross-wires are brought tangent to the edges. The novice will be surprised at the rapidity of the sun's motion when viewed through a high-

power telescope. In the forenoon, the sun is moving upwards and to the right. Therefore, for morning work, the telescope is set so that the lower edge of the sun is somewhat below the horizontal cross-wire, and the right-hand edge is exactly on the vertical wire, as shown in Fig. 18 (*a*). If the telescope has stadia wires, care must be taken that neither the upper nor the lower stadia wire is mistaken for the middle wire. Then, by means of the upper tangent screw, the vertical wire is kept on the right edge of the sun until the lower edge reaches the horizontal wire. In this position the readings of both the horizontal and vertical circles are recorded. The approximate time should also be noted for finding the declination of the sun and for reference. This completes a single observation.



(*a*) (*b*)

Fig. 19

In order to obtain more accurate results, the same operation should be repeated three or four times as rapidly as possible, keeping the lower plate clamped throughout. Then the telescope is plunged, the instrument is turned through 180° in azimuth, and another series of observations is taken. However, in each pointing, the telescope is set so that the left edge of the sun is a little to the left of the vertical wire and the upper edge is exactly on the horizontal wire, as shown in Fig. 18 (*b*). By means of the tangent screw to the telescope, the horizontal wire is kept tangent to the upper edge of the sun until the left edge reaches the vertical wire. The readings of the horizontal and the vertical circles and the time are recorded. It should be noticed that in each case the setting is made so that one edge of the sun clears the fixed wire and the other moves toward the moving wire.

The same number of observations should be taken in each position of the telescope. After the last observation, the telescope in its normal position is directed again to the reference point *M*; if the reading of the horizontal circle differs appreciably from zero, it shows that the plates have slipped, and the observations must be rejected.

In the afternoon the sun moves downwards and to the right. Therefore, at that time, the positions of the sun are as shown in Fig. 19; in (*a*) is represented the field of view with the telescope normal and in (*b*) is shown that for the plunged telescope.

70. Calculating the Azimuth.—In computing the azimuth, the average of the vertical circle readings is taken as the observed altitude, the average of the horizontal circle readings is taken as the angle between the lines to the sun and to the reference point, and the average of the times is taken as the time of observation. The sun's path is considered straight between the first and the last observation since the interval of time is usually short. Near the meridian the curvature is relatively great, and near the prime vertical the altitude is so low that the refraction is uncertain. Hence, the best observations are obtained either from 8 to 10 A. M. or from 2 to 4 P. M.

It should be noted that the reversal of the telescope between the observations eliminates the index error of the vertical circle, the error of level in the horizontal axis of the telescope, and the error of collimation of the telescope. By sighting in diagonal corners of the field of view, and taking the averages of the observations, the corrections (both horizontal and vertical) due to the semi-diameter of the sun, are eliminated. The actual readings of the horizontal circle for the inverted telescope differ from those for the normal position by about 180°. To simplify the notes, 180° is often added to (or subtracted from) the horizontal plate reading when the instrument is inverted.

EXAMPLE.—The following observations were taken at New York on January 2, 1924; the latitude of New York is $+40° 48' 35''$ and the longitude is $+4^h 55^m 50^s$. The transit was set at a point *P* and a back-

sight was taken to M with the vernier reading zero. Find the azimuth of PM.

Telescope	Edges Observed	Standard Time A. M.	Vertical Circle	Horizontal Circle
Direct	Right	8^h 33^m	18° 4′	273° 5′ 30″
	and	8 35	18° 46′	273° 33′
	Lower	8 38	19° 39′	274° 10′
Inverted	Left	8 40	20° 51′	274° 6′ 30″
	and	8 42	21° 32′	274° 33′ 30″
	Upper	8 45	22° 27′	275° 9′

SOLUTION.—The conditions are shown in Fig. 20.

Average of vertical circle readings	20° 13′ 10″
Refraction	−2′ 33″
Parallax	+8″
Correct altitude of sun's center	20° 10′ 45″

Zenith distance=90°−altitude=69° 49′ 15″.

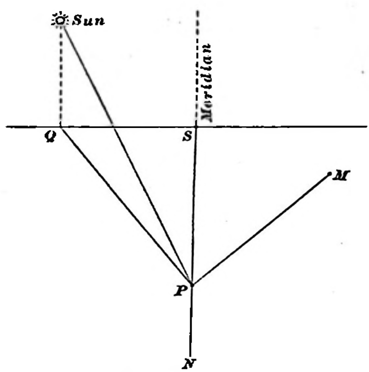


FIG. 20

The average of the watch times is 8^h 39^m and the corresponding Greenwich time is $8^h\ 39^m + 5^h = 1^h\ 39^m = 1.65^h$ P. M.

The declination of the sun is $23°\ 0'\ 16.3'' - 12.43 \times 1.65 = -22°\ 59'\ 56''$.

Then, $z = 69°\ 49'\ 15''$; $l = 40°\ 48'\ 35''$; $d = -22°\ 59'\ 56''$; $\frac{1}{2}(z+l+d) = 43°\ 48'\ 57''$; $\frac{1}{2}(z+l-d) = 66°\ 48'\ 53''$.

Since d is negative, it is subtracted in $\frac{1}{2}(z+l+d)$ and added in $\frac{1}{2}(z+l-d)$.

When the values are substituted in the formula of Art. 56,

$$\sin \tfrac{1}{2}a = \sqrt{\frac{\cos 43°\ 48'\ 57'' \sin 66°\ 48'\ 53''}{\sin 69°\ 49'\ 15'' \cos 40°\ 48'\ 35''}}$$

Since the observation is in the morning, the acute angle is used; hence,

$$\tfrac{1}{2}a = 75°\ 4'\ 20'', \text{ and } a = 150°\ 8'\ 40''$$

The average of the horizontal circle readings is 274° 6′ 15″. As the horizontal reading is greater than the azimuth of the sun, the azimuth of *PM* is 150° 8′ 40″+360°−274° 6′ 15″=236° 2′ 25″. Ans.

EXAMPLE FOR PRACTICE

The following observations were made on the sun at Philadelphia January 6, 1924; the latitude and longitude of Philadelphia are +39° 58′ 2″ and + 5^h 1^m 7^s. The transit was set at a point *A* and a backsight was taken to *B* with the vernier reading zero. Find the azimuth of *A B*.

Telescope	Edges Observed	Standard Time P. M.	Vertical Circle	Horizontal Circle
Direct	Left	3^h 15^m	22° 1′	99° 52′
	and	3 18	21° 6′	100° 37′
	Lower	3 20	20° 28′	100° 55′ 30″
Inverted	Right	3 23	20° 5′	102° 10′
	and	3 25	19° 33′	102° 34′
	Upper	3 27	18° 58′	103° 1′

Ans. 110° 50′ 55″

71. Modification of Method for Vertical Arc.—If the transit has only a vertical arc, the telescope cannot be inverted. Then, in the morning, the sun is observed in either position shown in Fig. 18. In the afternoon the setting is made as shown in Fig. 19 (*b*), so that the vertical wire moves to the right and the tangent screw operates against the spring. If the observation were made as in Fig. 19 (*a*), the horizontal wire would have to move downwards and the tangent screw would be loosened, thus preventing dependable settings. Several observations should be made on the same edges of the sun, and then the instrument should be sighted along the reference line again to see if the plates slipped.

The vertical circle readings must be corrected for index error, refraction, and semi-diameter. The reading of the horizontal circle must be corrected for semi-diameter. The correction to the vertical circle reading is simply the angular semi-diameter of the sun; this value should be subtracted from the reading if the upper edge of the sun has been observed, but

added to the reading if the wire has been placed tangent to the lower edge of the sun.

The correction to be applied to the reading of the horizontal circle is found by dividing the sun's semi-diameter by the cosine of its altitude. This correction is to be added to the reading of the horizontal circle if the wire is placed tangent to the left edge of the sun, but subtracted from the reading of the horizontal circle if the wire is placed tangent to the right edge of the sun.

EXAMPLE.—The following observations were taken in the afternoon, in latitude 39° 58′:

Vertical Circle	Horizontal Circle	Diagram of Field
22° 48.5′	237° 41.0′	
22° 12.5′	238° 11.0′	
21° 44.5′	238° 34.0′	
21° 19.0′	238° 55.0′	
20° 49.5′	239° 19.5′	
20° 28.0′	239° 38.0′	

The declination of the sun was +14° 45′ 40″, and the semi-diameter of the sun was 15′ 54″; the index error was eliminated. Find the azimuth of the sun.

SOLUTION.—

Mean of the vertical circle readings..........	21° 33′ 40″
Refraction (Table VII)	−2 24
Parallax (Table VIII)	+8
Semi-diameter, which is to be subtracted (see field diagram)	−15 54
True altitude	21° 15′ 30″
Zenith distance=90°−true altitude............	68 44 30
Mean reading of horizontal circle..............	238° 43′ 5″
Correction for semi-diameter=15′ 54″÷cos 21° 15′ 30″=954″÷0.932, to be subtracted (see field diagram)	−17 4
True horizontal angle to the center of the sun	238° 26′ 1″

To find the azimuth of the sun:

$$z=68°\ 44'\ 30''; \ l=39°\ 58'\ 0''; \ d=14°\ 45'\ 40''$$

$$\tfrac{1}{2}(z+l+d)=61°\ 44'\ 5''; \ \tfrac{1}{2}(z+l-d)=46°\ 58'\ 25''$$

By substitution of these values in the formula,

$$\sin \tfrac{1}{2}a=\sqrt{\frac{\cos 61^\circ\ 44'\ 5''\ \sin 46^\circ\ 58'\ 25''}{\sin 68^\circ\ 44'\ 30''\ \cos 39^\circ\ 58'}}$$

$$\tfrac{1}{2}a=135^\circ\ 52'\ 46'';\ a=271^\circ\ 45'\ 32''$$

The obtuse angle corresponding to $\sin \frac{1}{2} a$ is taken, because the observation was made in the afternoon. Then, the azimuth of the mark is

$$271^\circ\ 45'\ 32''-238^\circ\ 26'\ 1''=33^\circ\ 19'\ 31''.\ \text{Ans.}$$

EXAMPLE FOR PRACTICE

The following observations were taken in the morning in latitude 40° 36′:

Vertical Circle	Horizontal Circle	Diagram of Field
43° 09′ 43° 35′	64° 42′ 0″ 65° 10′ 30″	

The declination of the sun was +19° 43′ 10″, and the semi-diameter was 15′ 49″; the index error was +2′. Find the azimuth.

Ans. 35° 48′ 18″

DETERMINATION OF LATITUDE

72. Introduction.—In determining the meridian, a knowledge of the latitude is required. In case a reliable map is not available and in other cases when desirable, the latitude of a place may be determined from the altitude of a celestial body at the instant of its transit and the declination of the body.

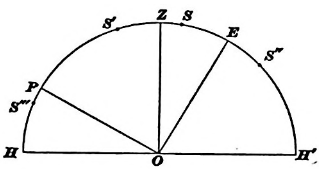


Fig. 21

73. General Formulas. Let Fig. 21 represent a section of the celestial sphere cut by the plane of the meridian. The earth is at O; HZH' is the meridian; HH', the horizon; Z, the zenith; EO, the plane of the equator; and P, the north pole. Then, HP is the altitude of the pole and EZ is the latitude of the observer.

There are four conditions to be considered: (1) A celestial body whose declination is positive and greater than the latitude of the observer, will cross the meridian between the zenith and pole, as at S'. (2) A body whose declination is positive and less than the latitude, will cross the meridian between the zenith and equator, as at S. (3) A body whose declination is negative will cross the meridian between the equator and the horizon as at S''. (4) A circumpolar star at lower culmination will cross the meridian between the pole and the horizon as at S'''. In the following formulas, let l be the latitude of the observer; h, the altitude of the body; d, its declination; and p, its polar distance:

For the first case,

$$HP = HS' - PS'$$

or,

$$l = h - p \qquad (1)$$

For the second case,

$$EZ = ES + ZS = ES + H'Z - H'S$$

Since $H'Z = 90°$,

$$EZ = 90° + ES - H'S$$

or,

$$l = 90° + d - h \qquad (2)$$

For the third case,

$$EZ = 90° - ES'' - H'S''$$

or,

$$l = 90° - d - h \qquad (3)$$

It must be remembered that in this third case, the numerical value of the declination is subtracted and the negative sign is neglected.

For the fourth case,

$$HP = HS''' + PS'''$$

or,

$$l = h + p \qquad (4)$$

It should be borne in mind that the conditions just described apply only for an observer in the northern hemisphere. However, Fig. 21 and formulas **1** to **4** can also be used for the southern hemisphere. In this case, P represents the south pole, but HP is its altitude which is equal to the south latitude of the observer. Then, formula **1** is used when the declination is negative and greater than the latitude of the observer;

formula **2** applies when the declination is negative and less than the latitude; formula **3** is employed when the declination is positive; and formula **4** is for a circumpolar star at lower culmination. In each case only the numerical value of the declination is considered; it is added if the sign preceding d in the formula is $+$, and subtracted if the sign before d is $-$.

74. Making the Observation.—From the preceding explanation it is evident that the body to be observed can be either a star or the sun. The body is assumed to be on the meridian when it has its maximum altitude. This is not quite true for the sun, since its declination is constantly changing, but the error is very small and can be neglected for observations made with the engineers' transit. Polaris can be observed at either lower or upper culmination. The body changes its altitude very slowly when near the meridian, and the exact time of crossing is not necessary; but the time should be known approximately, so that the observer will not have to wait too long. The local time of culmination of Polaris can be found from Table II and the time at which the sun crosses the meridian is local apparent noon. In either case, the corresponding watch time can be readily determined.

75. A few minutes before the time of culmination, the horizontal cross-wire of the transit is set on the body and the motion of the body is followed by means of the tangent screw until the maximum altitude is reached, that is, until the body no longer appears to rise in the telescope; then the vertical limb is read. If the instrument has a vertical circle, it is advisable to eliminate the errors of adjustment of the instrument by repeating the observation with the telescope inverted. Both observations should be made within 3 or 4 minutes of the time of culmination.

EXAMPLE 1.—The altitude of Polaris at upper culmination was measured with a transit having a vertical arc. The reading was 48° 36′, the index error was $+1'$, and the declination of Polaris was $+88°$ 55′. Find the observer's latitude.

SOLUTION.—The measured altitude is corrected for index error and refraction as follows:

Observed altitude	48° 36′
Index correction	− 1′
Refraction	− 0′ 50″
Corrected altitude	48° 34′ 10″

Since Polaris is at upper culmination, it will cross the meridian between the zenith and the pole, and formula **1** of Art. **73** applies. The polar distance of Polaris is $p=90°-88° 55'=1° 5'$. Then,

$$l=h-p=48° 34' 10''-1° 5'=47° 29' 10''. \text{ Ans.}$$

Example 2.—The meridian altitude of the lower edge of the sun was observed at Philadelphia on Jan. 3, 1924. The altitude with the telescope normal was 26° 55′ and with the telescope plunged it was 26° 52′. If the longitude of Philadelphia is $+5^h\ 1^m\ 7^s$, find (*a*) the standard time of the sun's transit; and (*b*) the latitude of Philadelphia.

Solution.—

(*a*) Local apparent time........................	$12^h\ 0^m\ 0^s$
Correction for longitude....................	$+5^h\ 1^m\ 7^s$
Greenwich apparent time, Jan. 3............	$5^h\ 1^m\ 7^s$ P. M.

The Greenwich apparent time is .209 da. after noon of Jan. 3.

From Table VI, the equation of time to be added to apparent time is $4^m\ 9.29^s+.209\times(4^m\ 37.22^s-4^m\ 9.29^s)=4^m\ 15^s$. The local mean time is, therefore, $12^h\ 4^m\ 15^s$ and the standard time is $12^h\ 4^m\ 15^s+1^m\ 7^s=12^h\ 5^m\ 22^s$ or $5^m\ 22^s$ P. M. Ans.

(*b*) Greenwich mean time is $5^h\ 1^m\ 7^s+4^m\ 15^s=5^h\ 5^m\ 22^s=5.09^h$ P. M. From Table VI, the declination of the sun is $22° 55' 4.3''-5.09\times13.57''=-22° 53' 55''$.

The corrected altitude of the sun's center is determined as follows:

Average observed altitude of lower edge.......	26° 53′ 30″
Correction for refraction (Table VII).........	− 1′ 52″
Correction for parallax (Table VIII)..........	+ 8″
Corrected altitude of lower edge..............	26° 51′ 46″
Semi-diameter (Table VI)......................	+ 16′ 18″
True altitude of sun's center..................	27° 8′ 4″

Since the declination is negative, the sun crosses the meridian between the equator and the horizon, and formula **3** of Art. **73** applies. Then,

$$l=90°-d-h=90°-22° 53' 55''-27° 8' 4''=39° 58' 1''. \text{ Ans.}$$

EXAMPLES FOR PRACTICE

1. The altitude of Polaris at lower culmination was measured with a transit having an index error of −2′, and the observed reading of the vertical limb was 50° 12′. Find the observer's latitude if the declination of Polaris was 88° 55′. Ans. 51° 18′ 12″.

2. The meridian altitude of the upper edge of the sun was observed at Chicago on Jan. 5, 1924. The altitude with the telescope normal was 25° 47′ and with the telescope plunged it was 25° 46′. If the longitude of Chicago is $+5^h\ 50^m\ 27^s$, find (*a*) the standard time of the sun's transit, and (*b*) the latitude of Chicago.

Ans. $\begin{cases} (a)\ 11^h\ 55^m\ 38^s \text{ A. M.} \\ (b)\ 41^\circ\ 49'\ 53''. \end{cases}$